

Wide Angle Scattering and Pair Distribution Functions

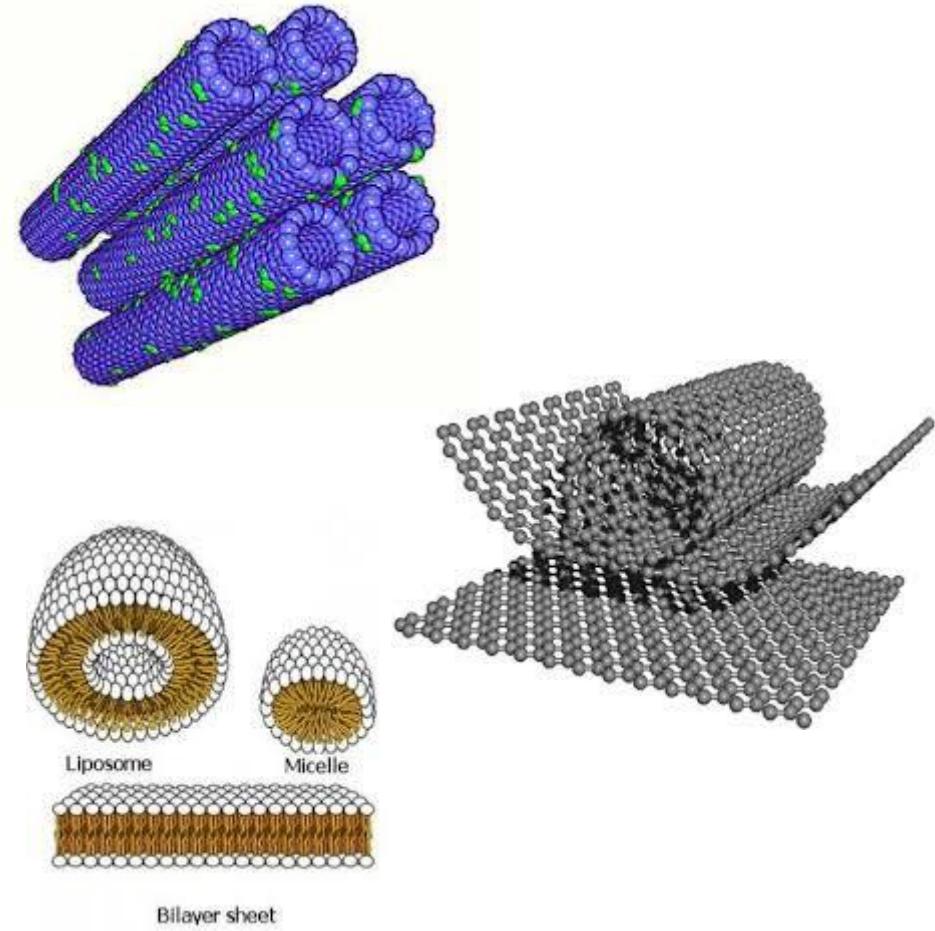
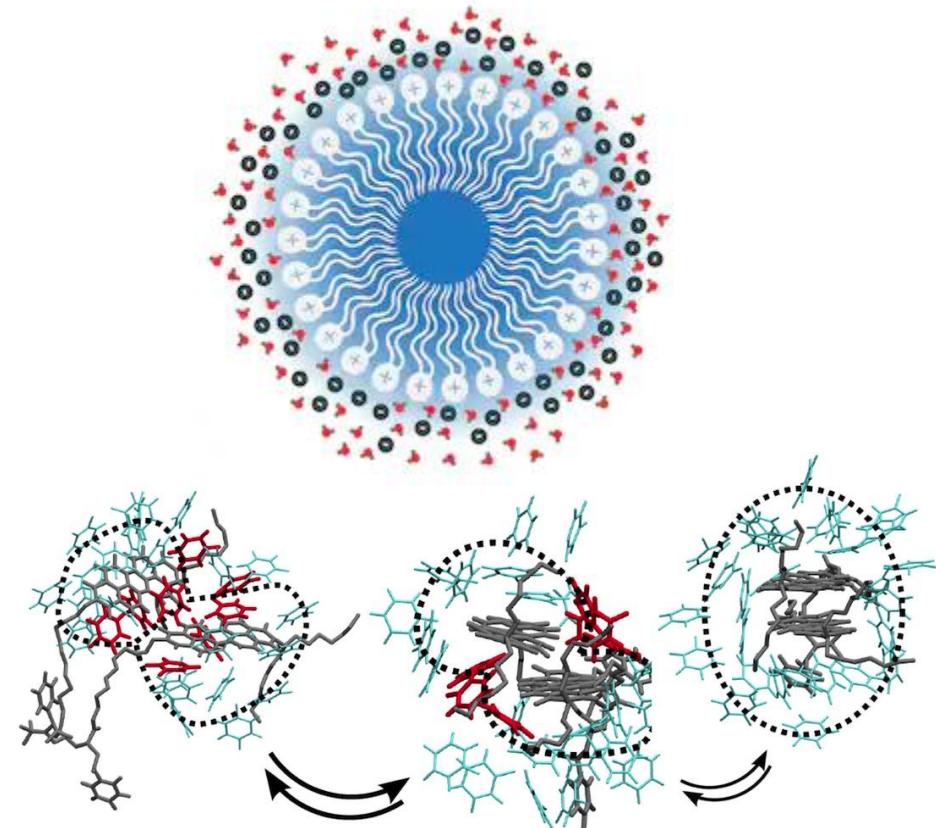
Dr Katharina Edkins
University of Strathclyde
[@k_edkins](https://twitter.com/k_edkins)

Overview

- Diffraction
 - What are the effects of increasing disorder?
- Small vs wide angle scattering
- Pair distribution function
- Examples

Wide vs small angle scattering

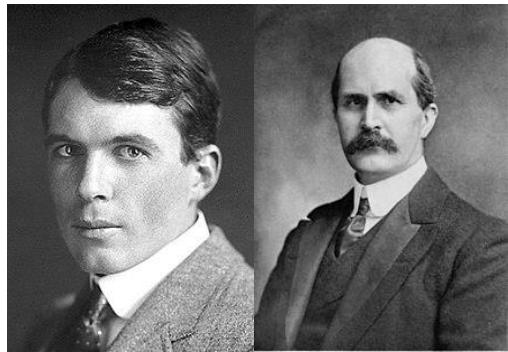
Wide Angle Scattering $< 10^{-9}$ m



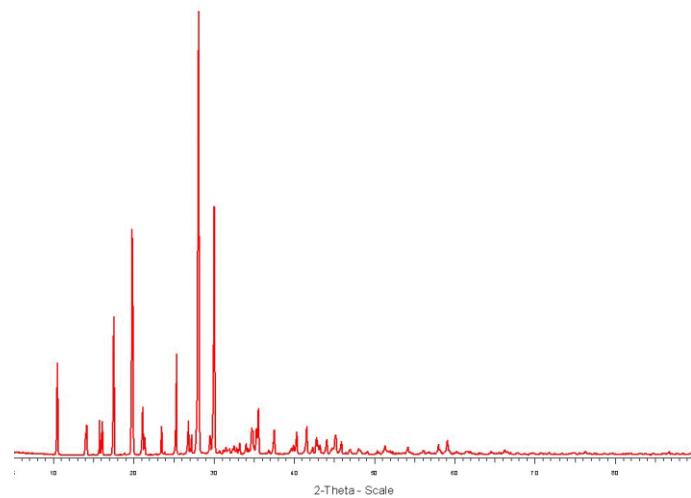
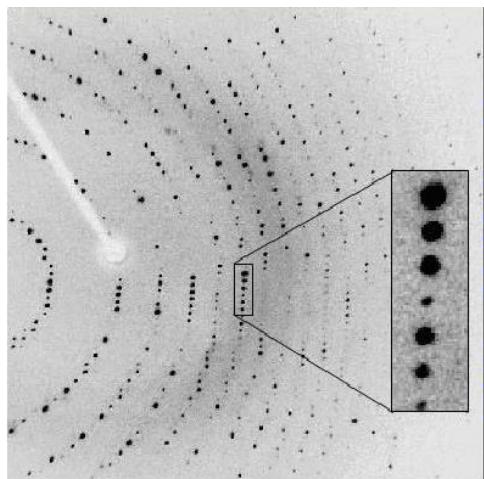
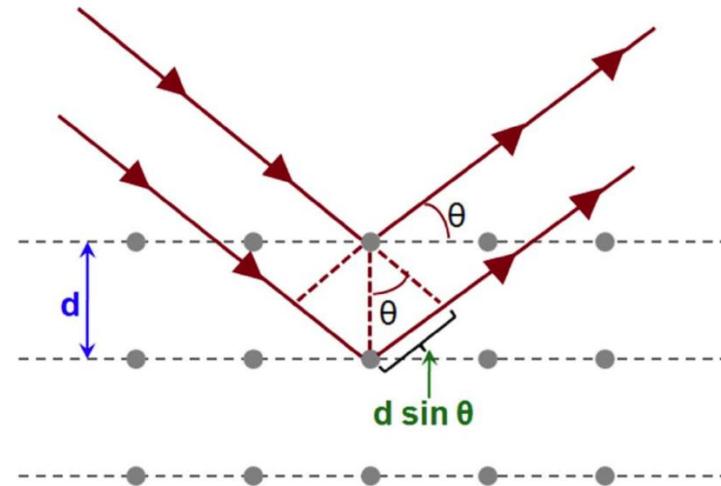
Small Angle Scattering $> 10^{-9}$ m

Diffraction and disorder

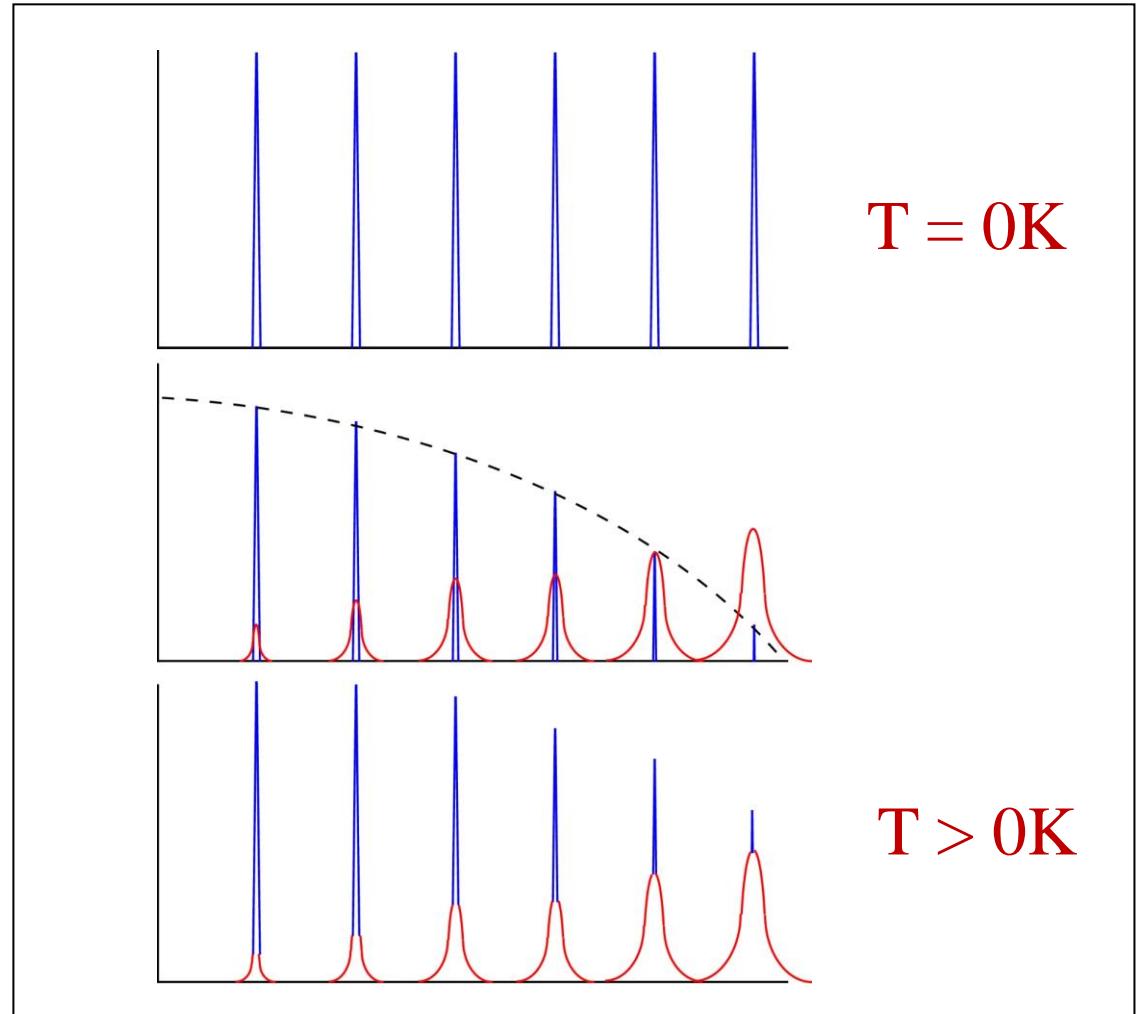
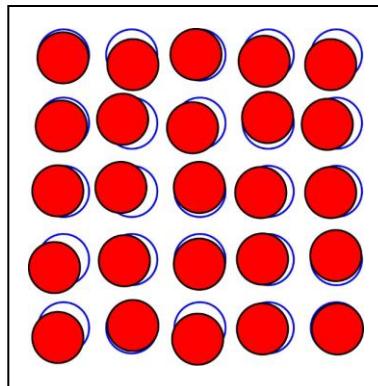
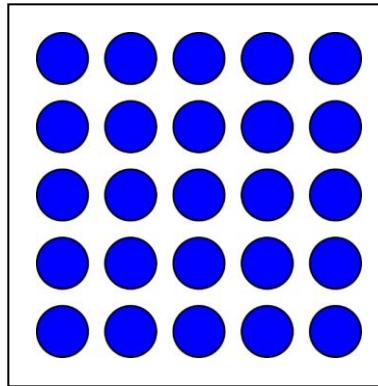
Diffraction



$$n\lambda = 2d \sin(\theta)$$

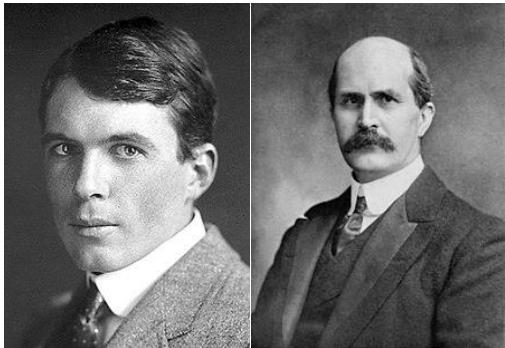


The effect of temperature

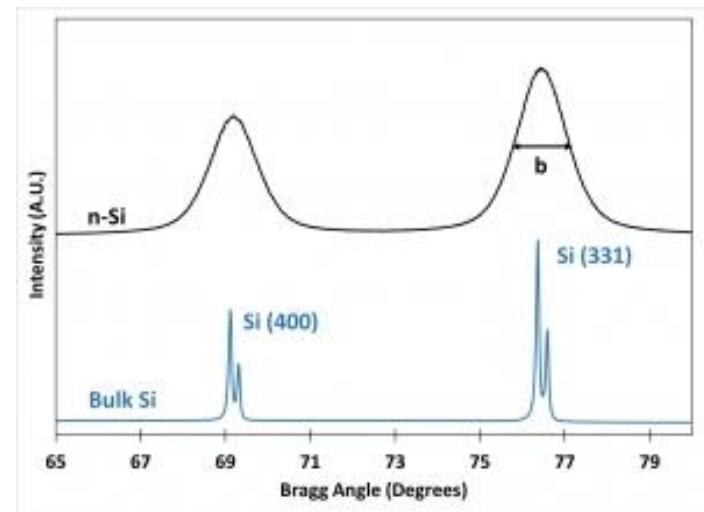
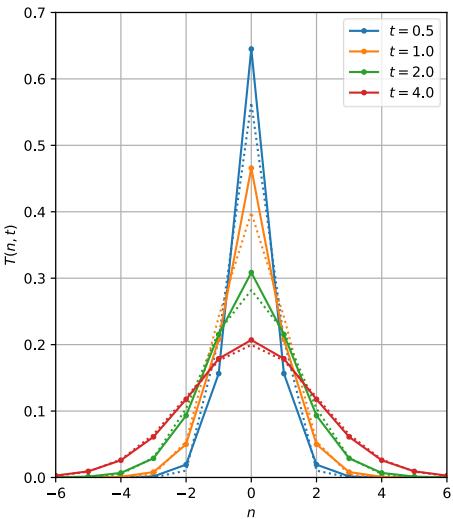
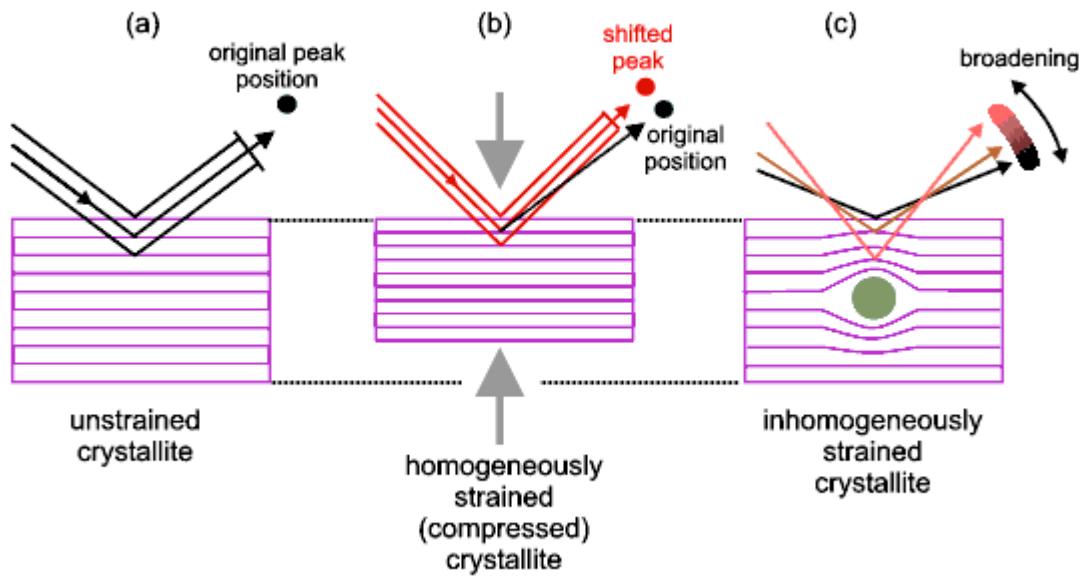


