# Neutron Compton Scattering

**Giovanni Romanelli – Università degli Studi di Roma Tor Vergata**

12° of September 2022 - 17° Oxford School on Neutron Scattering

# Neutron scattering off hydrogen in the Chadwick experiment:

*Elastic? Quasi-elastic? Inelastic? Else?*

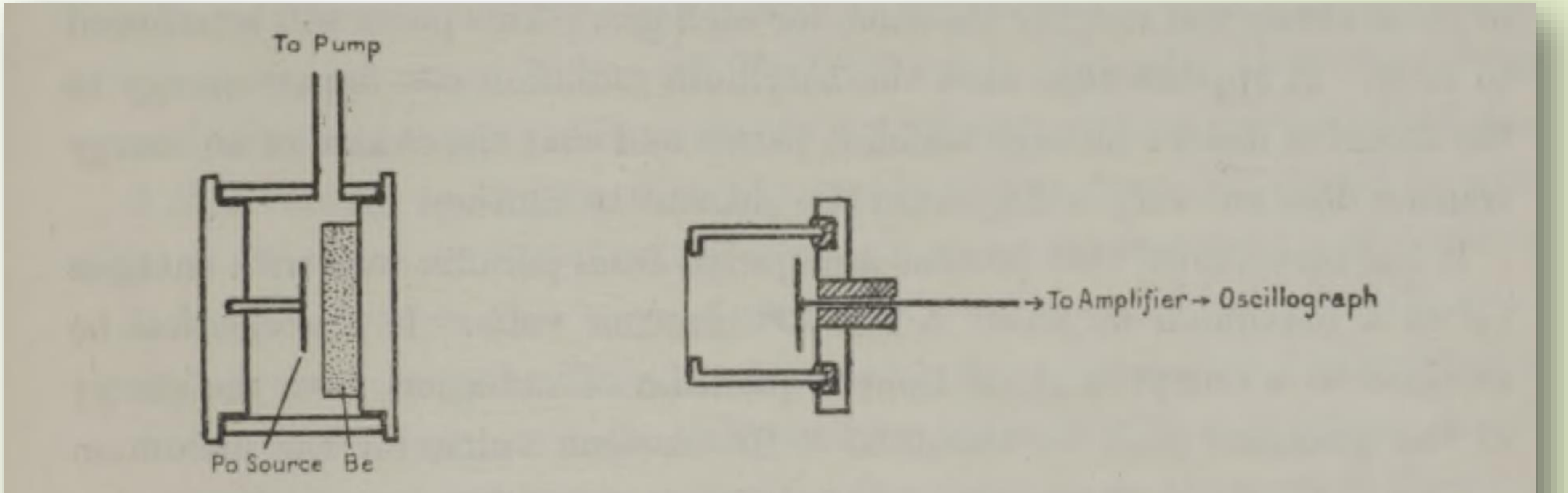


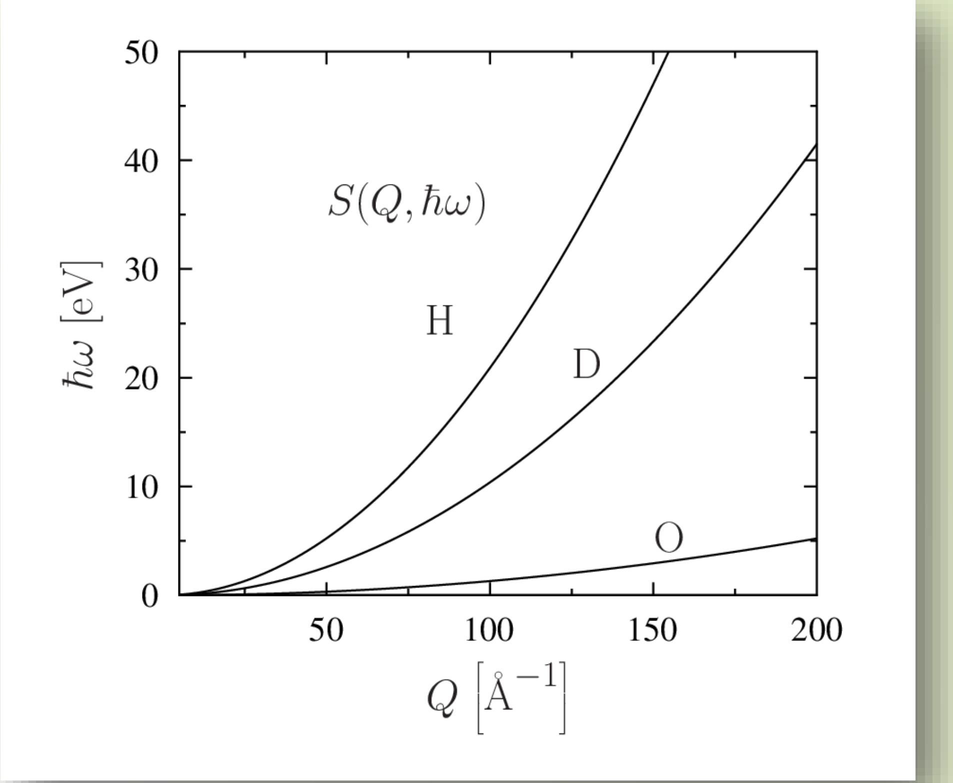
FIG. 1.

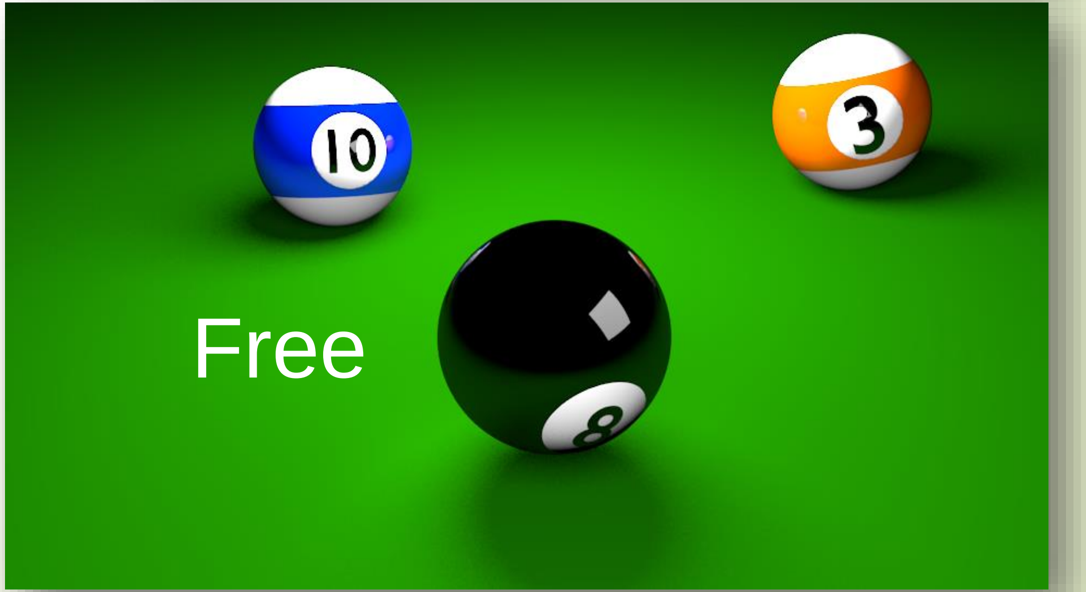
# The Impulse Approximation

$$k_0 + p = k_1 + p'$$

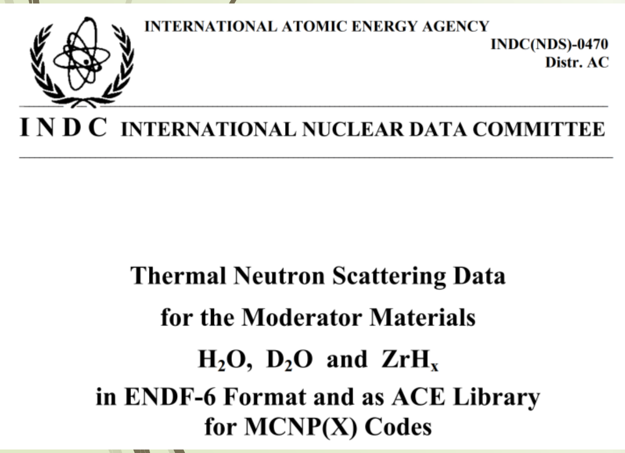
$$E_0 + \frac{p^2}{2M} = E_1 + \frac{p'^2}{2M}$$