Debye-Waller factor

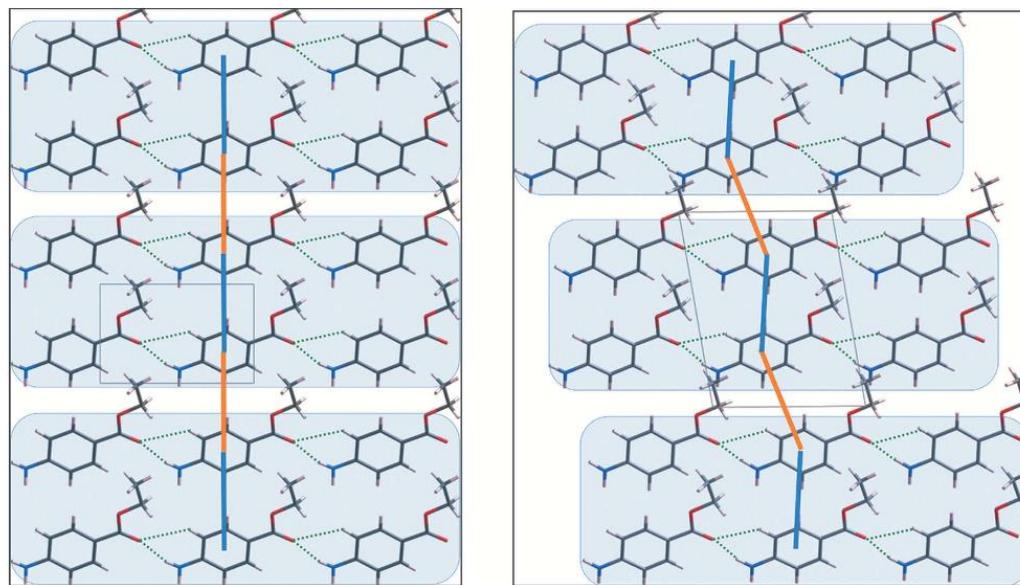
Diffraction



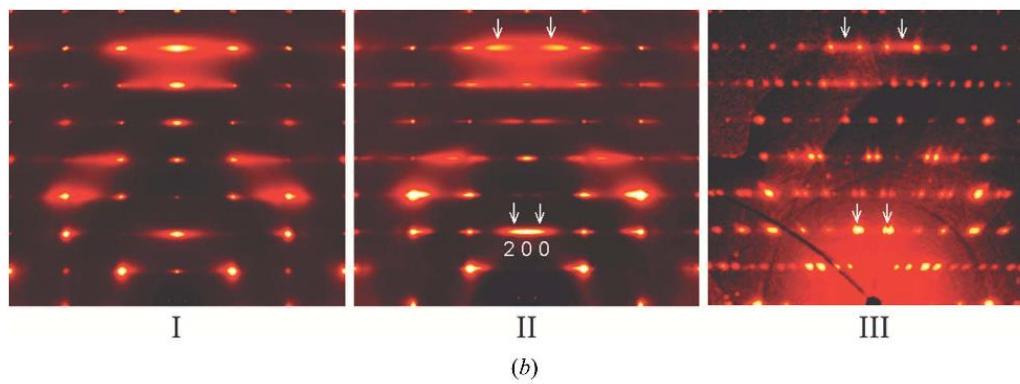
$$n\lambda = 2d \sin(\theta)$$



The effect of dislocation



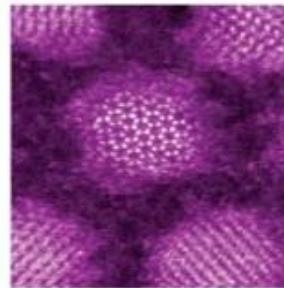
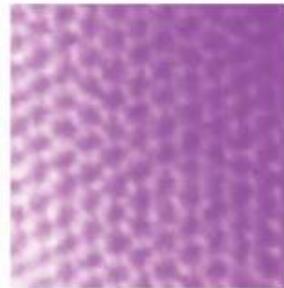
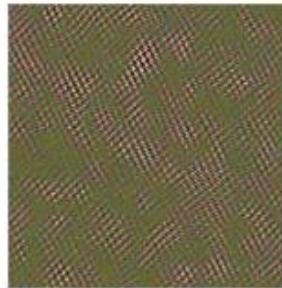
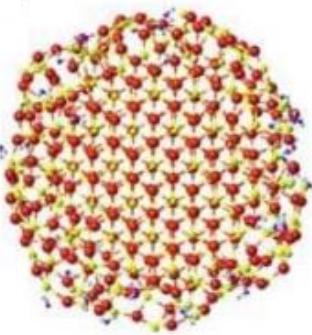
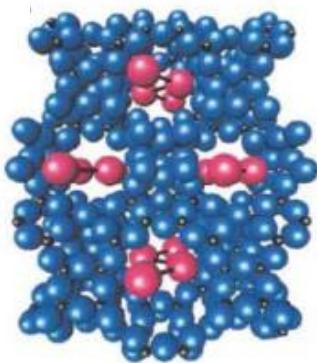
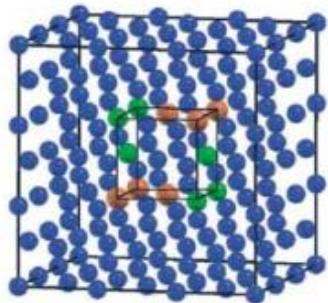
(a)



(b)

Single-crystal diffuse scattering studies on polymorphs of molecular crystals. I. The room-temperature polymorphs of the drug benzocaine.
E.J. Chan, T.R. Welberry, D.J. Goossens, A.P. Heerdegen, A. Beasley, P.J. Chupas
DOI:10.1107/S0108768109015857

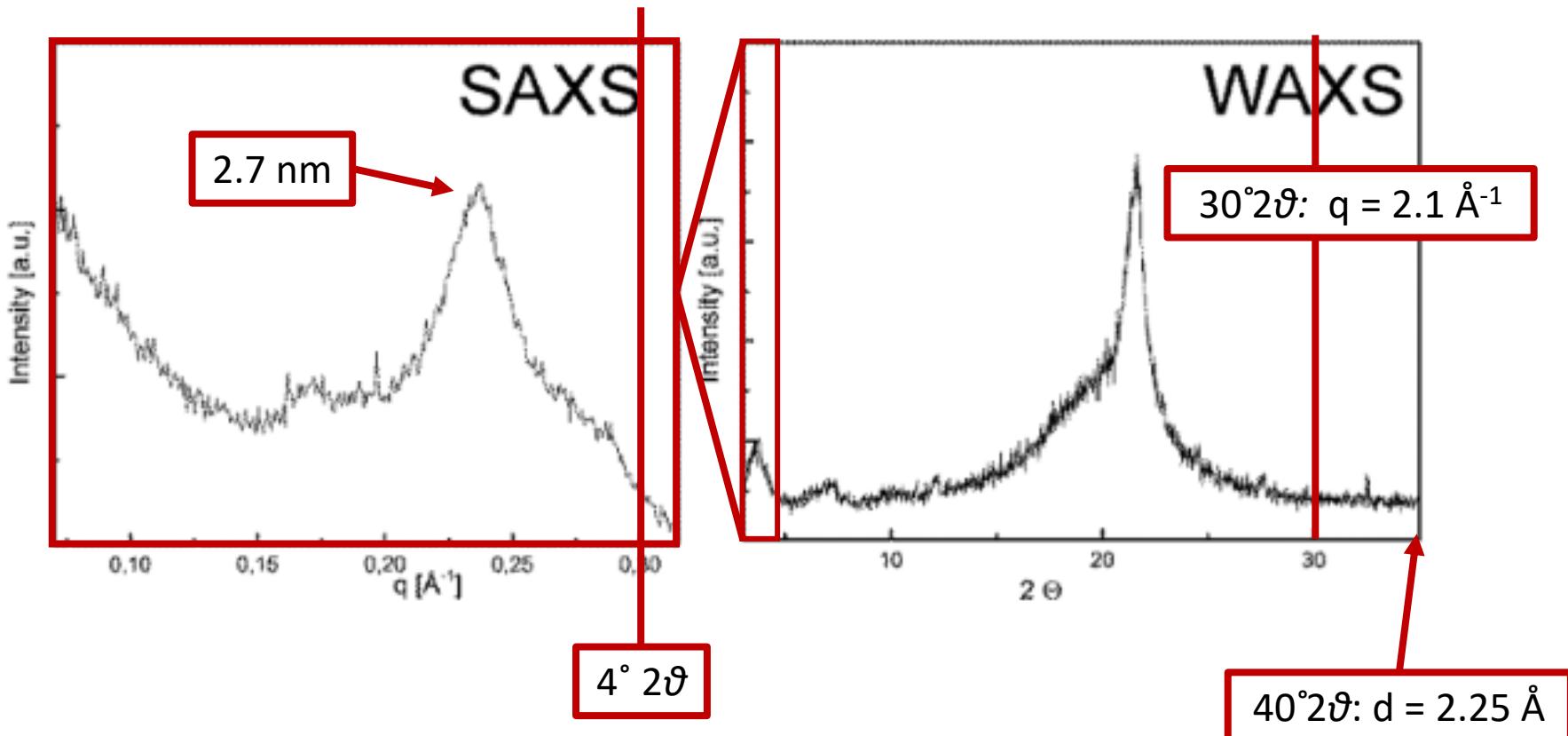
Nanoscale structure



Wide vs small angle scattering

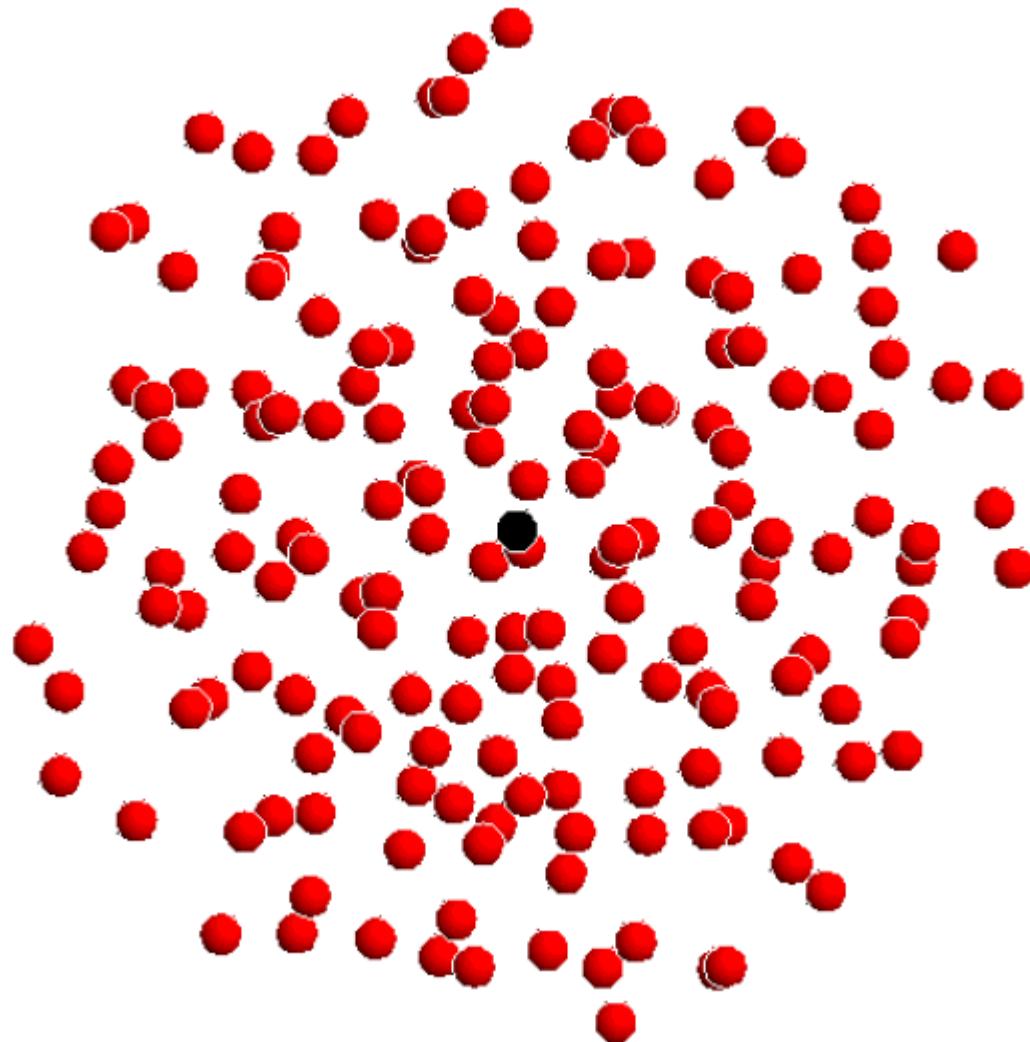
$$q = \frac{4\pi\sin(\theta)}{\lambda}$$

Amorphous poly(phosphoamidate), data acquired with Cu K α radiation ($\lambda = 1.54 \text{ \AA}$)



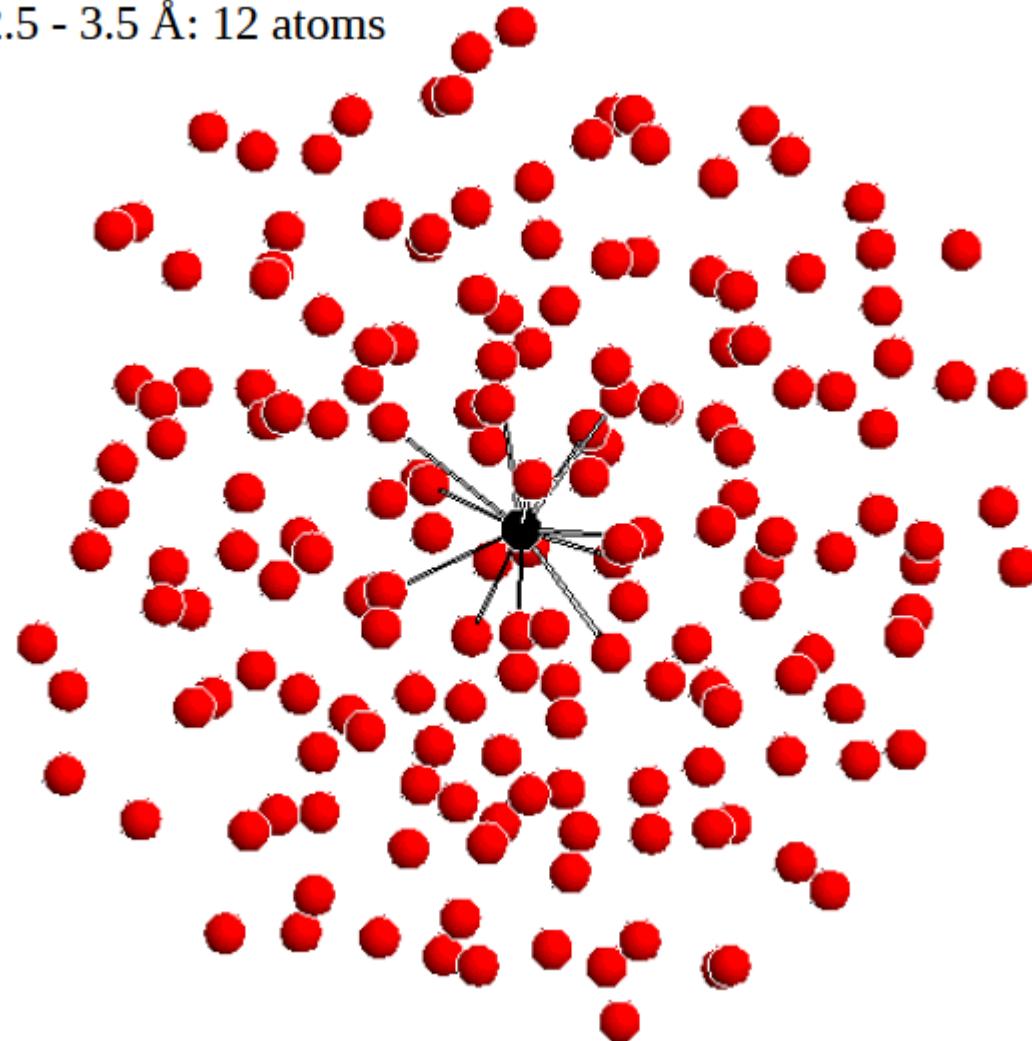
Pair distribution function

Pair distribution function



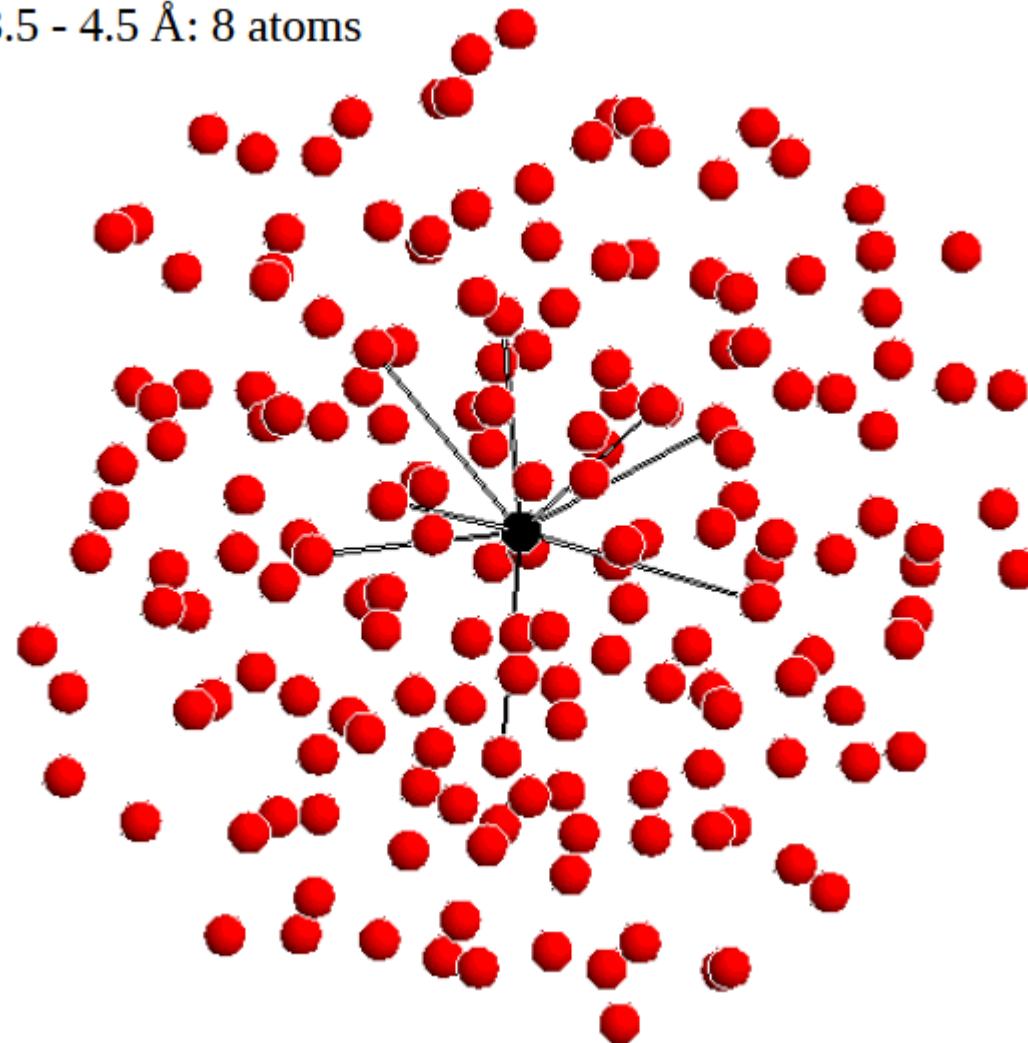
Pair distribution function

2.5 - 3.5 Å: 12 atoms



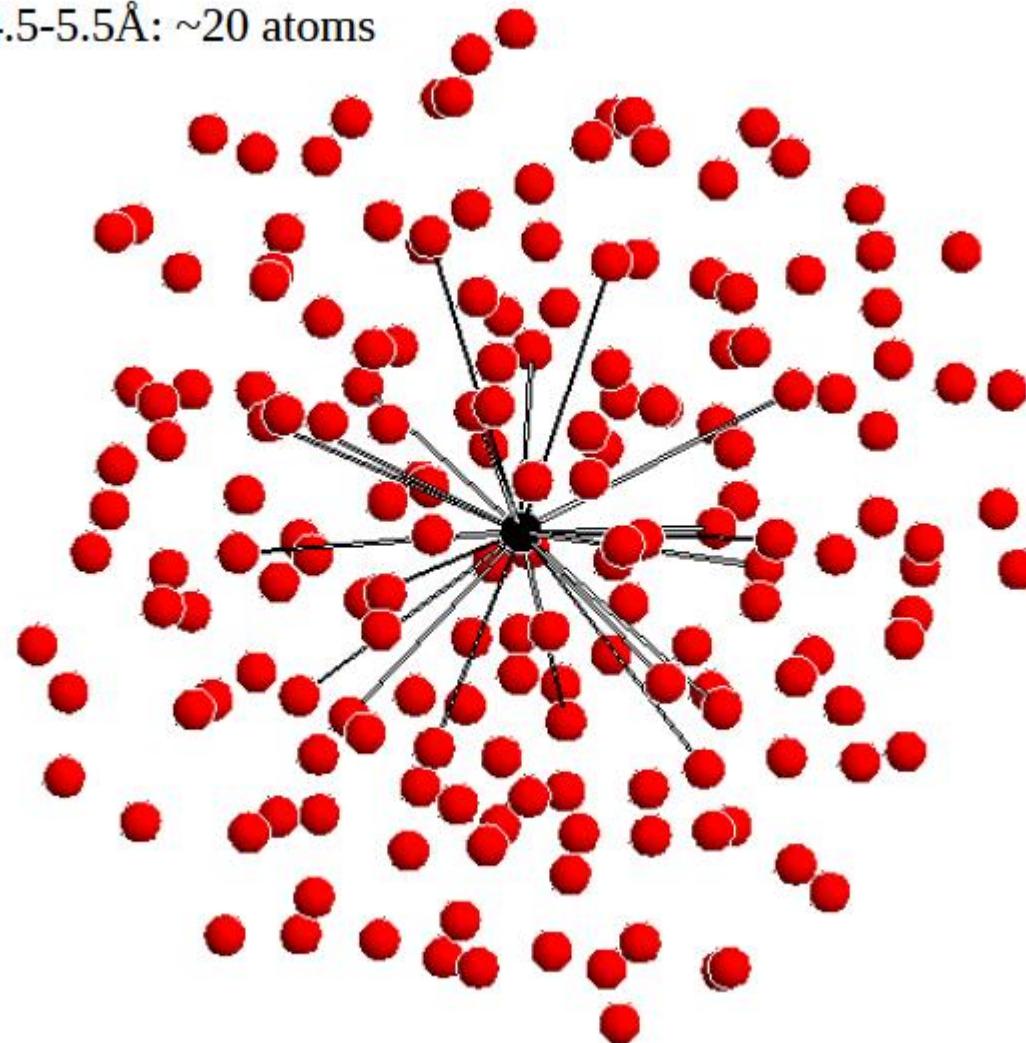
Pair distribution function

3.5 - 4.5 Å: 8 atoms



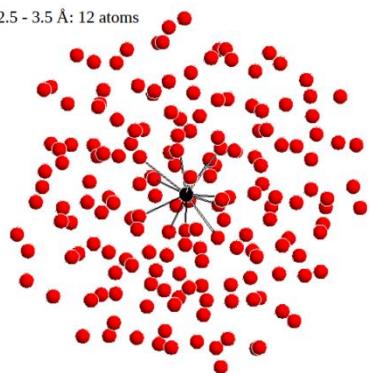
Pair distribution function

4.5-5.5Å: ~20 atoms

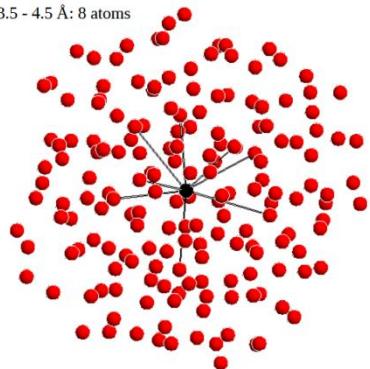


Pair distribution function

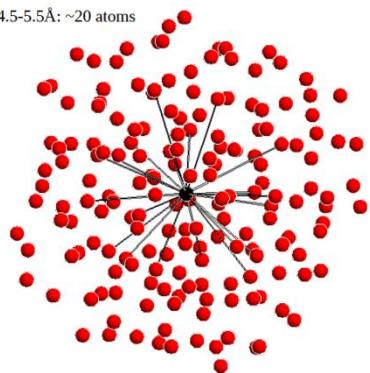
2.5 - 3.5 Å: 12 atoms



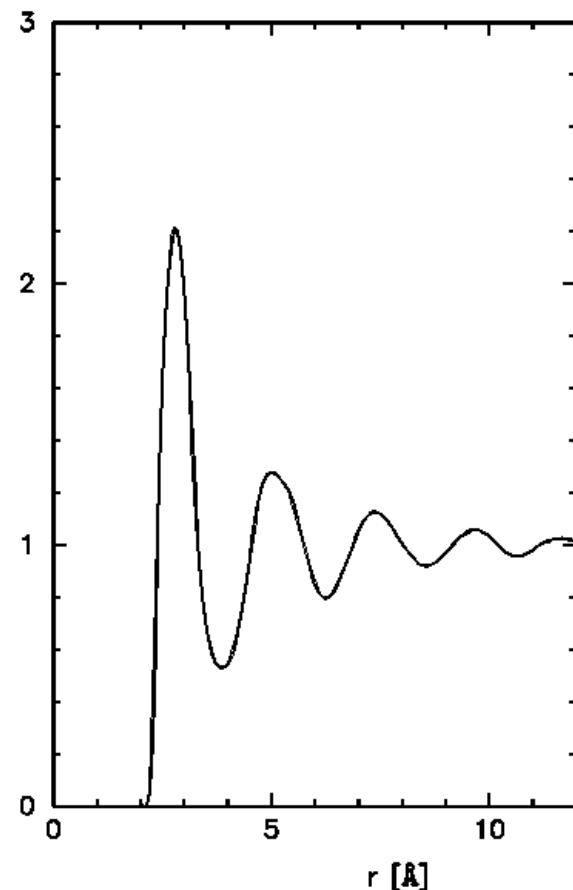
3.5 - 4.5 Å: 8 atoms



4.5-5.5 Å: ~20 atoms

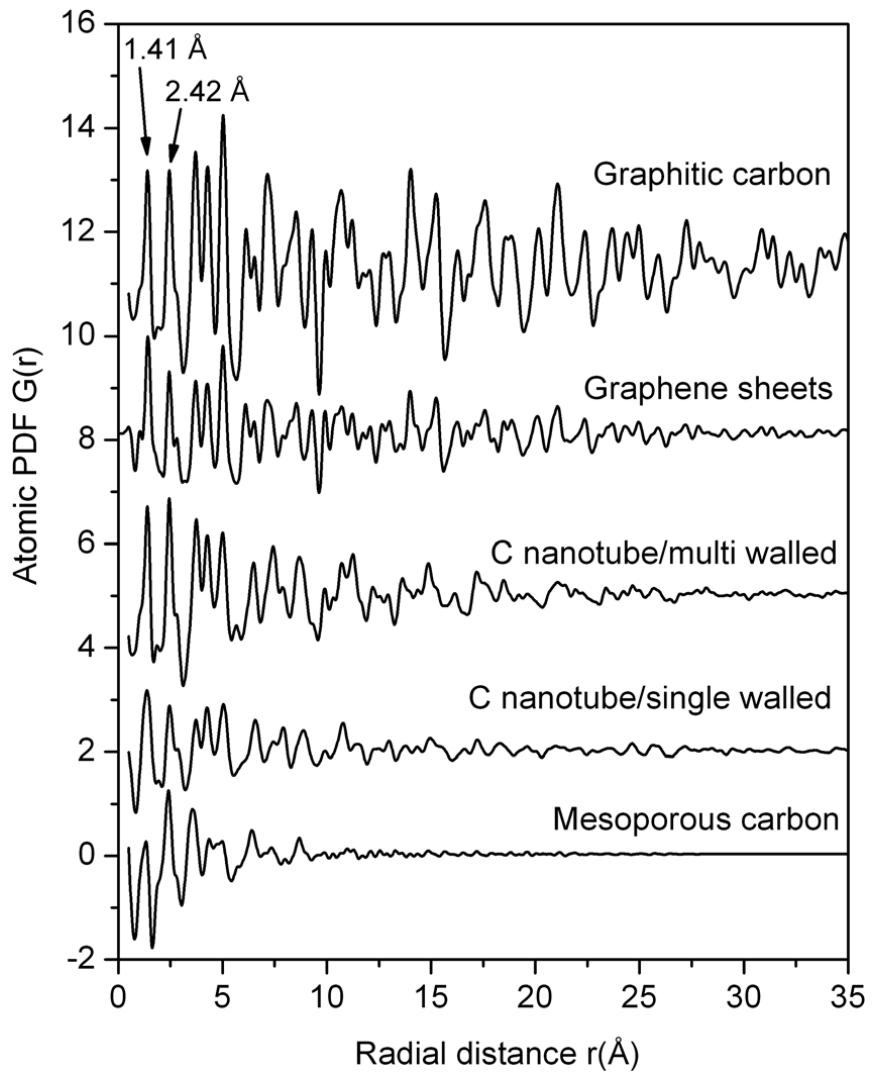


A "typical" liquid $g(r)$



Pair distribution function

$$G(r) = \frac{1}{\rho} \left(\sum_{i \neq 0} \delta(r - r_i) \right)$$
$$= V \frac{N-1}{N} (\delta(r - r_1))$$

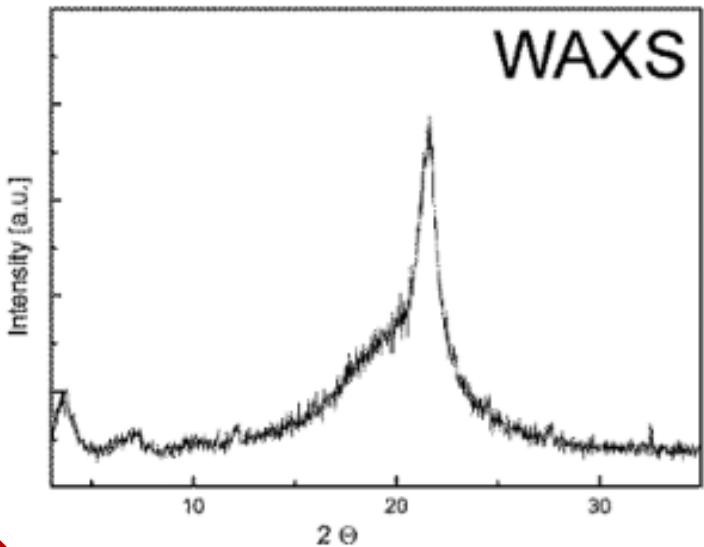


Pair distribution function

Measurement in reciprocal space gives structure factor $F(q)$

$$F(q) = \sum_{i,j} (2 - \delta_{i,j}) c_i c_j b_i b_j S_{i,j}(q)$$

$$S_{i,j}(q) = 1 + \frac{1}{N} \left(\sum_{i \neq j} e^{-iq(r_i - r_j)} \right)$$