$$\sigma_{free} = \frac{\sigma_{bound}}{\left(1 + \frac{m}{M}\right)^2}$$





# The total scattering cross section for $ZrH_2$



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I N D C INTERNATIONAL NUCLEAR DATA COMMITTEE

Thermal Neutron Scattering Data
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$H_2O$, $D_2O$ and $ZrH_x$
in ENDF-6 Format and as ACE Library
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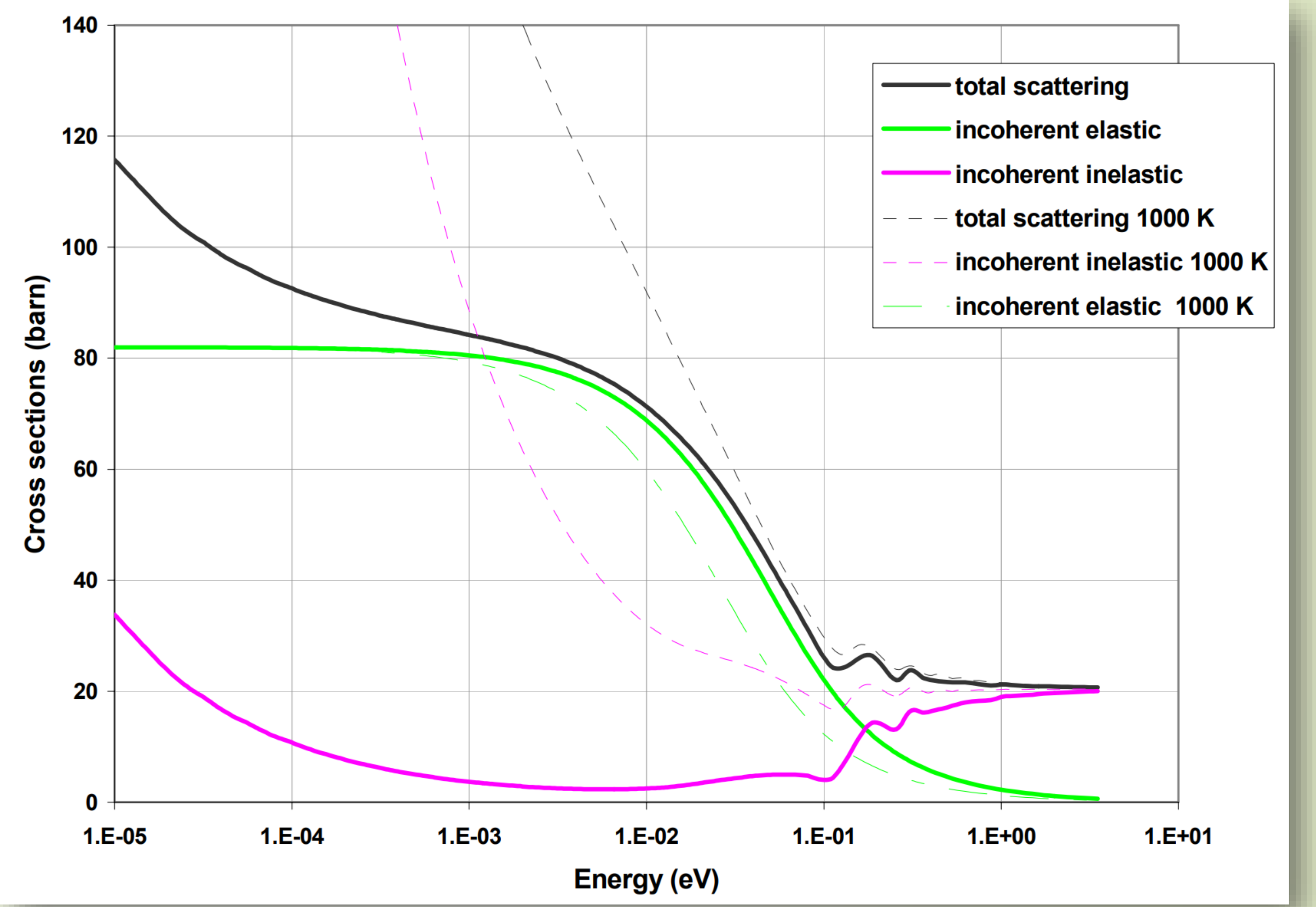
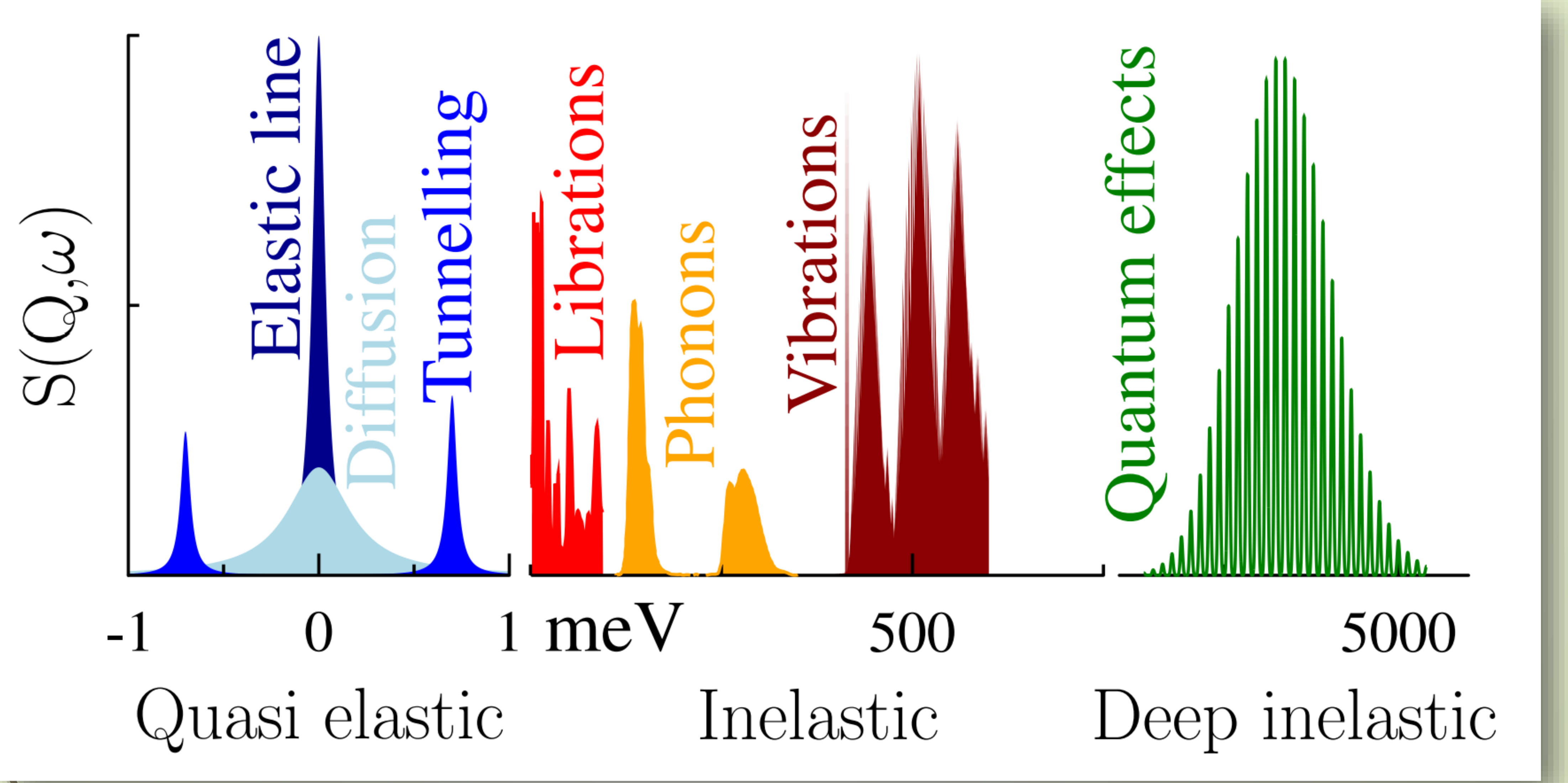


# From «elastic» to «deep inelastic»



# Recoil lines and West scaling variable

Experimental definiton

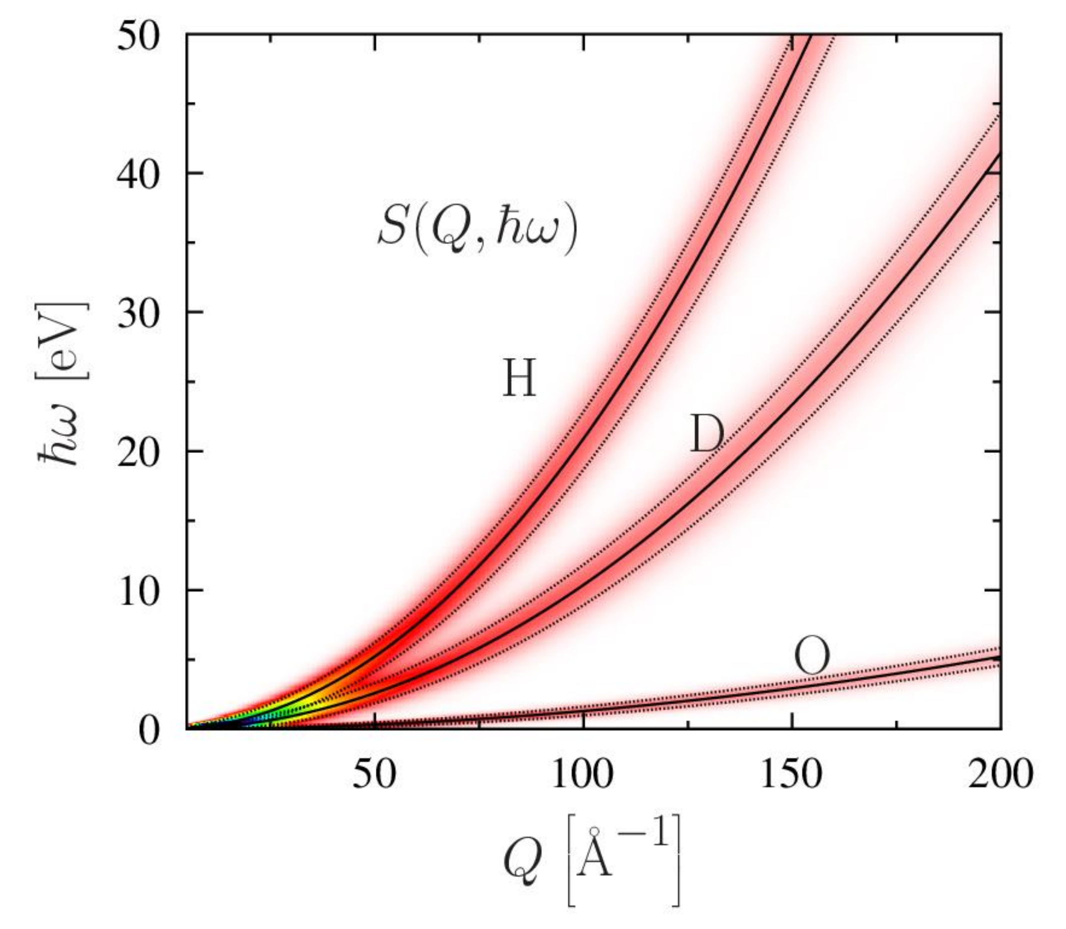
$$y = \frac{M}{\hbar Q}\left(\hbar\omega - \frac{\hbar^2 Q^2}{2M}\right)$$