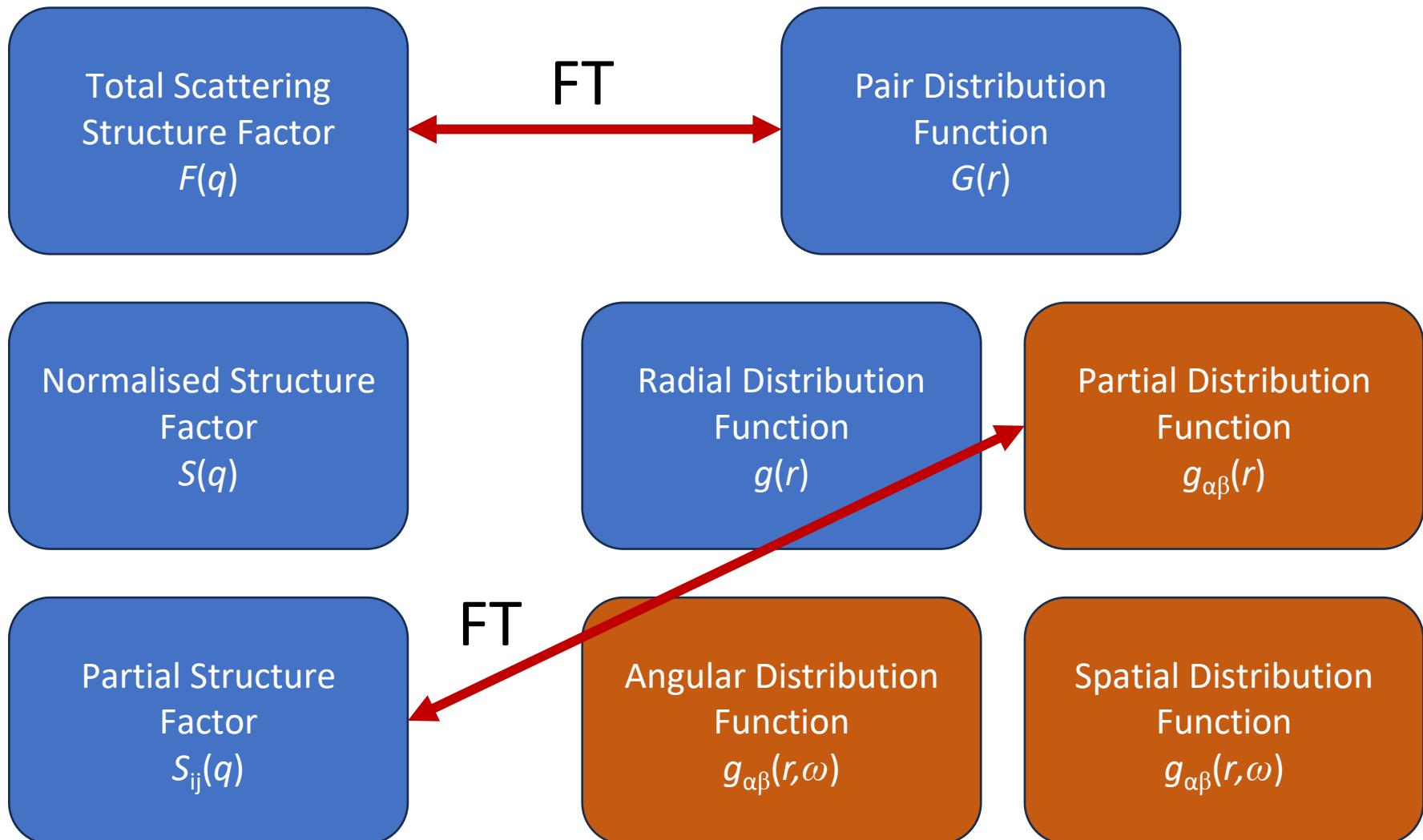


Partial structure factor

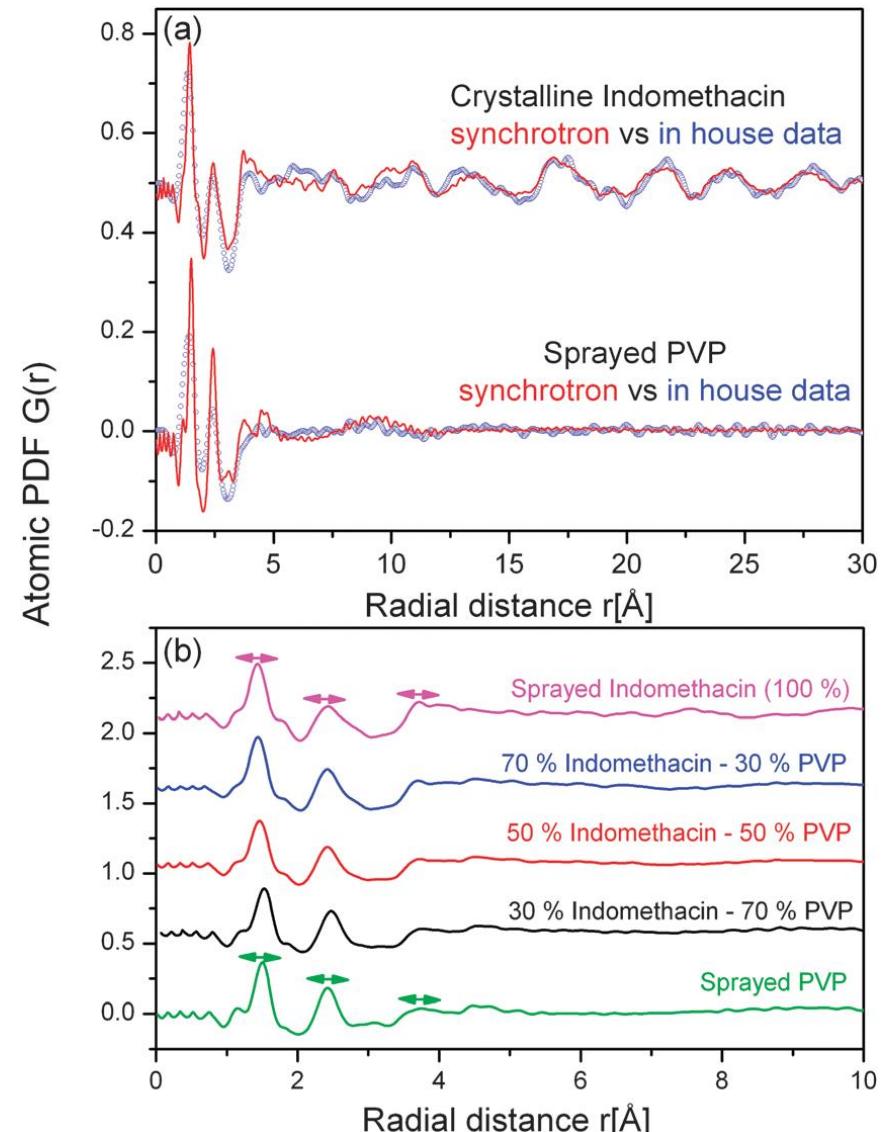
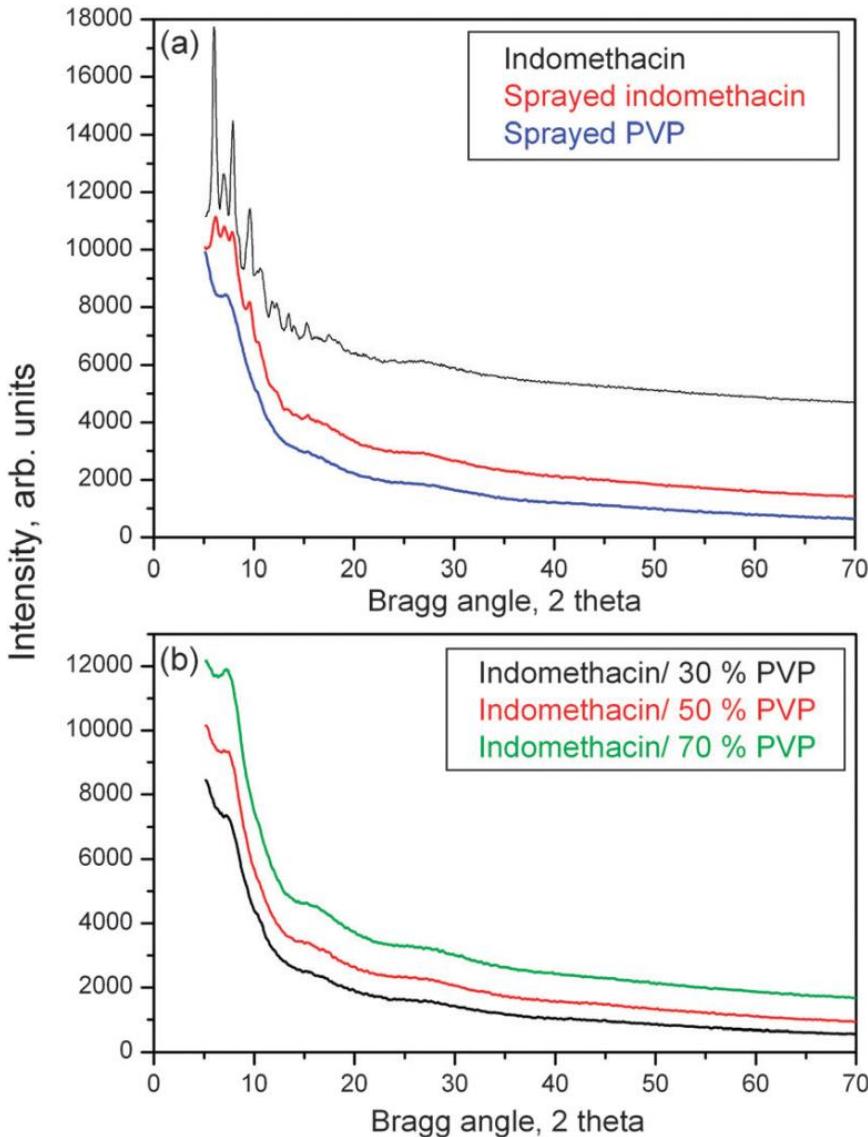
$F(q)$ and $S(q)$ are related to respective $G(r)$ and $g_{\alpha\beta}(r)$ by Fourier Transform

Reciprocal space

Real space

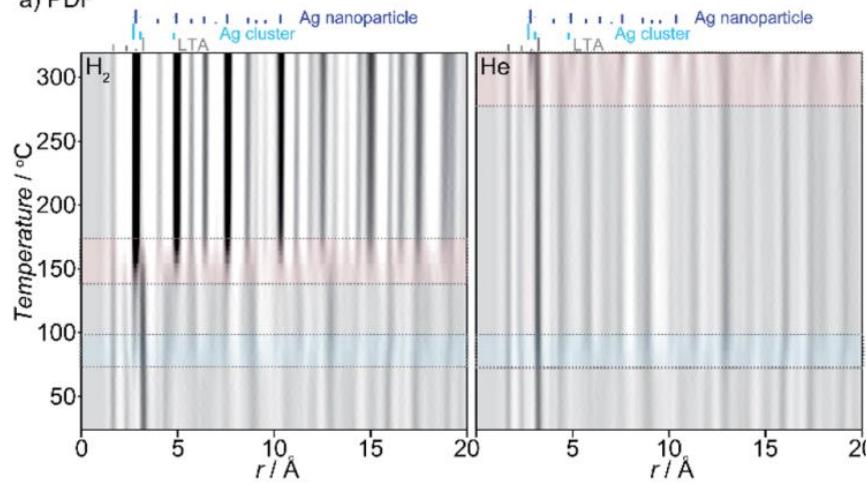


Indomethacin in polymer

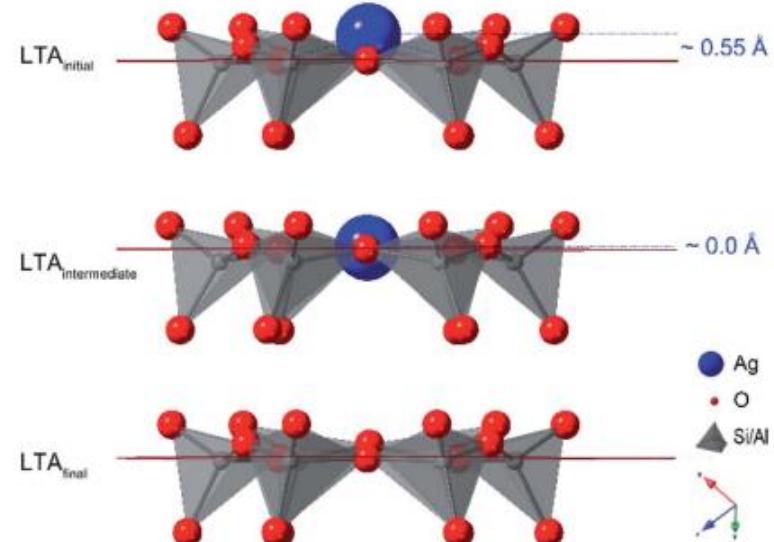
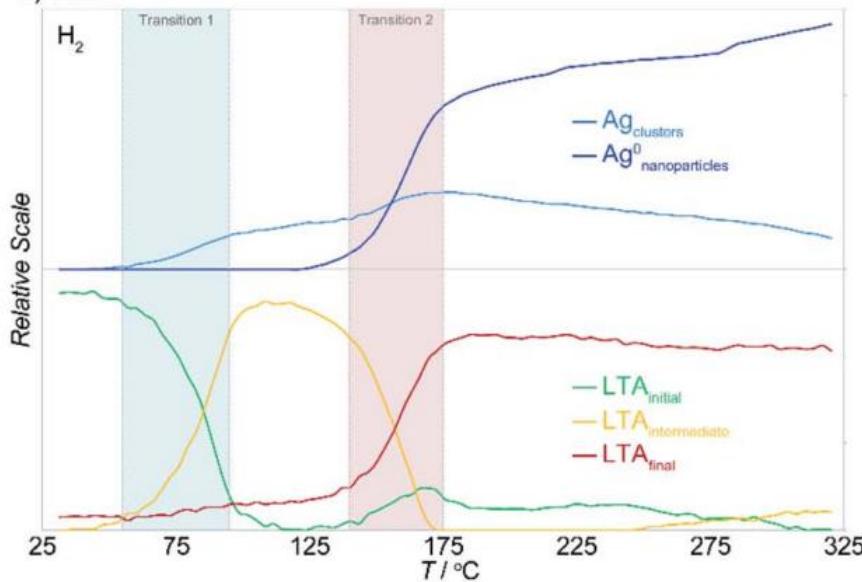


Silver nanoparticles in zeolite

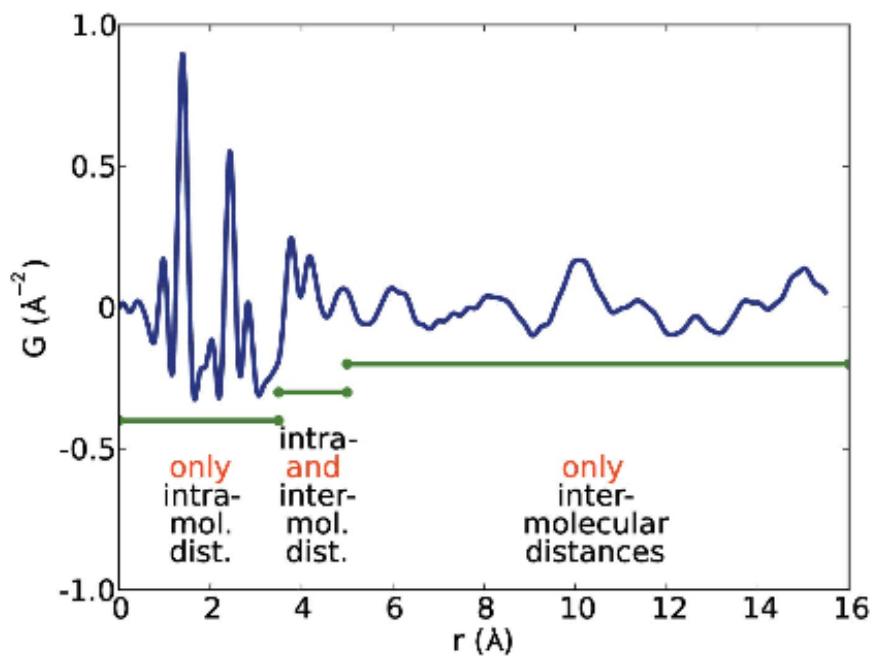
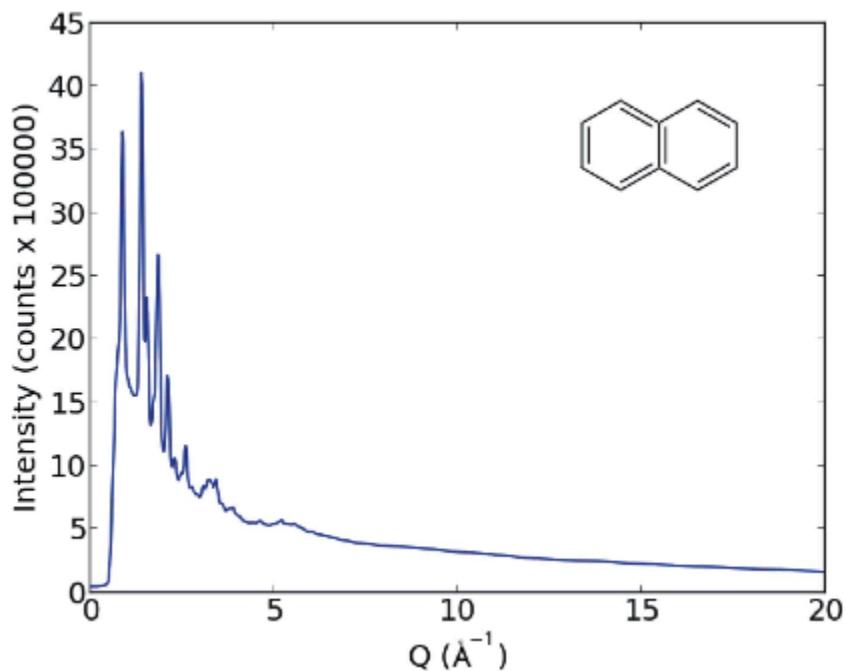
a) PDF



a) PDF

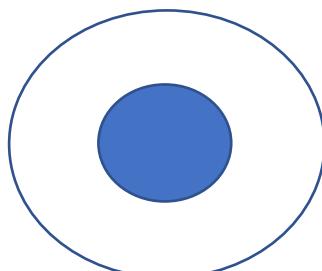
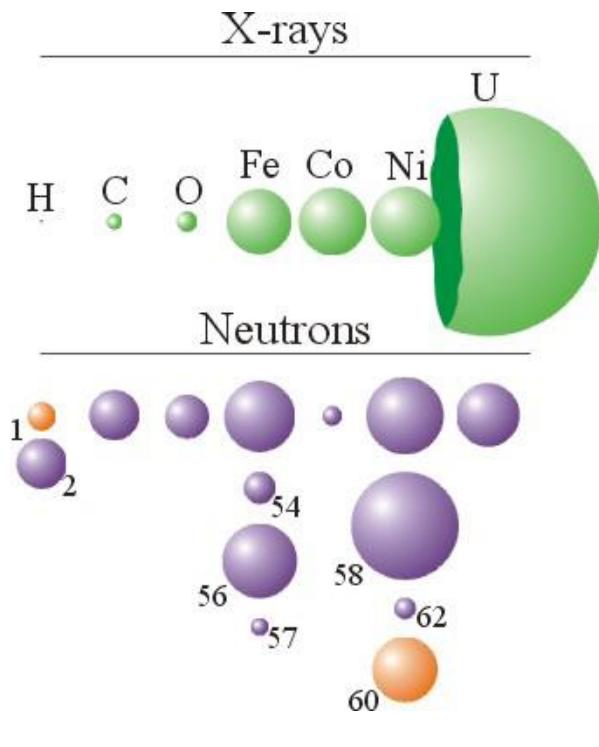


Structure solution from WAXS

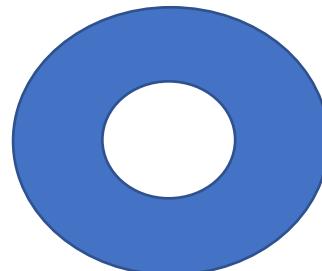


Problem: fitting of too many parameters to one structure factor!

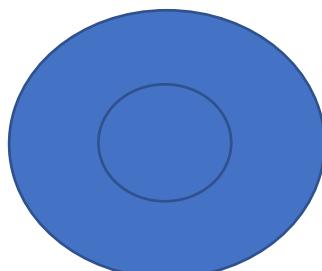
Wide-angle neutron scattering



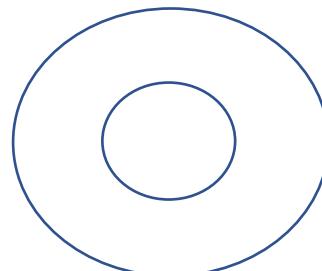
H/D



D/H



H/H



D/D

5 mol% H/D exchange on part
of the sample we want to see

WANS

$$F(q) = \sum_{i,j} (2 - \delta_{i,j}) c_i c_j b_i b_j S_{i,j}(q)$$

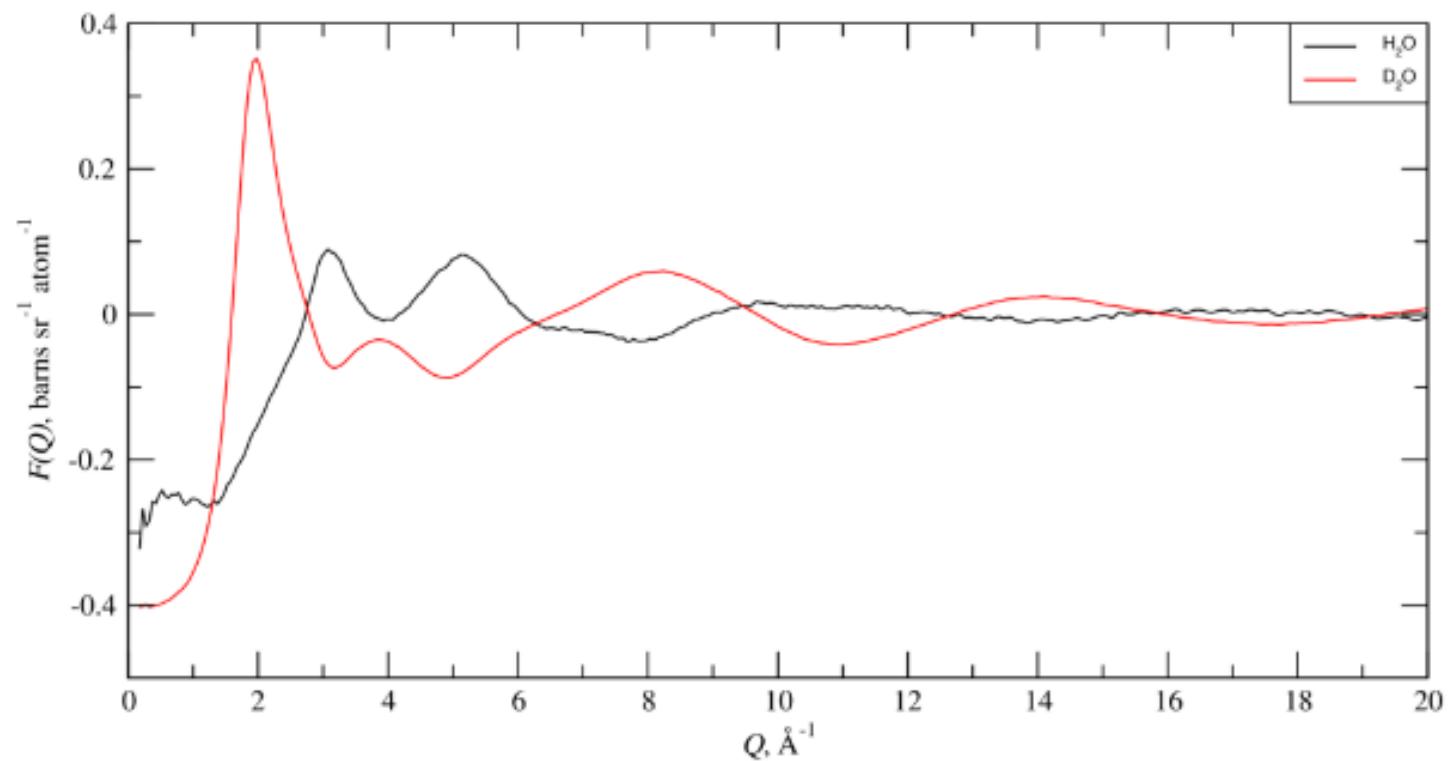
Water

i	j	c_i	c_j	$b_i, \text{ fm}$	$b_j, \text{ fm}$	$c_i c_j b_i b_j = w_{ij}, \text{ fm}$
O	O	1/3	1/3	0.5804	0.5804	0.0374
O	H	1/3	2/3	0.5804	-0.3741	-0.0482
H	H	2/3	2/3	-0.3741	-0.3741	0.0622

Heavy water

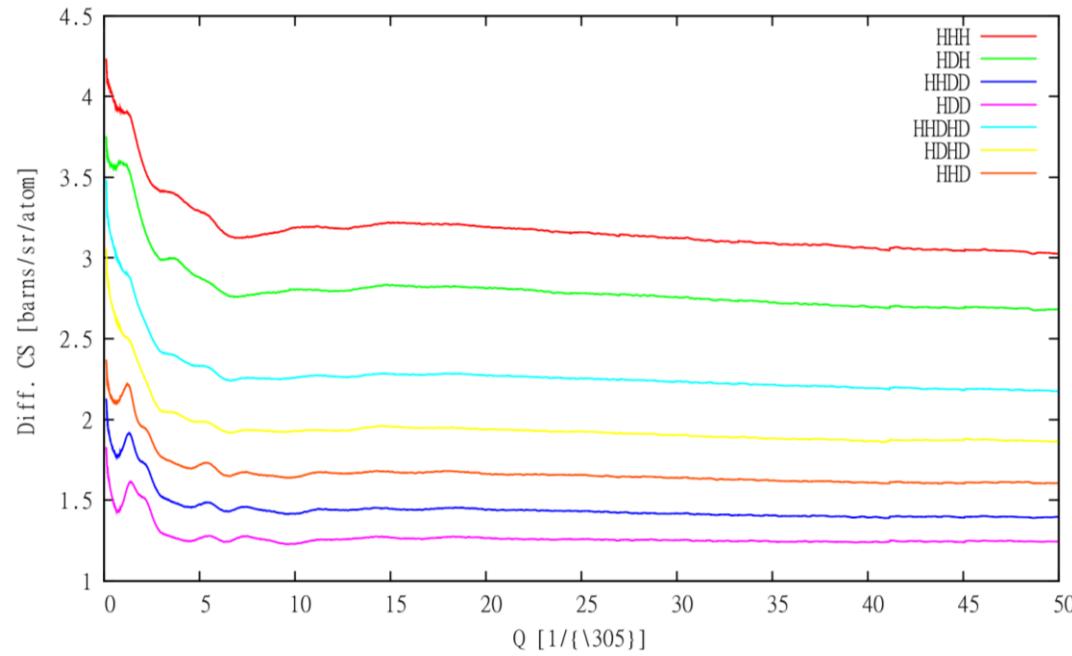
i	j	c_i	c_j	$b_i, \text{ fm}$	$b_j, \text{ fm}$	$c_i c_j b_i b_j = w_{ij}, \text{ fm}$
O	O	1/3	1/3	0.5804	0.5804	0.0374
O	D	1/3	2/3	0.5804	0.6674	0.0861
D	D	2/3	2/3	0.6674	0.6674	0.1980

WANS

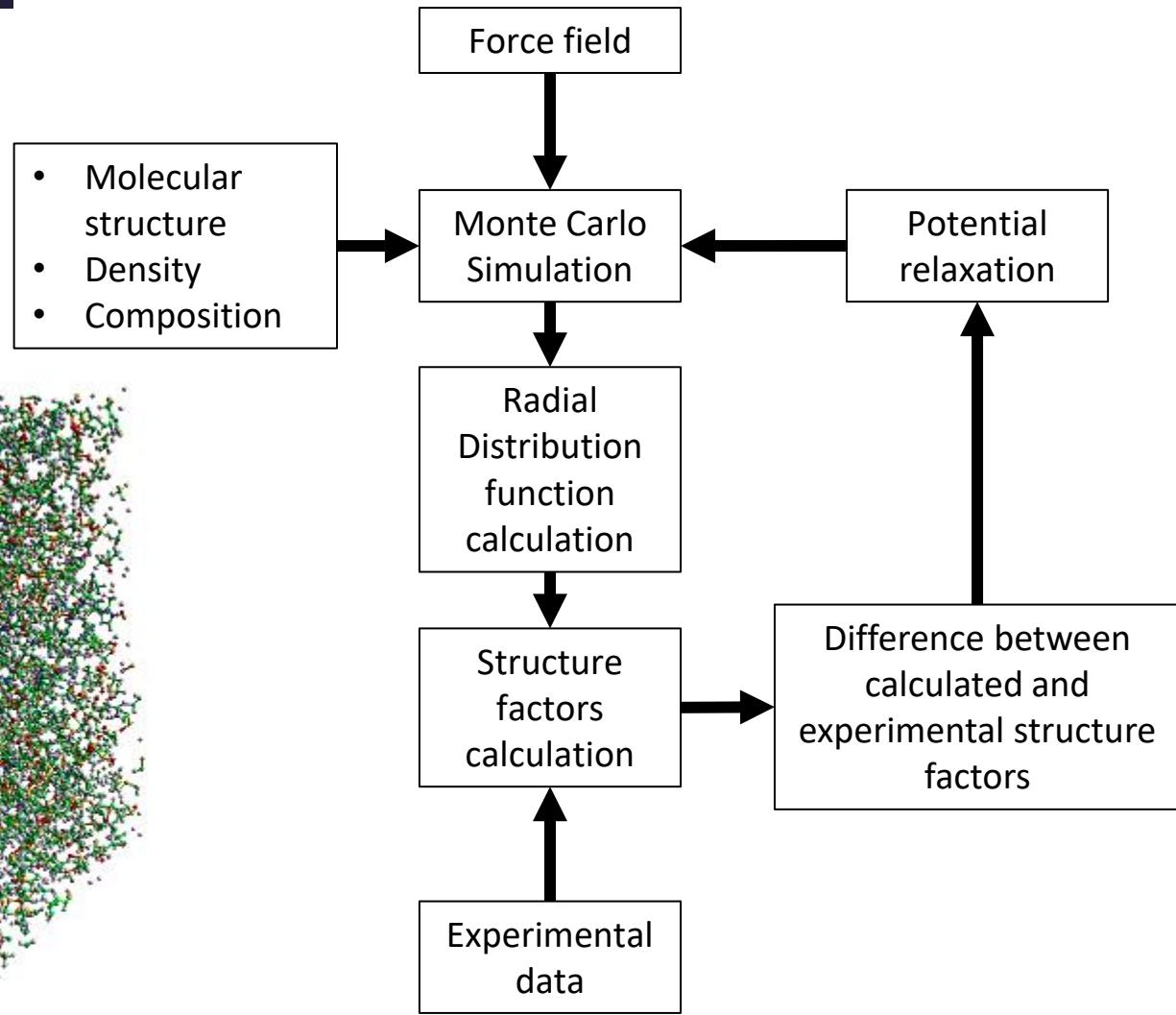
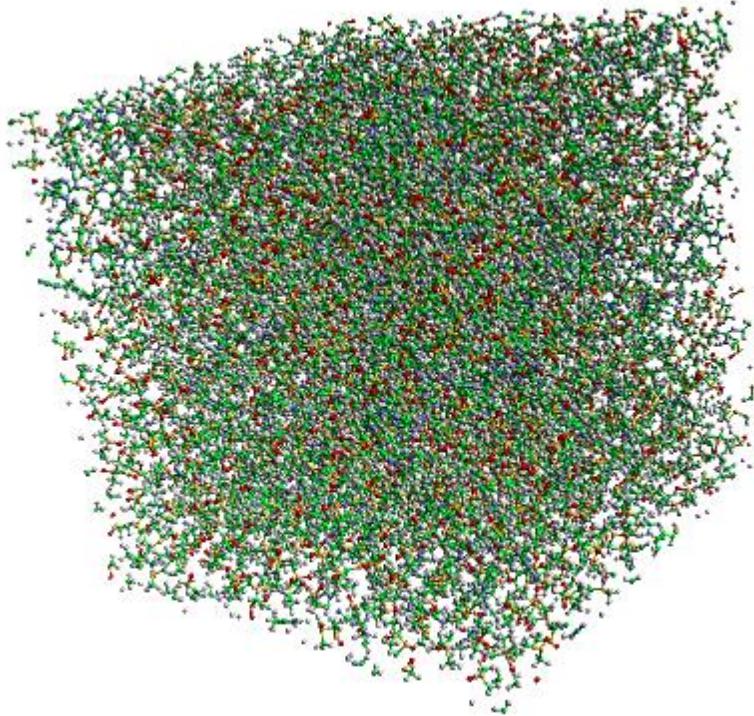


WANS

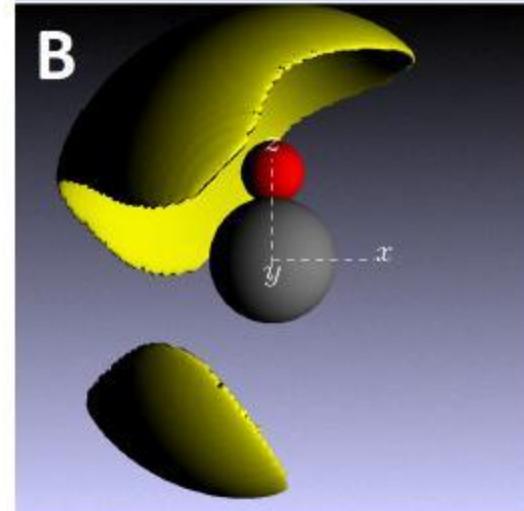
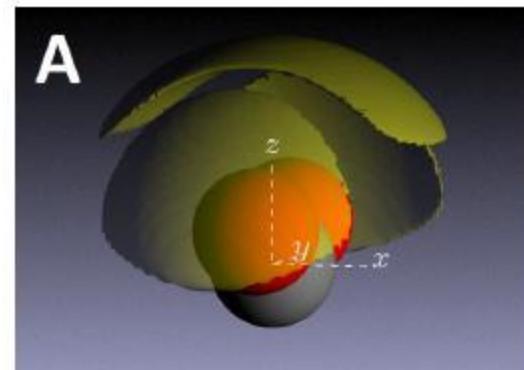
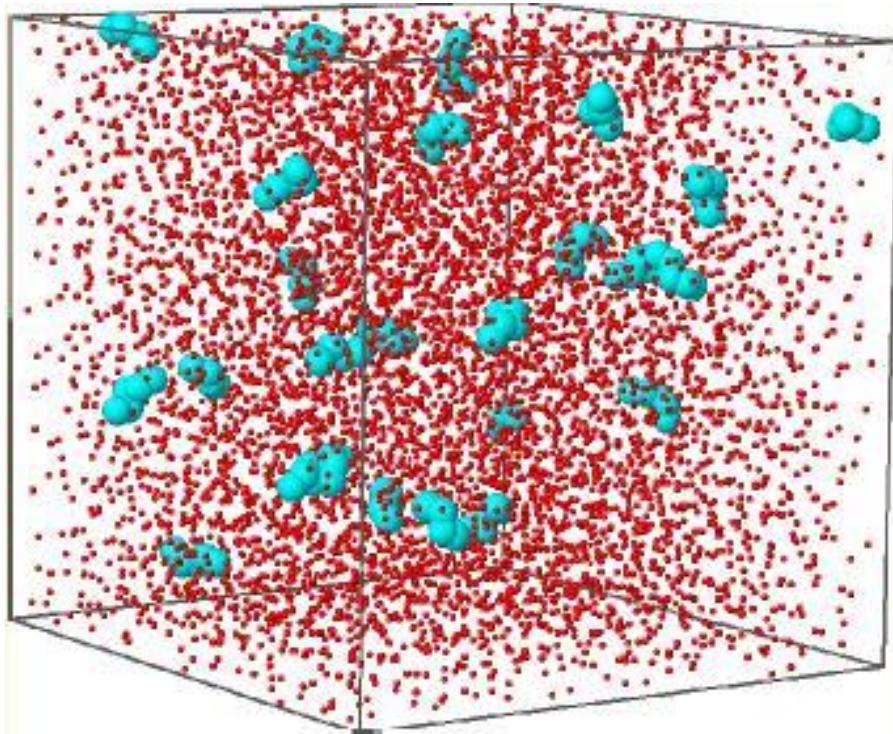
incoherency ↑



- Each isotopic mixture gives one structure factor
- Different correlations can be extracted from each structure factor