Physical interpretation

$$\hbar y = \boldsymbol{p} \cdot \widehat{\boldsymbol{Q}}$$

Momentum distribution

$$\langle p^2 \rangle = \sigma^2 = \int p^2 n(\boldsymbol{p}) d\boldsymbol{p}$$



# Some useful formulas for the Compton profile

$$S_{IA}(\mathbf{q},\omega) = \frac{M}{\hbar q} J(y,\hat{q})$$

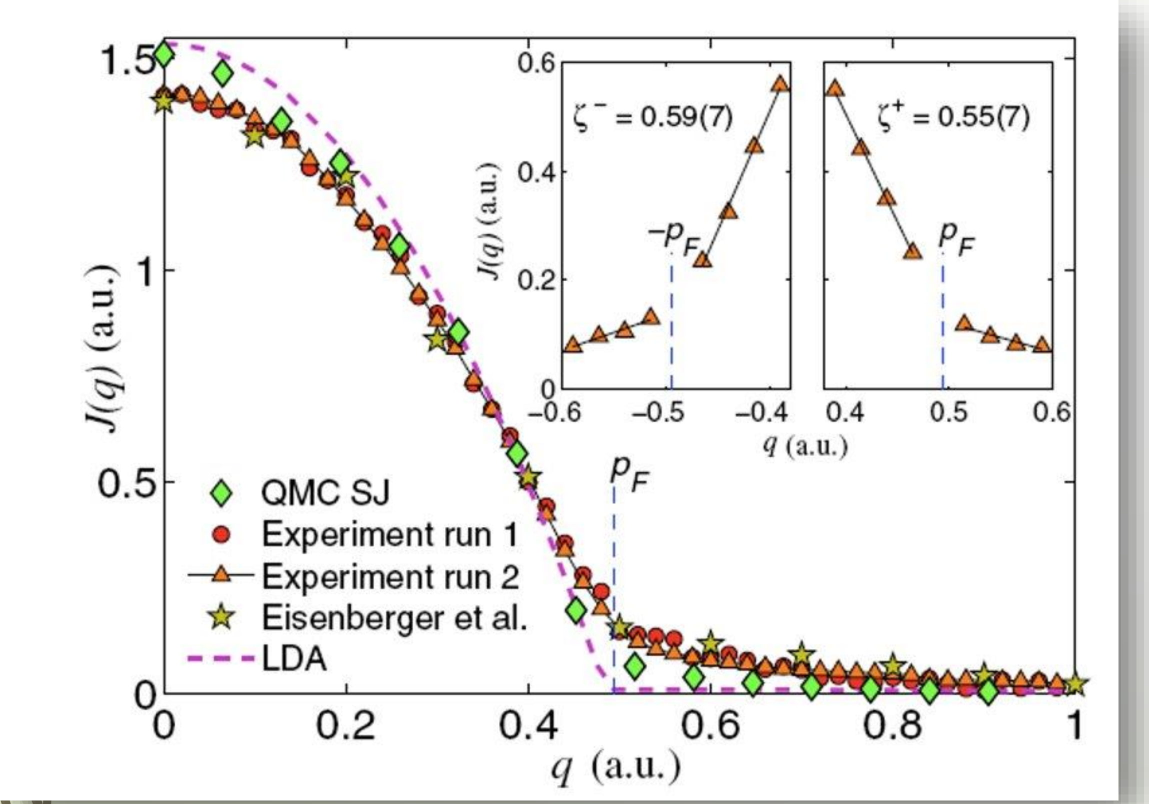
$$J(y,\hat{q}) = \hbar \int n(\mathbf{p}')\delta(\hbar y - \mathbf{p}'\cdot\hat{q})\, d\mathbf{p}'$$

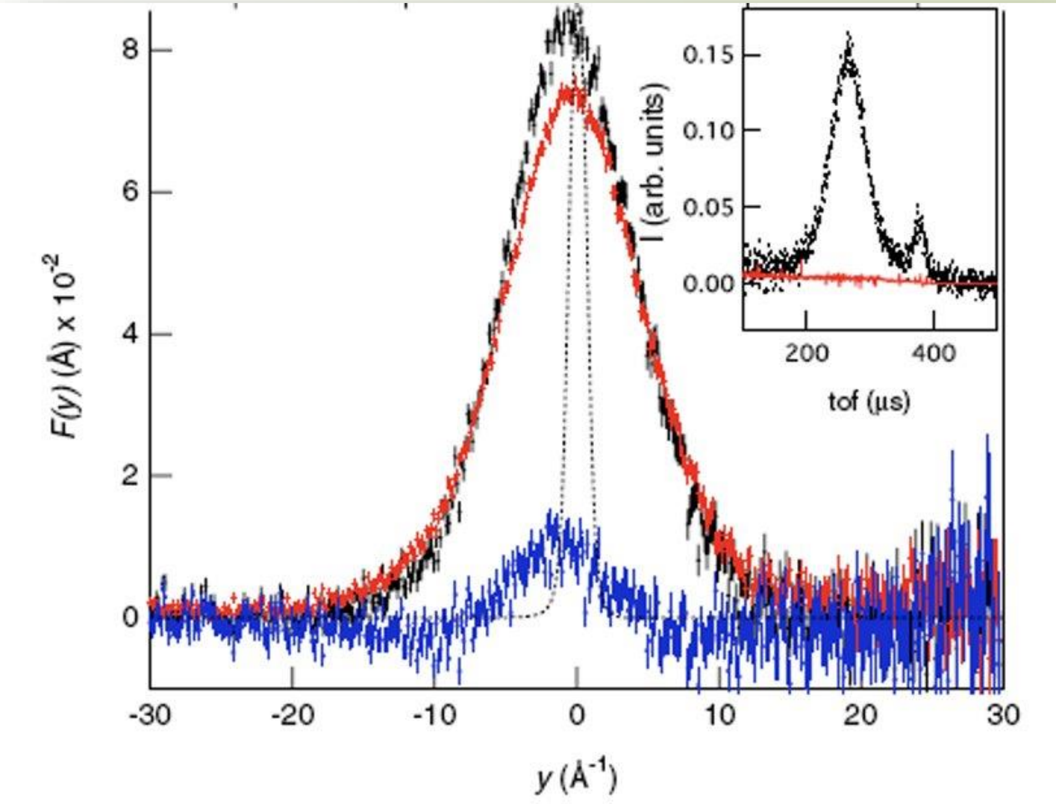
$$J(y) = 2\pi\hbar \int_{|\hbar y|}^{\infty} p\, n(p)\, dp$$

# Different types of Compton scattering

Electrons (ID16, ESRF)

Protons (VESUVIO, ISIS)