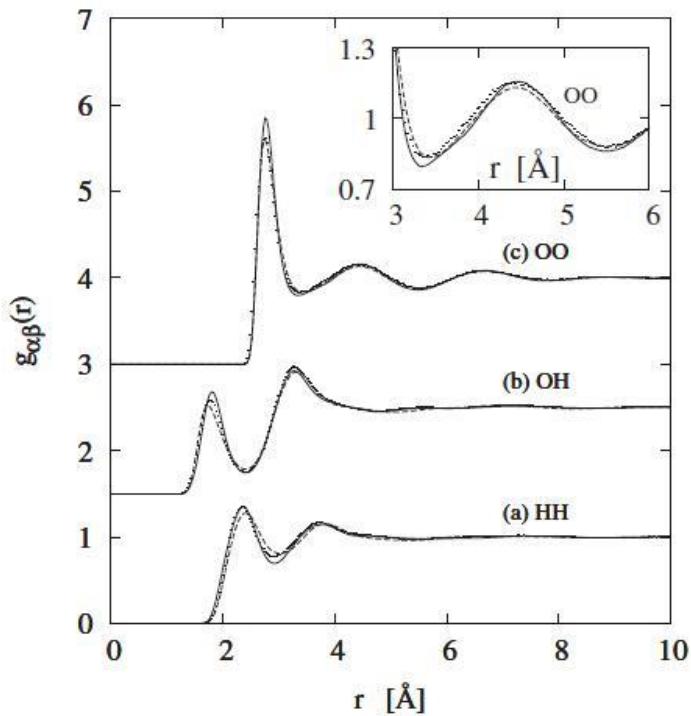
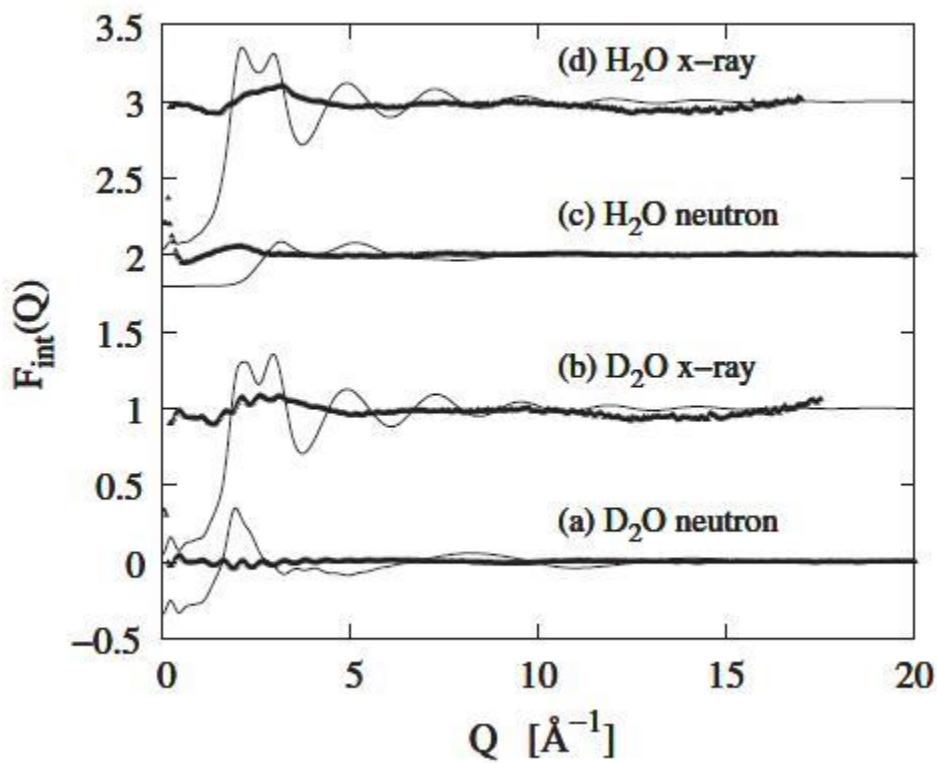
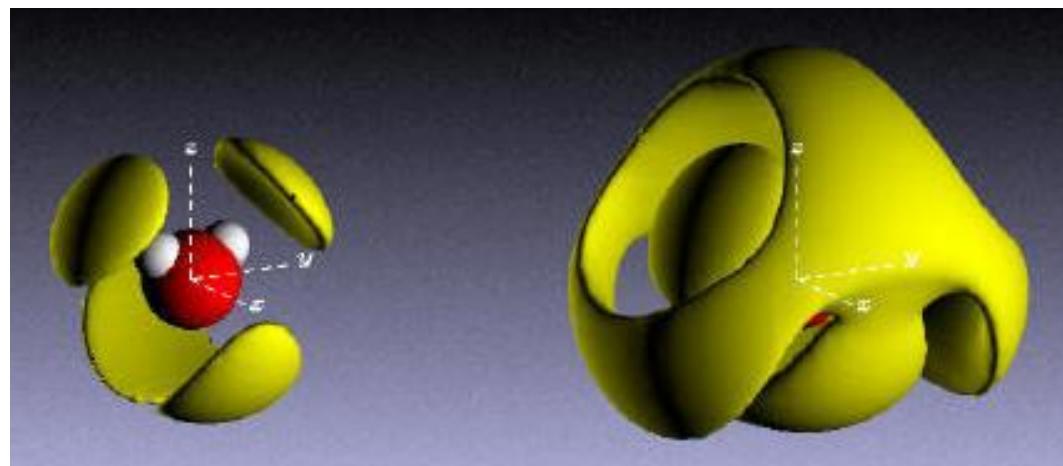


Generation of structural models

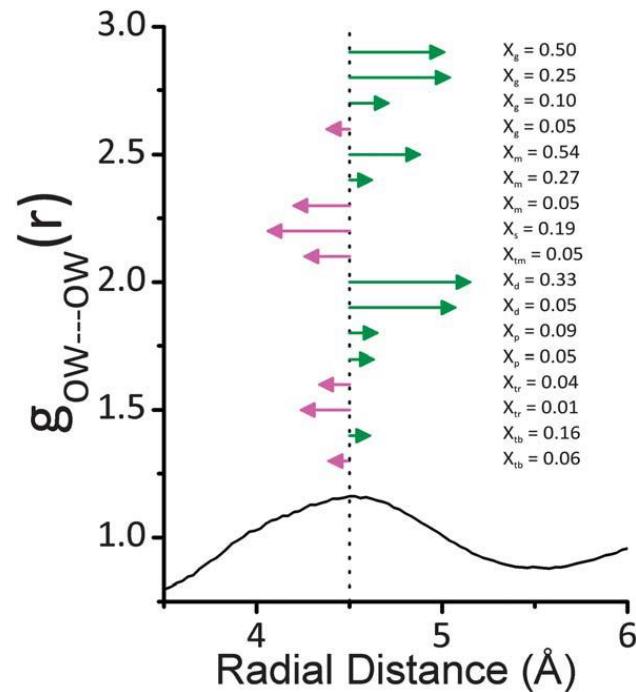
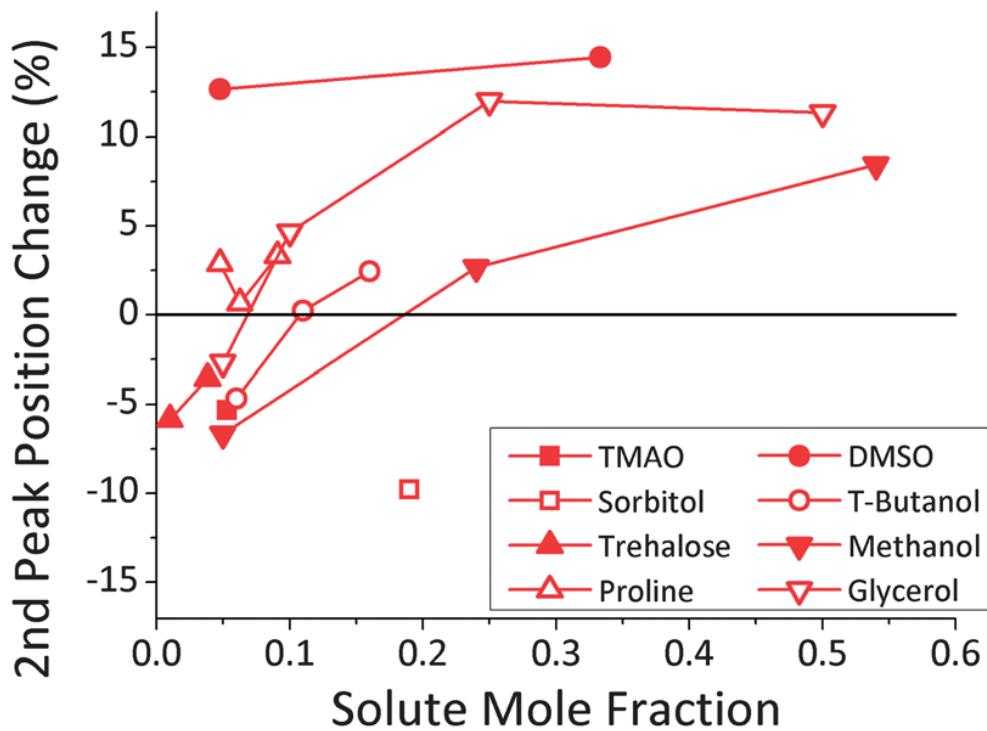
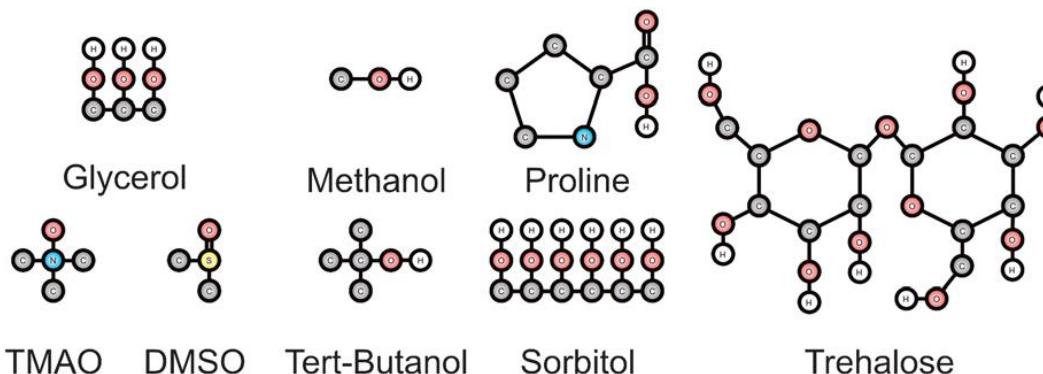


Applications

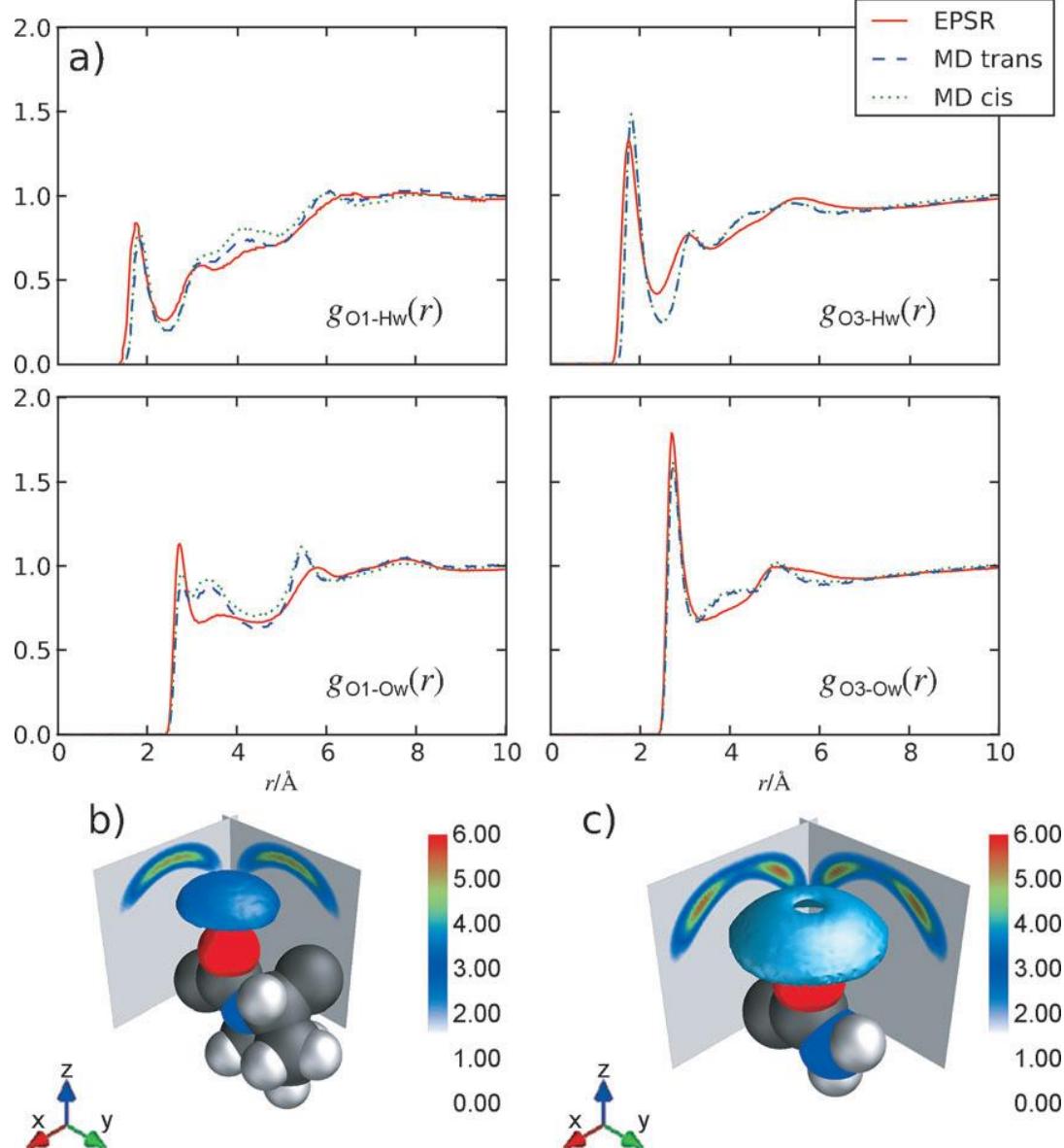
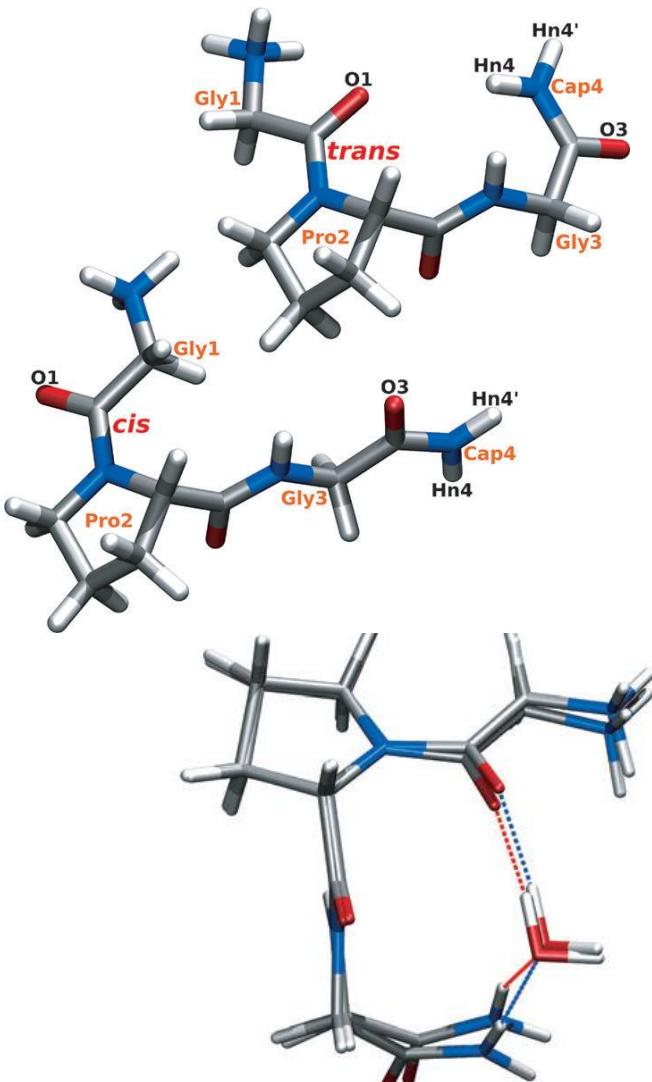
Water