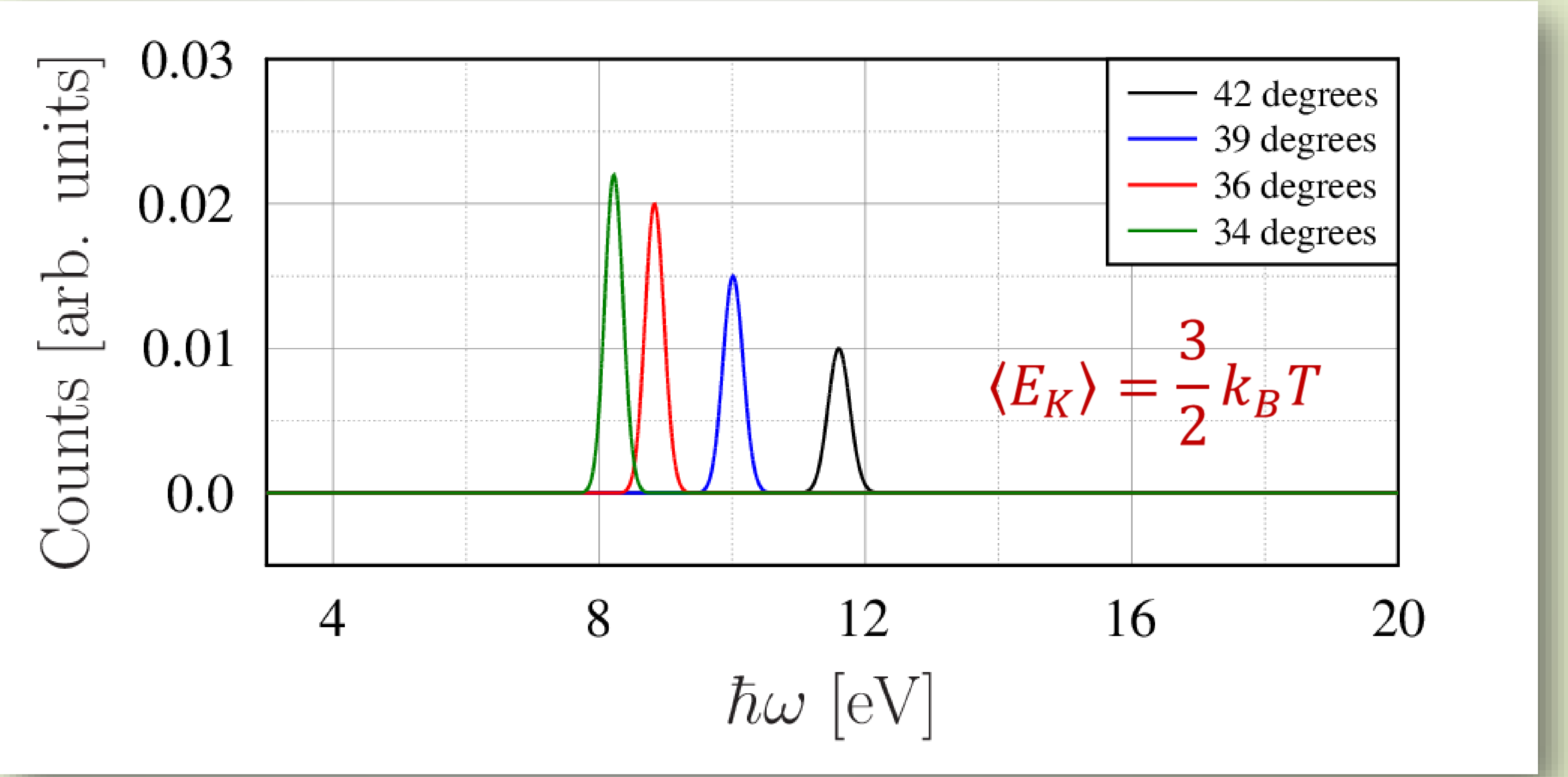




Compton profile of Na; p- resolution≈13% FWHM
S. Huotari et al, PRL 105, 086403 (2010)

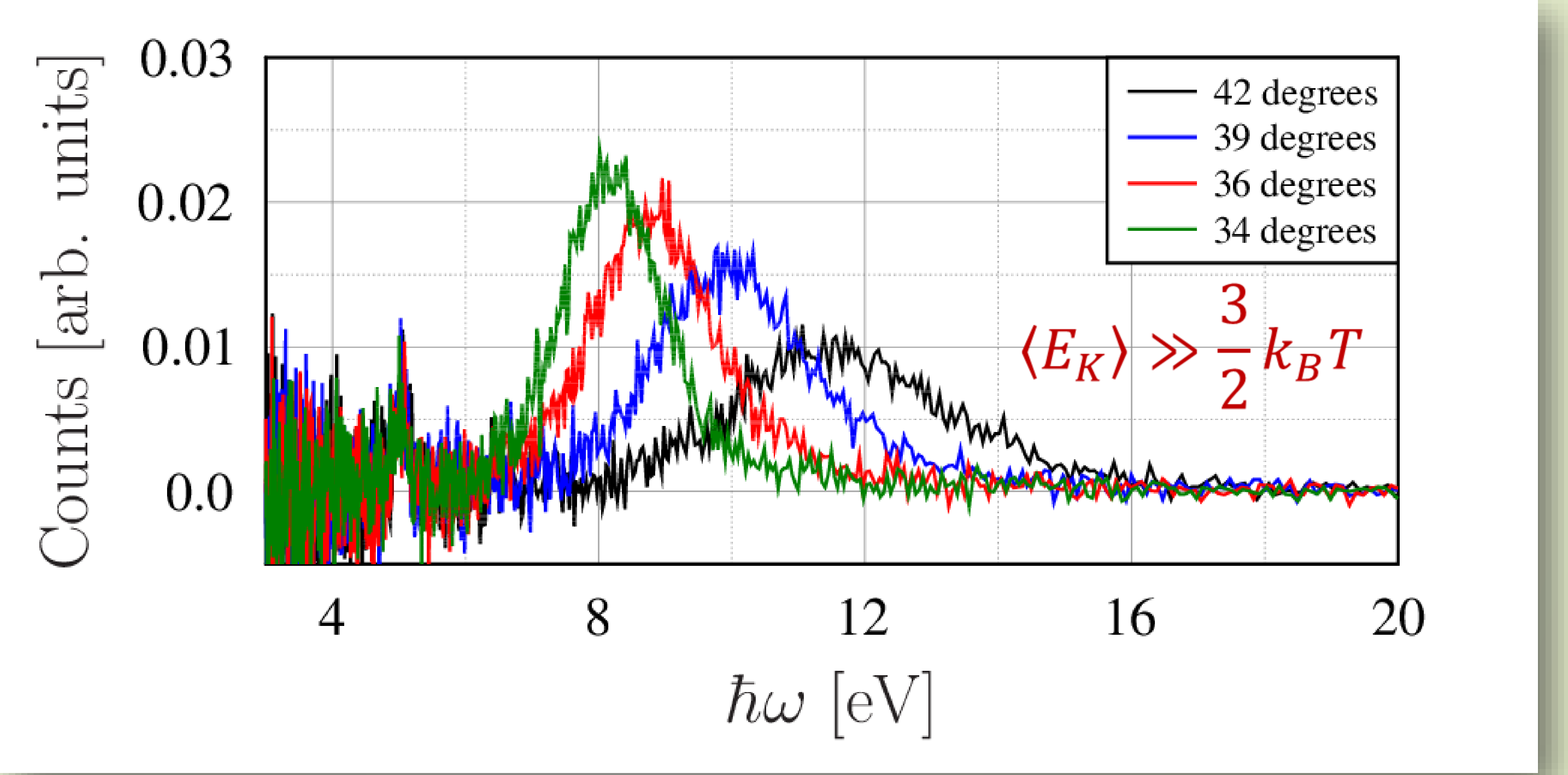
Neutron Compton profile of water; p-resolution≈14%
A. Pietropaolo et al, PRL 100, 127802 (2008)

# Scattering from classical particles



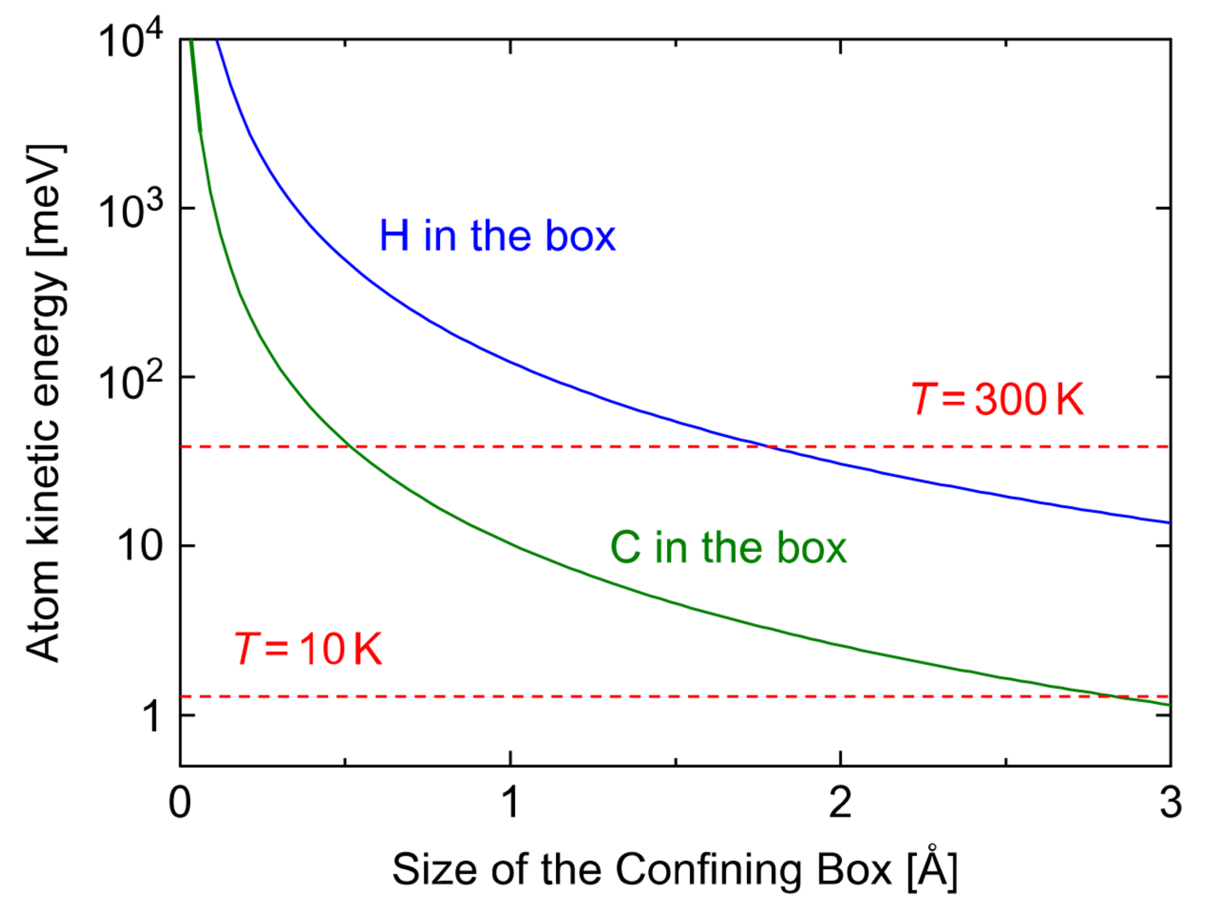
$$\langle E_K \rangle = \frac{3}{2} k_B T$$

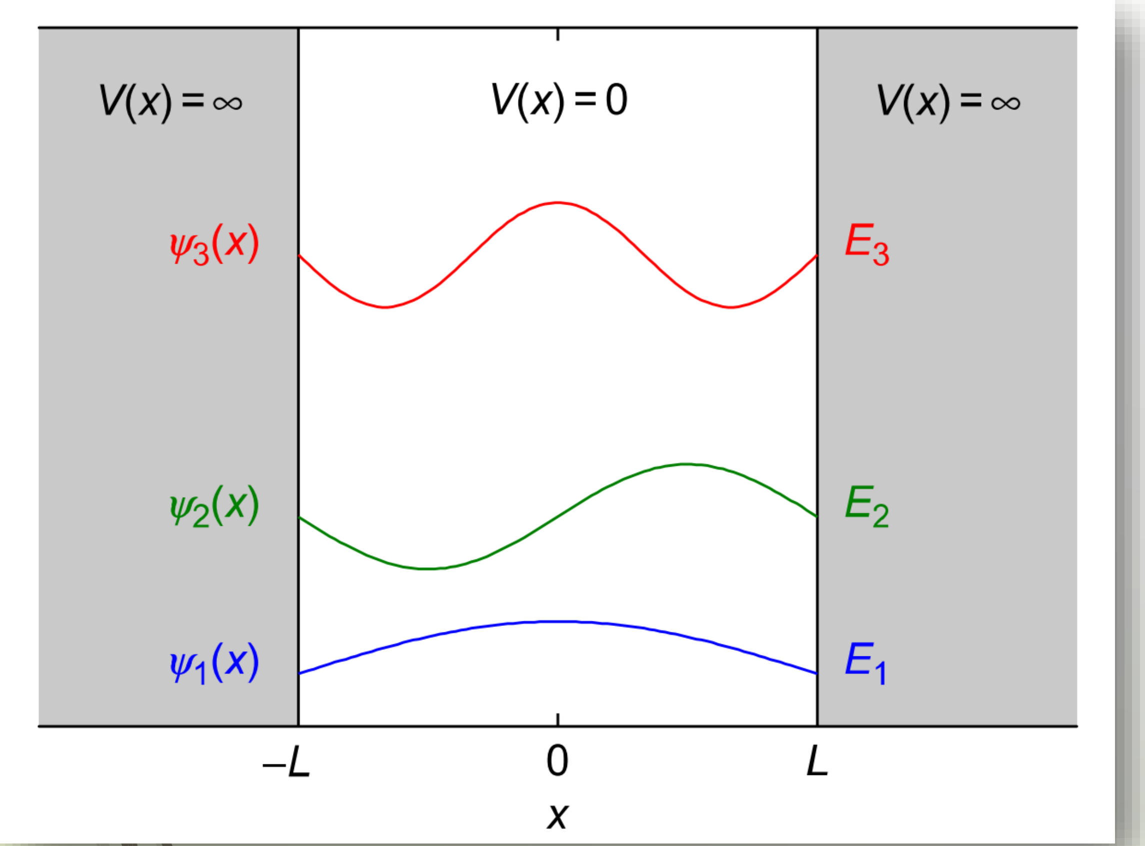
# Scattering from quantum particles



$$\langle E_K \rangle \gg \frac{3}{2} k_B T$$

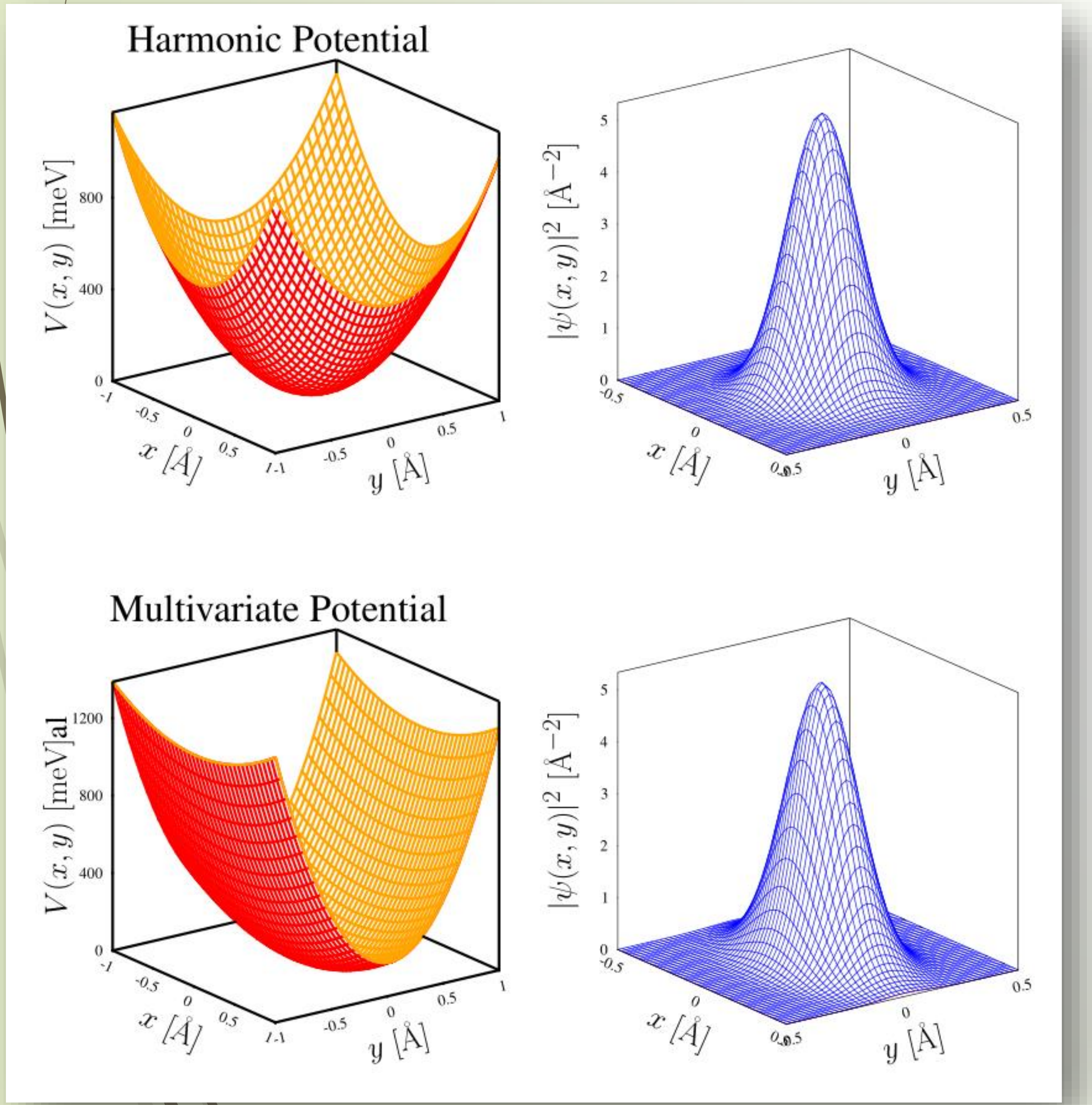
# Zero-point energy and local potential

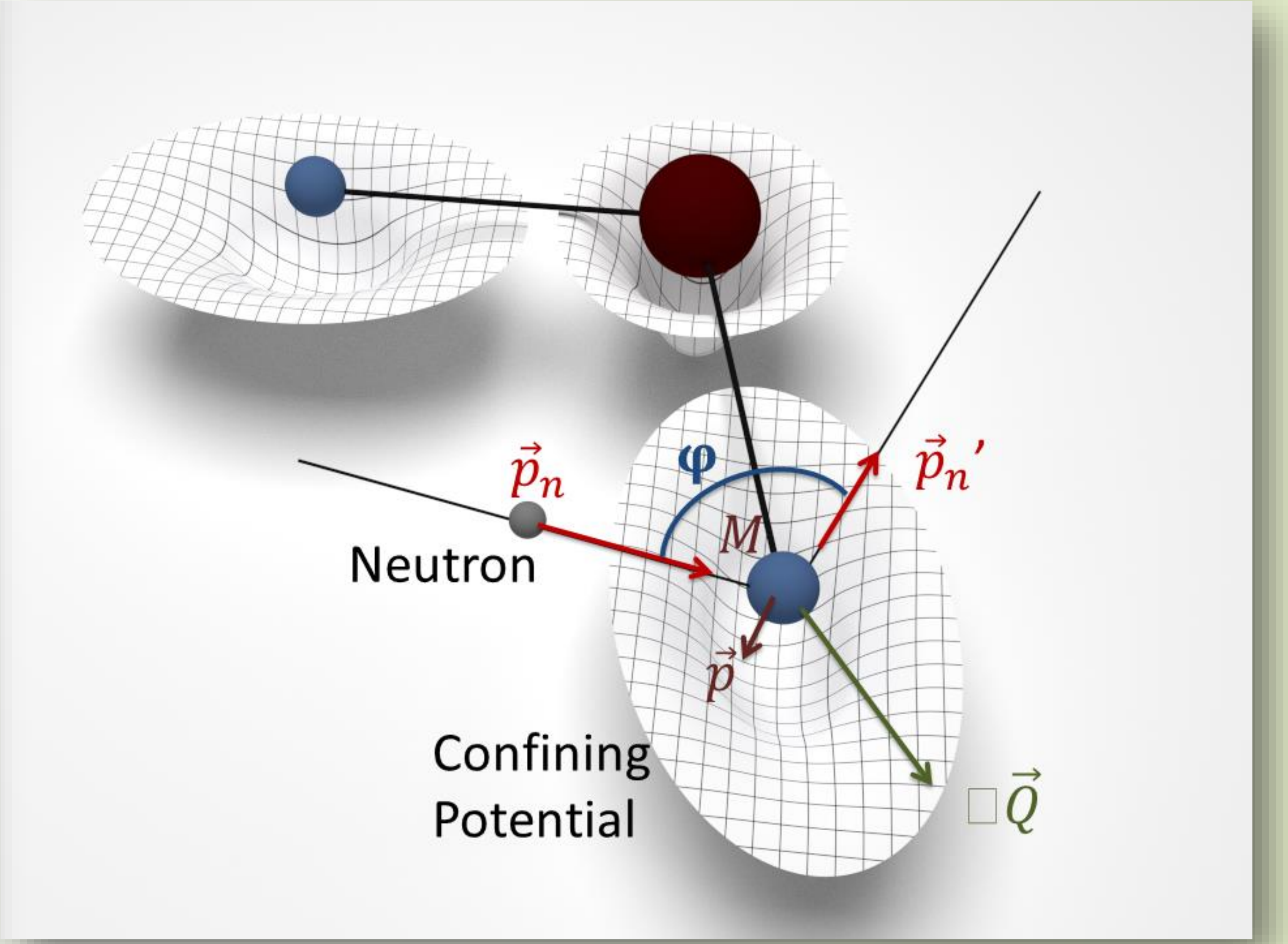




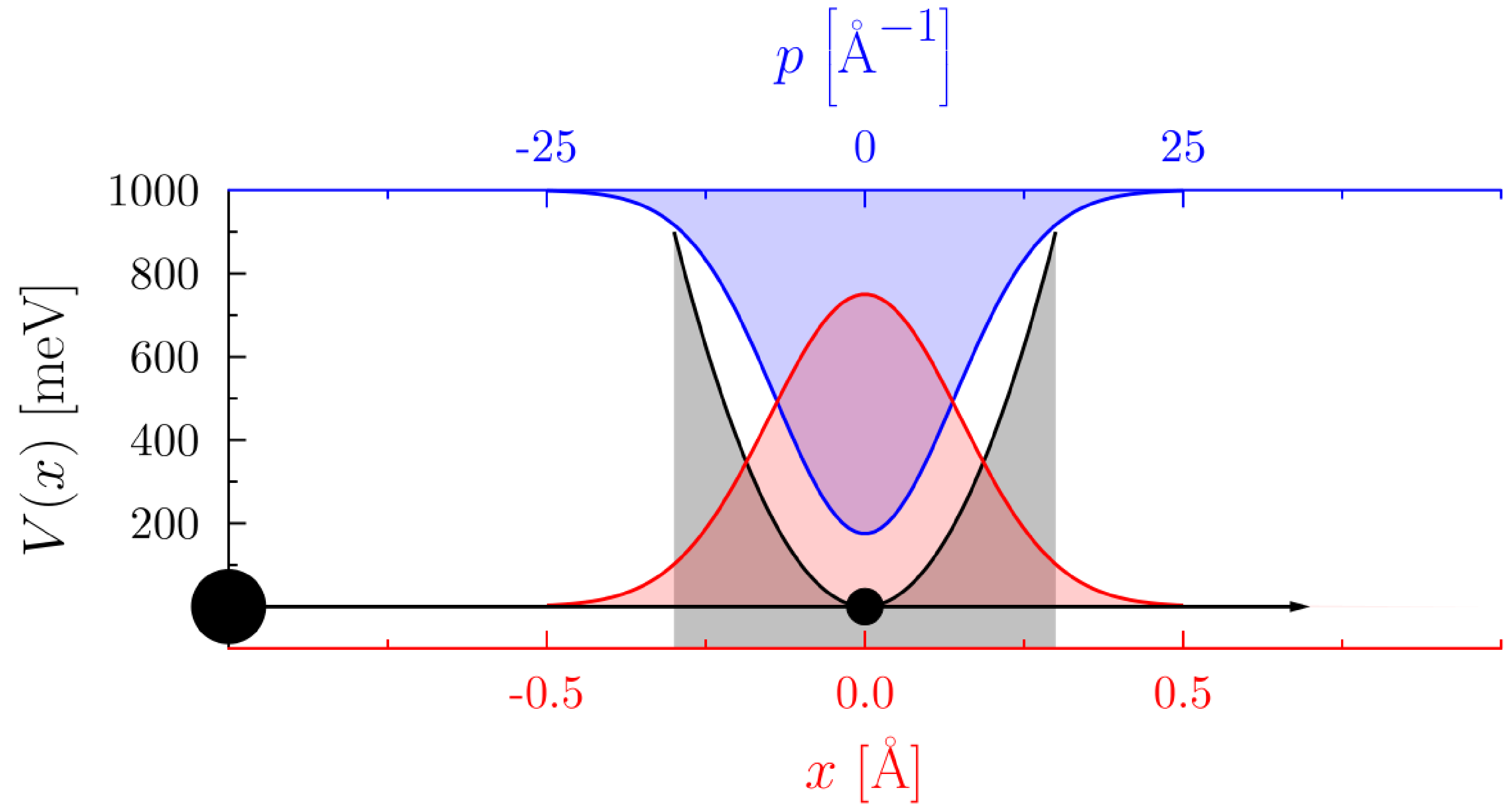