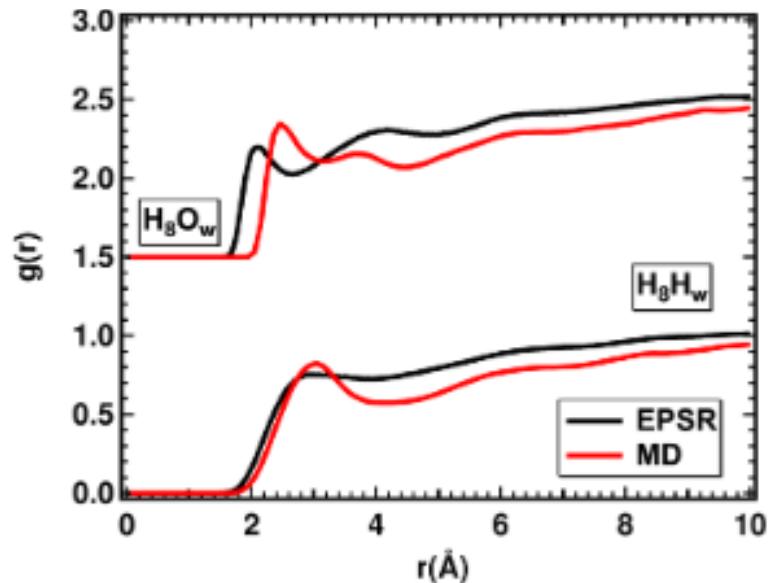
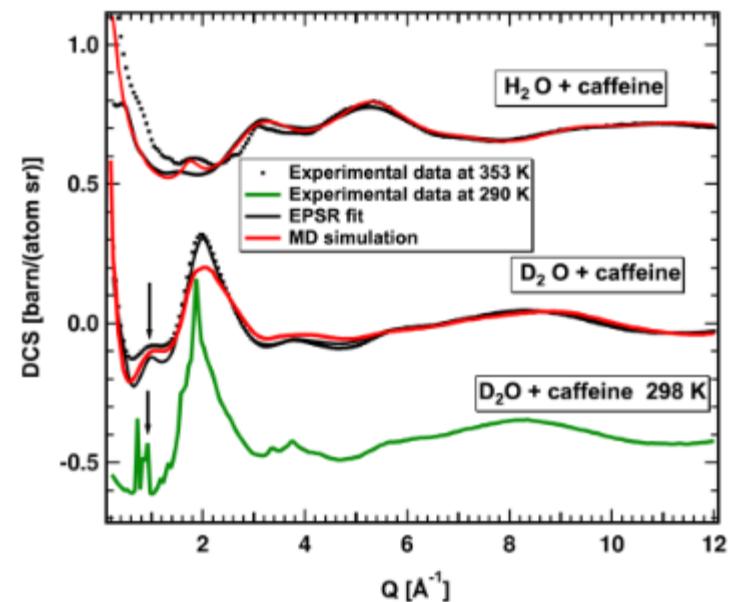
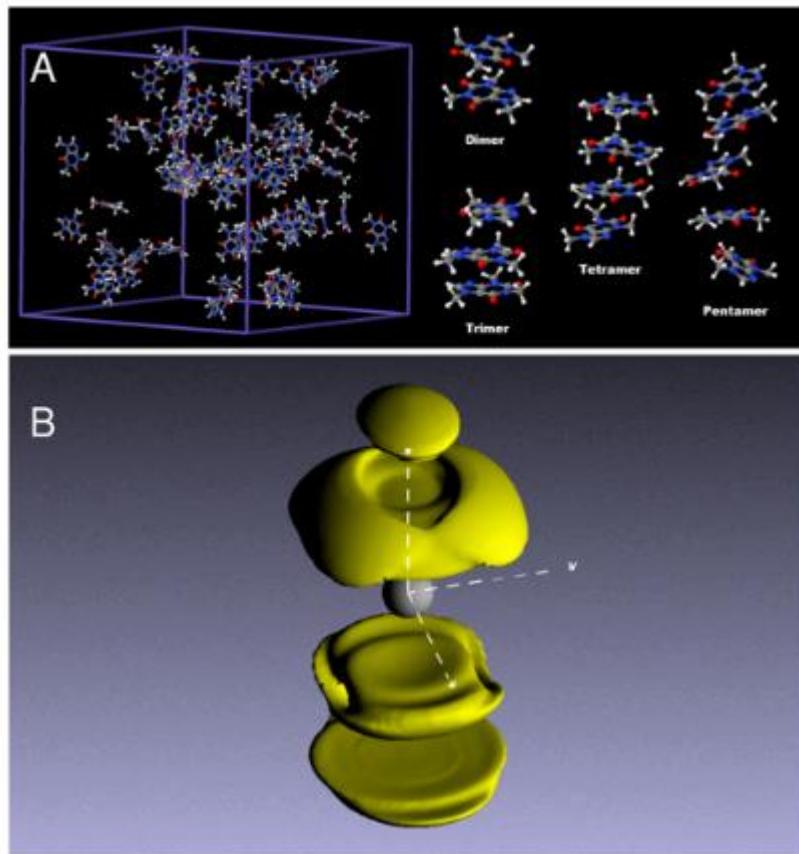
Antifreeze effect

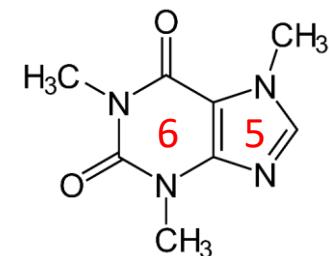
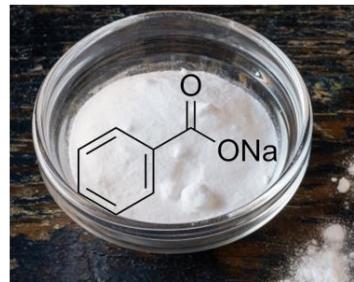
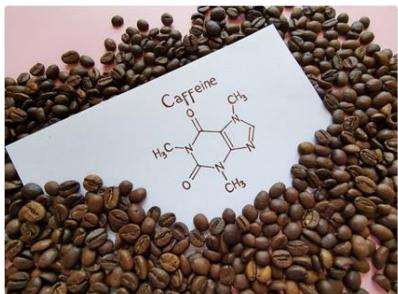


Small peptides

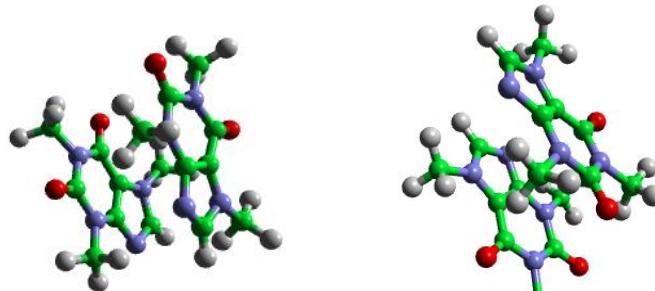
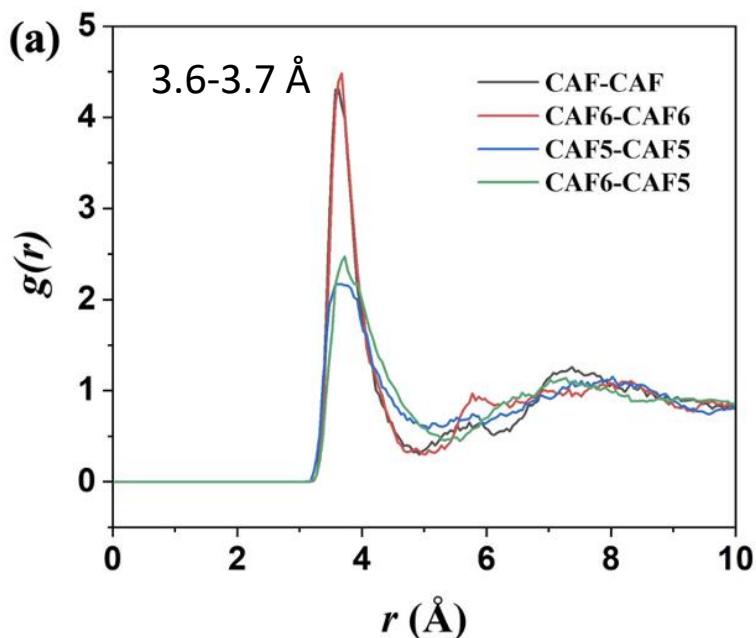


Drug compounds





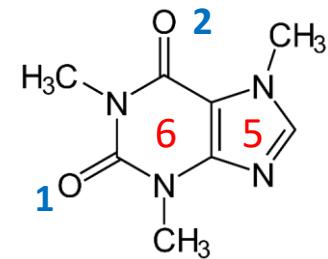
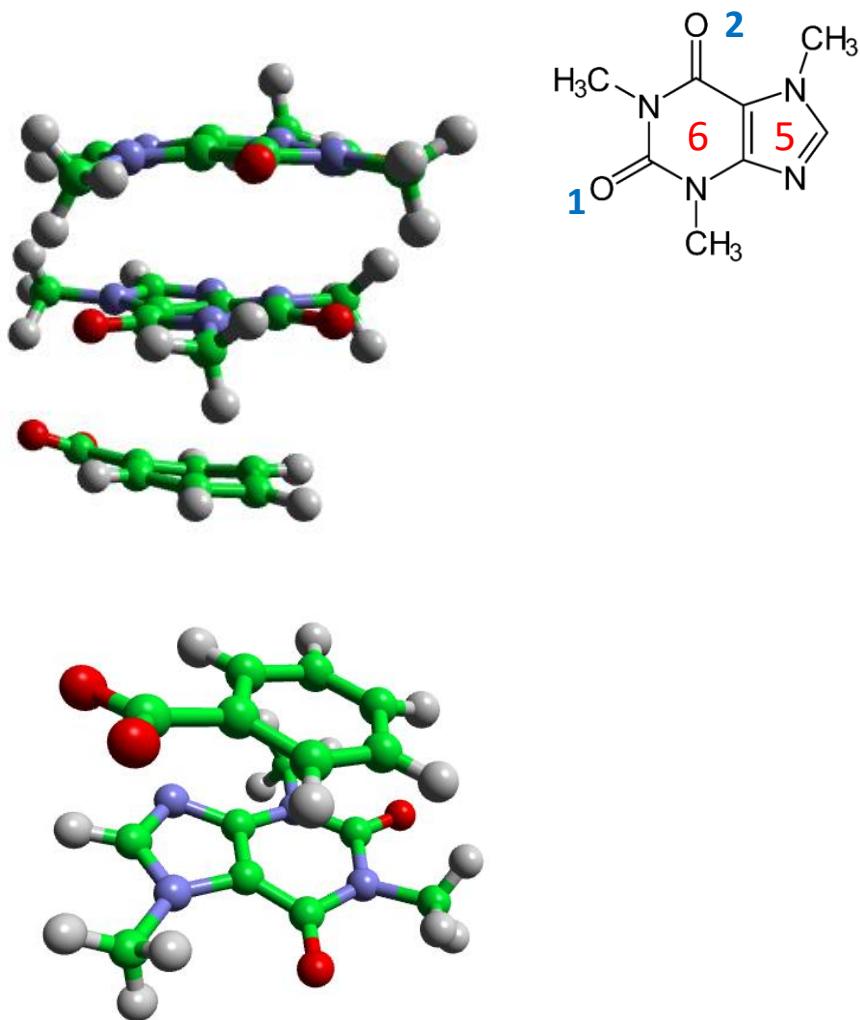
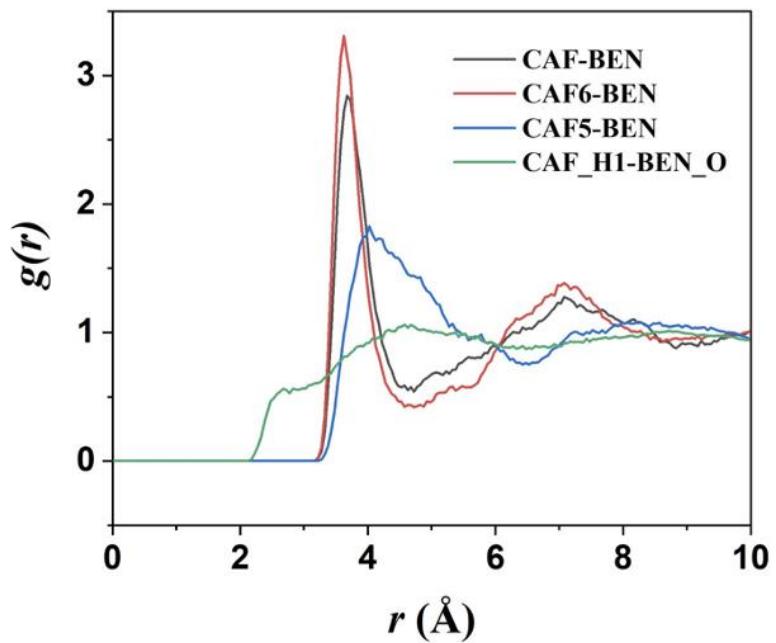
Pure CAF: 4.1 Å

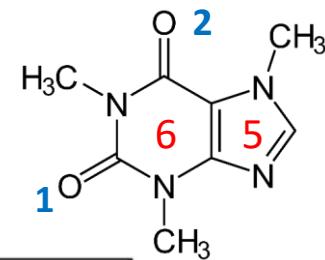


Correlation
CAF-CAF
CAF6-CAF6
CAF5-CAF5
CAF6-CAF5

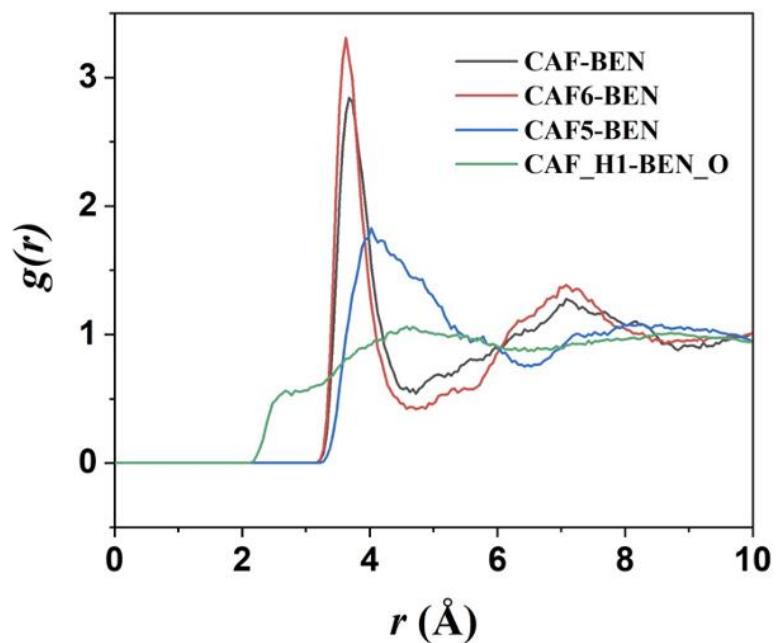
Co-ordination number (atoms)
0.39 ± 0.05
0.39 ± 0.06
0.27 ± 0.03
0.47 ± 0.05

CAF-Benzoate

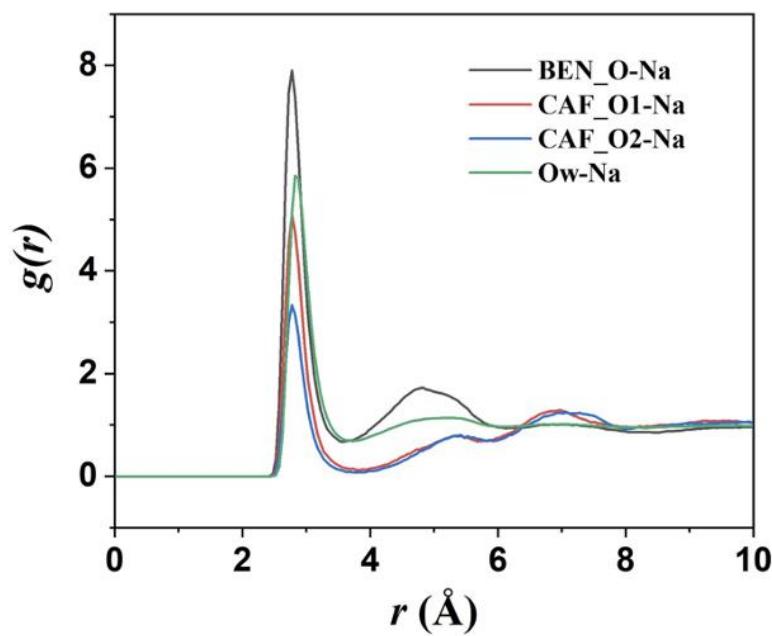


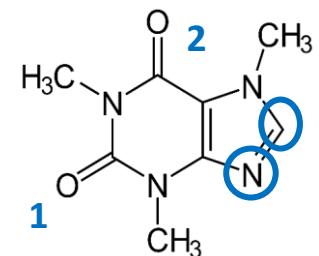


CAF-Benzoate

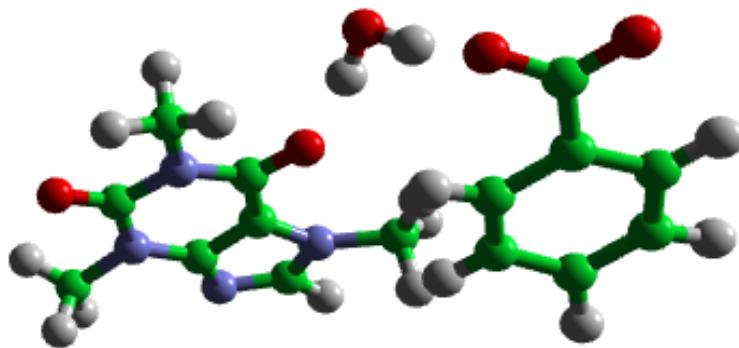
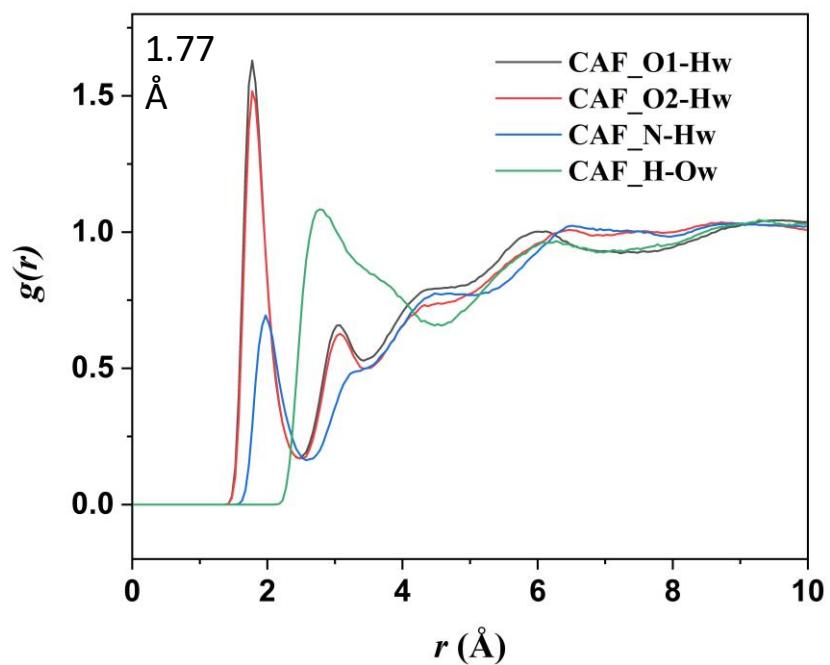