$$\mathcal{E}_n = \hbar\omega_n = \frac{n^2\pi^2\hbar^2}{2ML^2}.$$

# Probing the anisotropy of the local potential





# Delocalisation in real and momentum spaces



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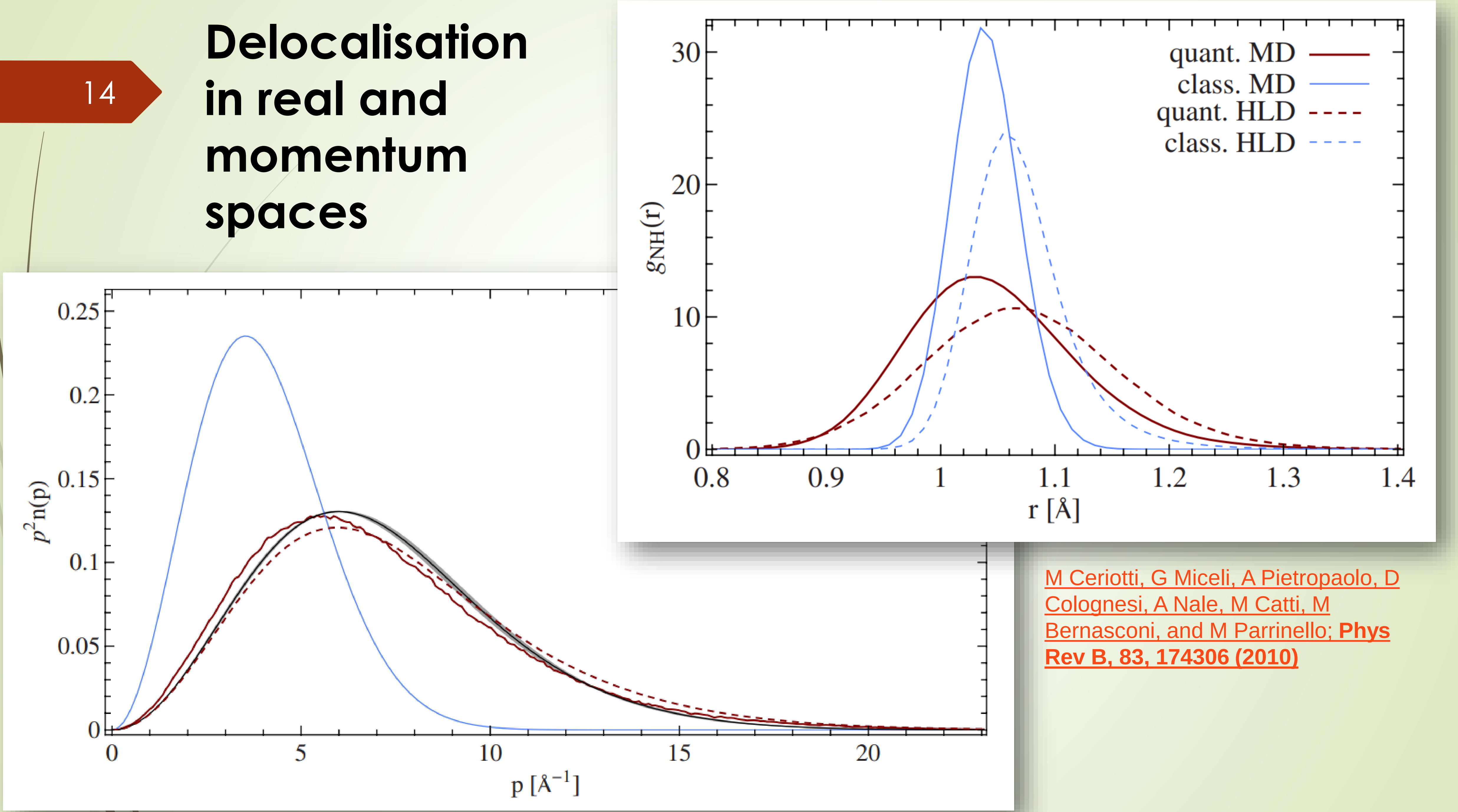
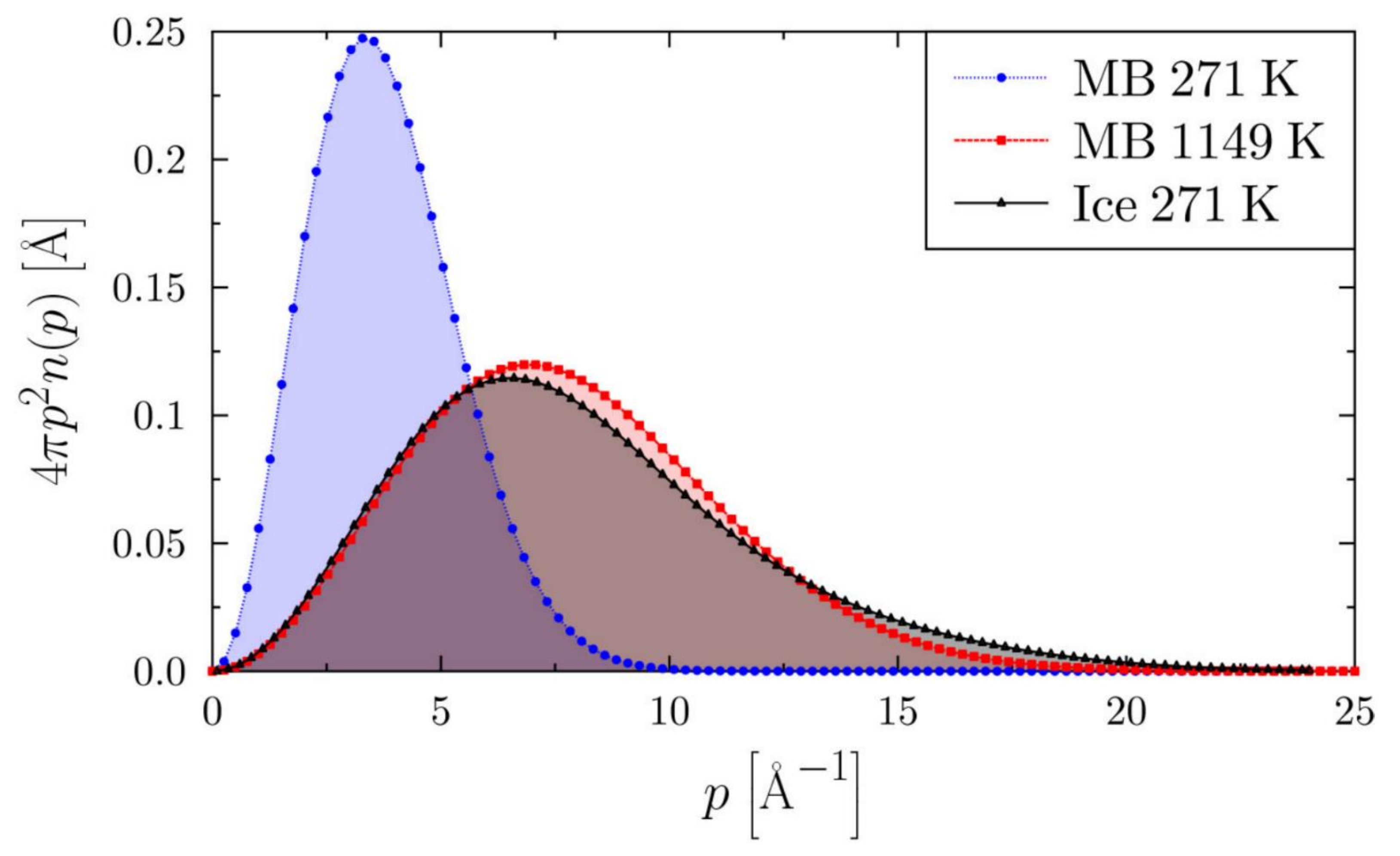


M Ceriotti, G Miceli, A Pietropaolo, D Colognesi, A Nale, M Catti, M Bernasconi, and M Parrinello; **Phys Rev B, 83, 174306 (2010)**

# Anisotropy of the local potential

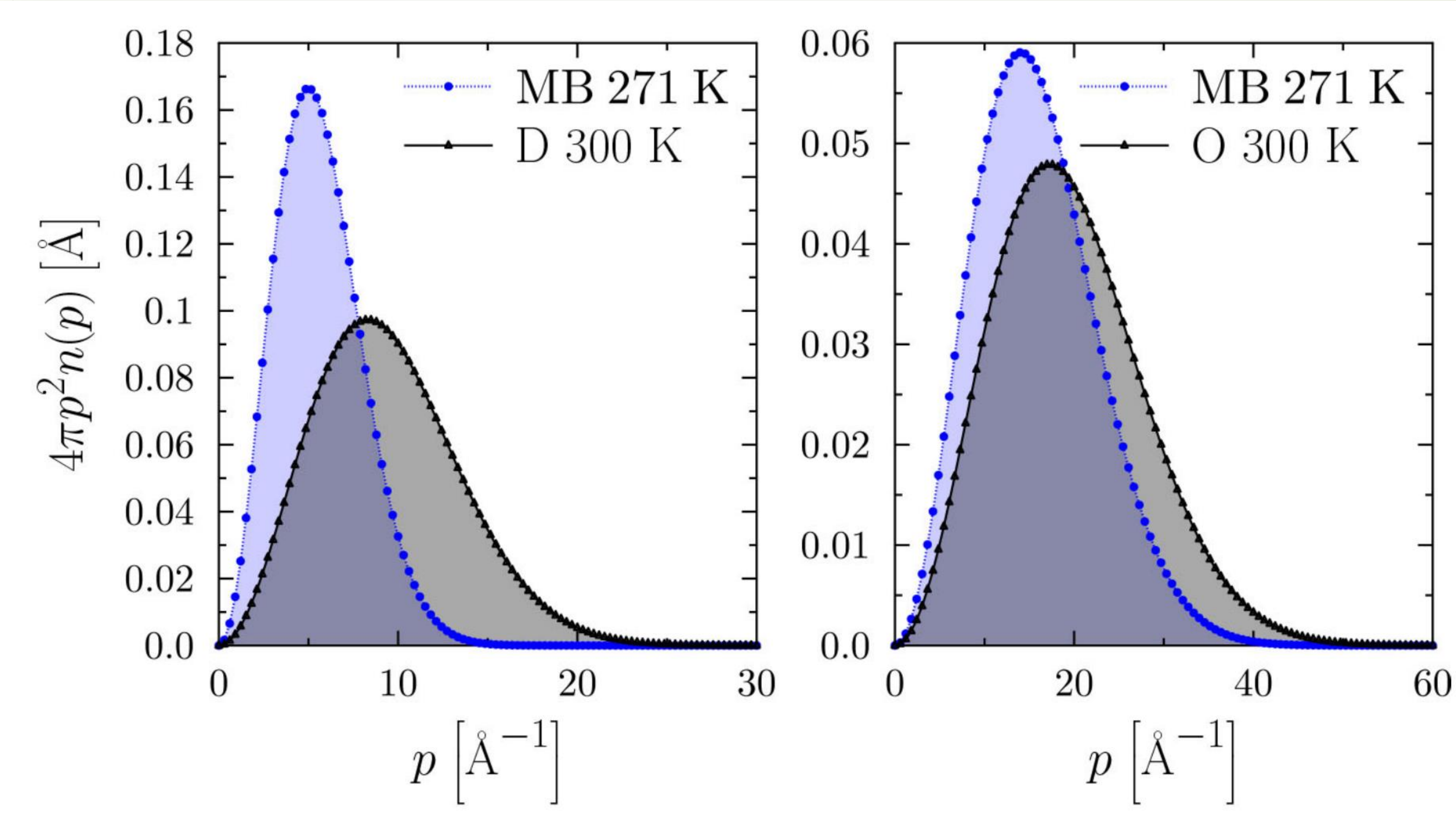
$$n(\boldsymbol{p}) = exp\left(-\frac{p_x^2}{2\sigma_x^2} - \frac{p_x^2}{2\sigma_x^2} - \frac{p_x^2}{2\sigma_x^2}\right)$$