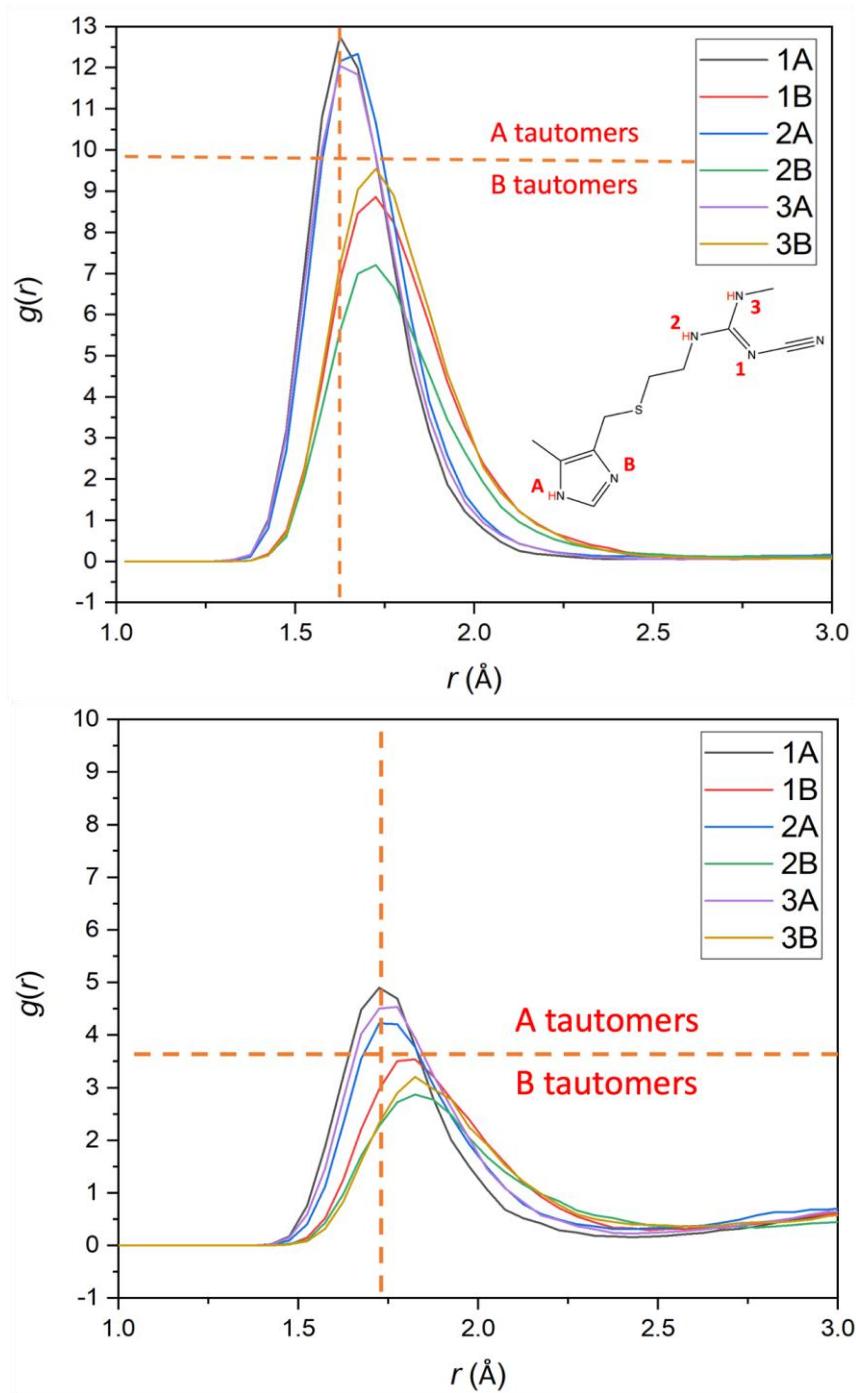
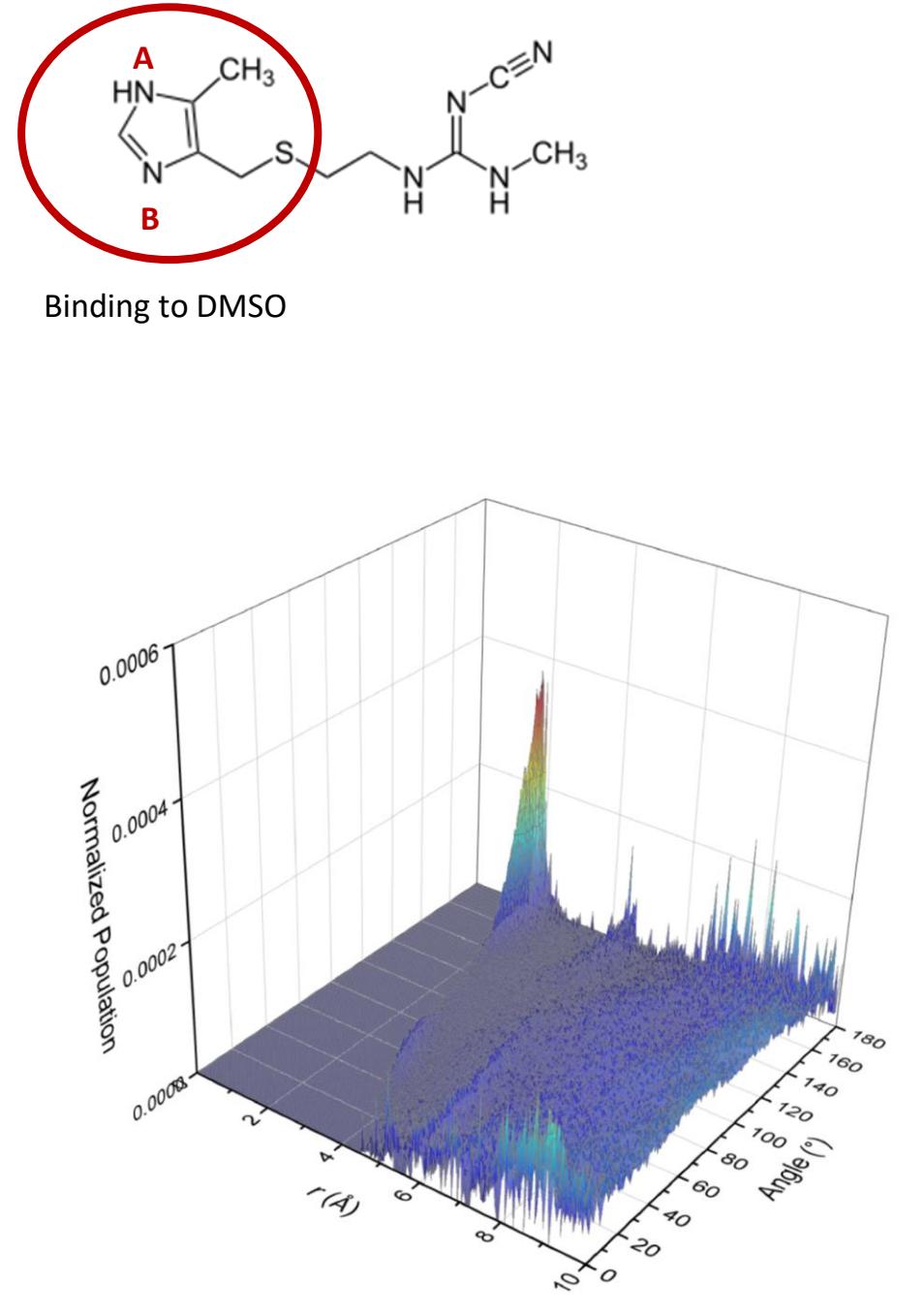
CAF/Benzoate- Na^+



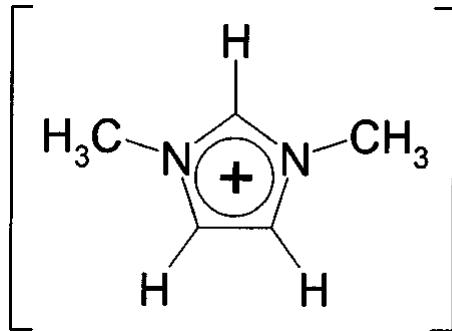


Pure CAF in water: 2.05 Å

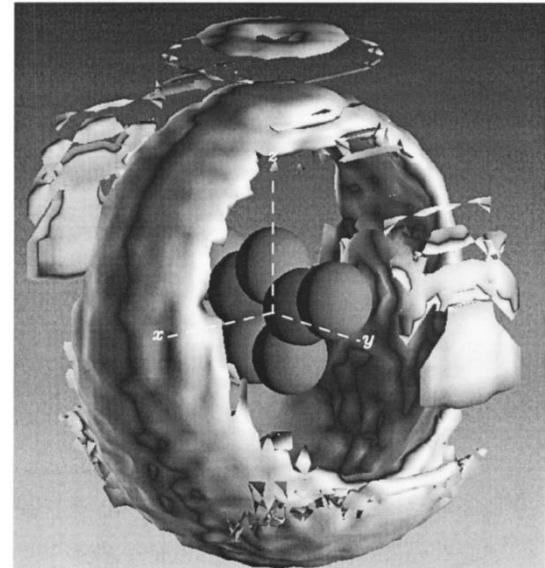
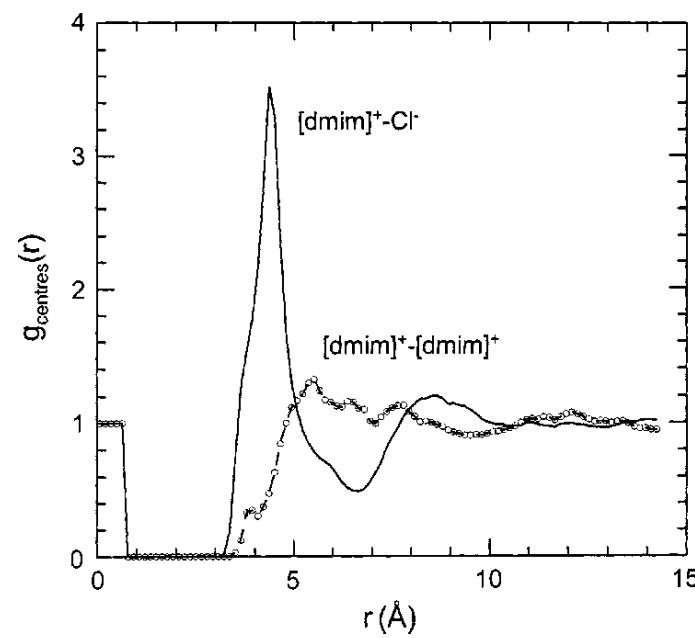
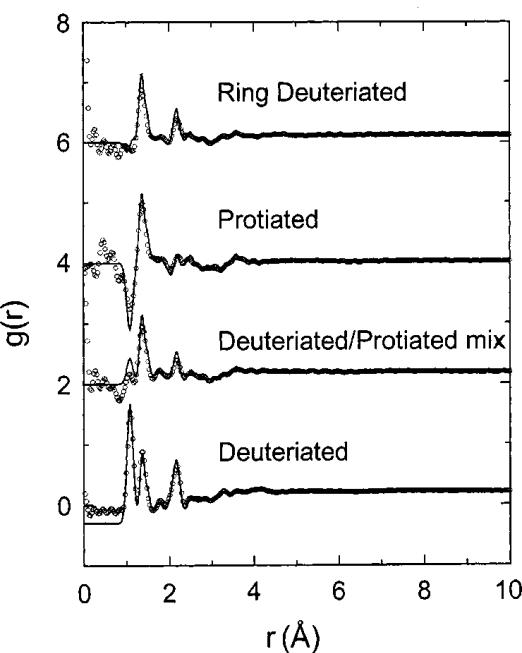




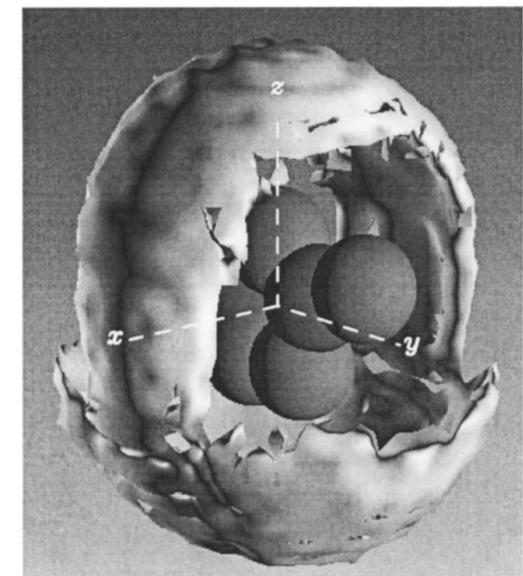
Ionic liquid/ salt melt



Cl⁻

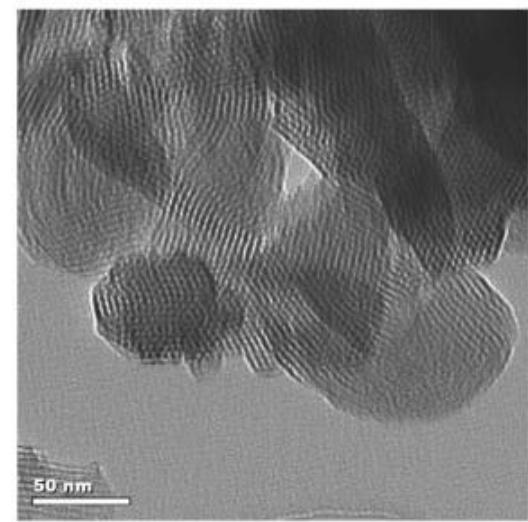


Imidazole-imidazole distance

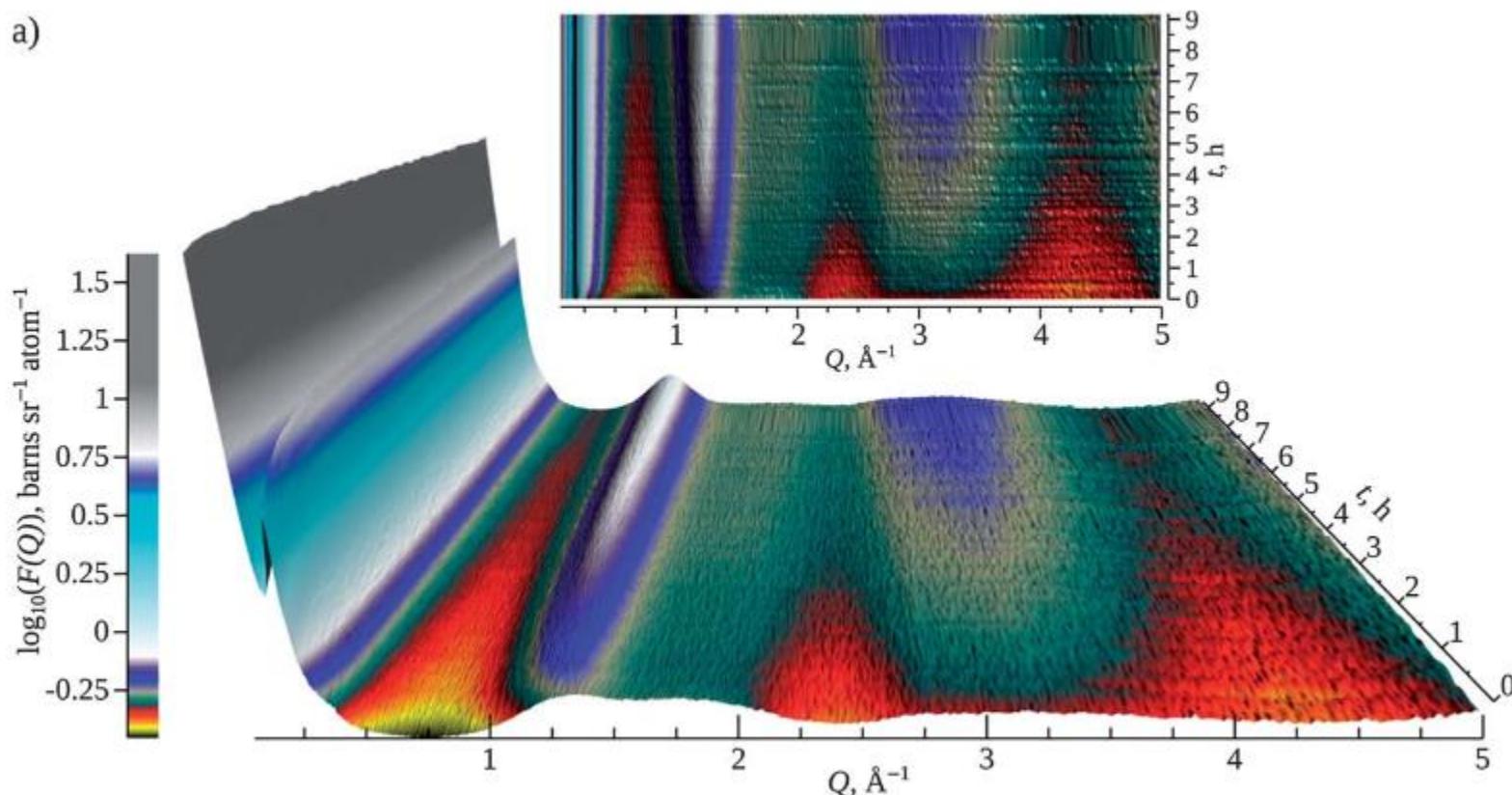


Imidazole-chloride distance

Heterogeneous catalysis



a)



Conclusion

- Wide angle scattering gives atomic resolution data
- Potential samples are
 - disordered crystalline materials
 - Nanomaterials
 - Solutions
 - Glasses
- Using WANS and WAXS in combination with Monte Carlo simulation can give a structural snap-shot of the disordered phase