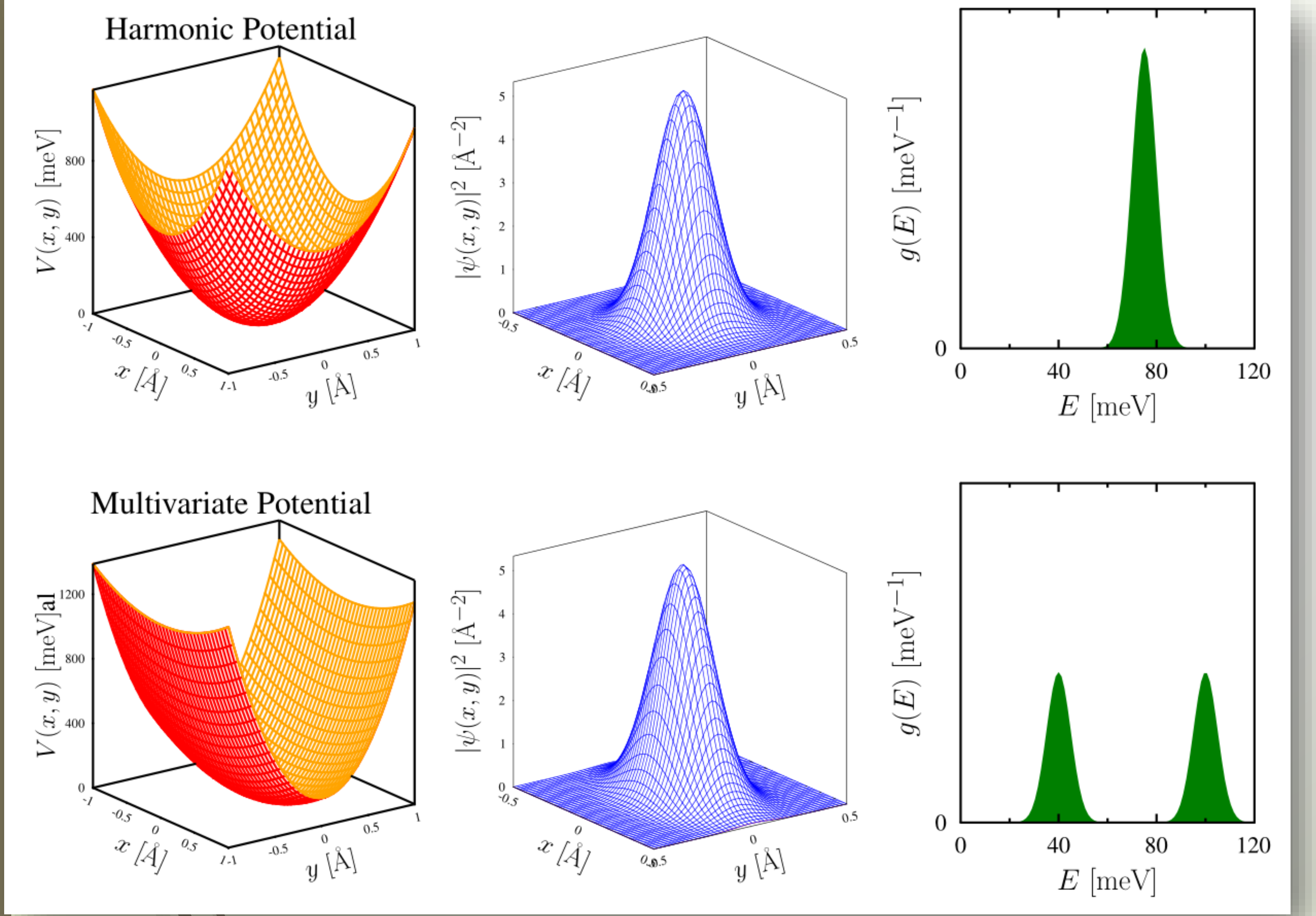
C Andreani, R Senesi, M Krzystyniak, G Romanelli, F Fernandez-Alonso; Rivista del Nuovo Cimento, 291-340 (2018)

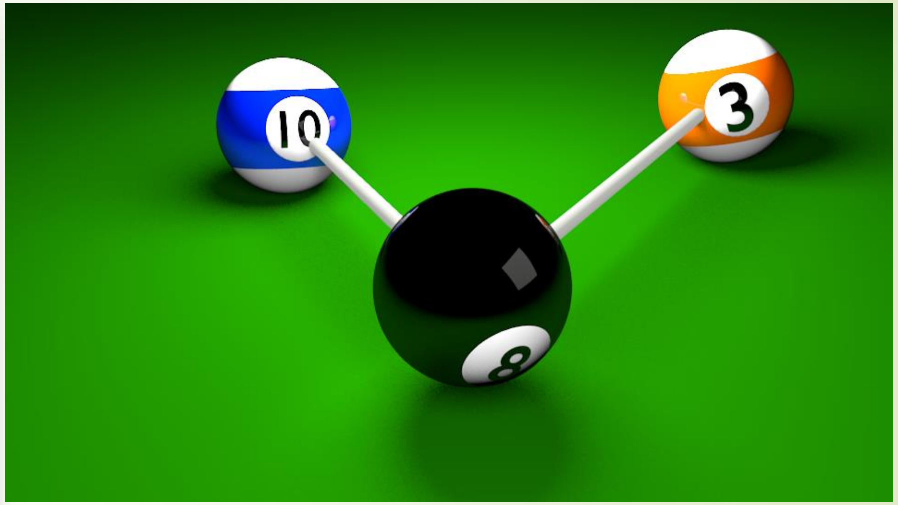
# Nuclear kinetic energy for heavier masses

C Andreani, R Senesi, M Krzystyniak, G Romanelli, F Fernandez-Alonso; Rivista del Nuovo Cimento, 291-340 (2018)



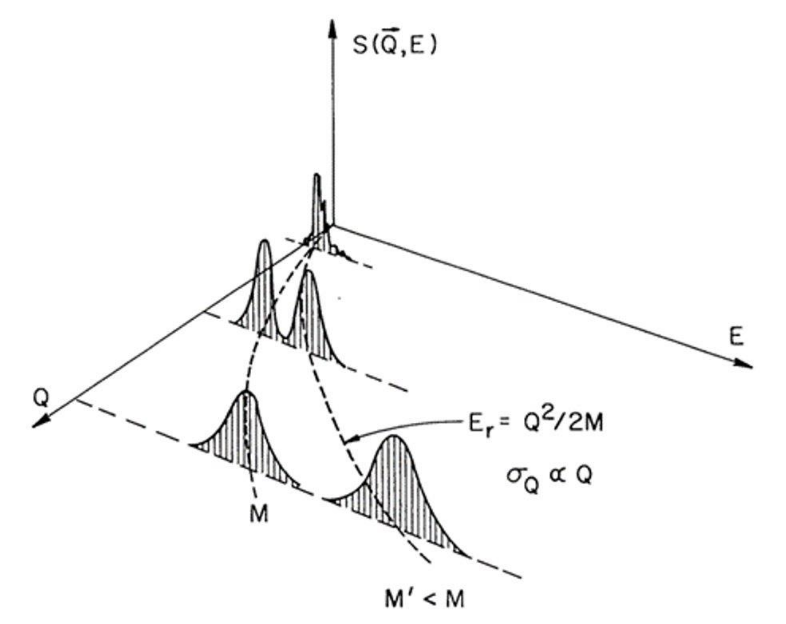
# Nuclear kinetic energy from vibrational modes

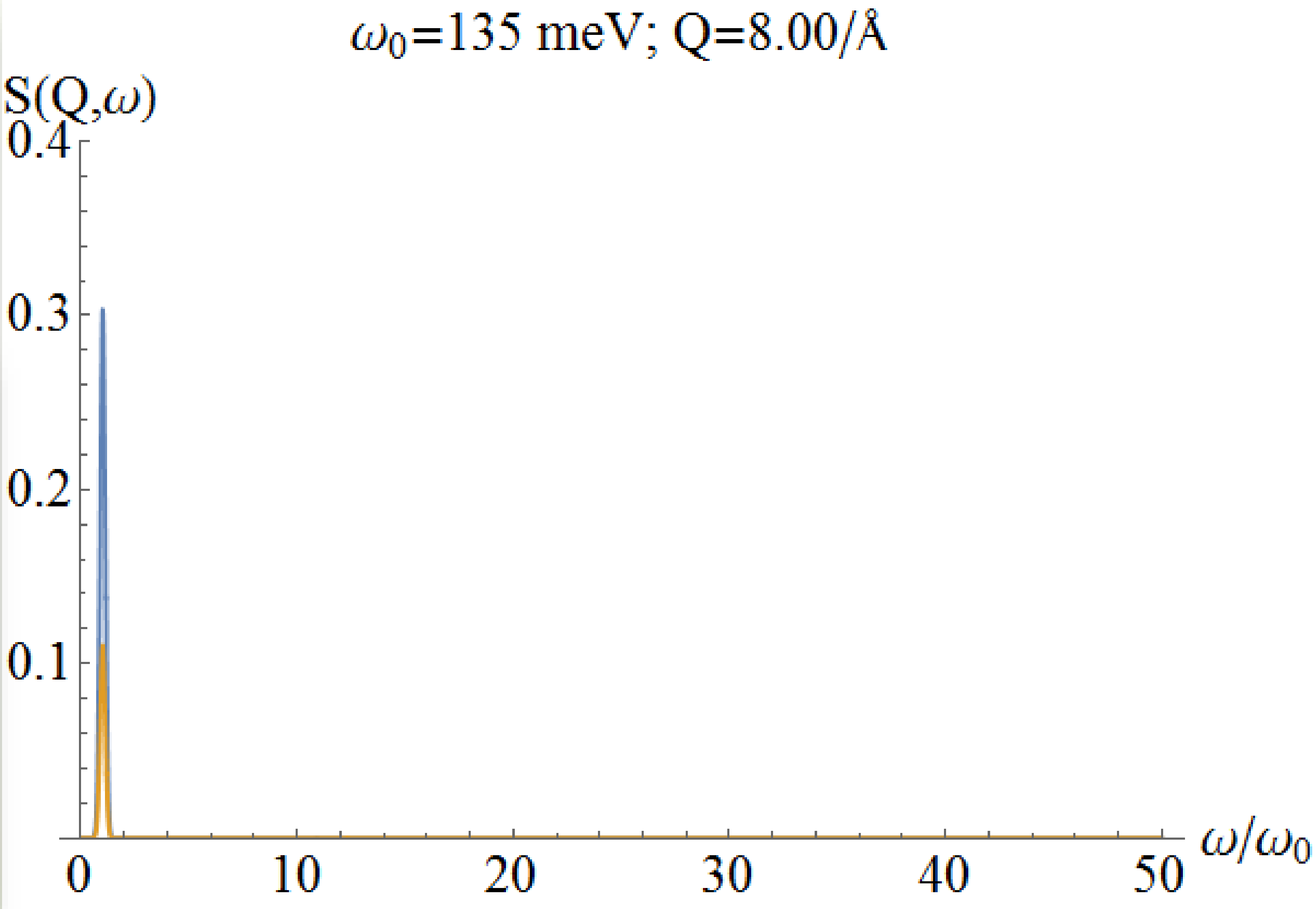




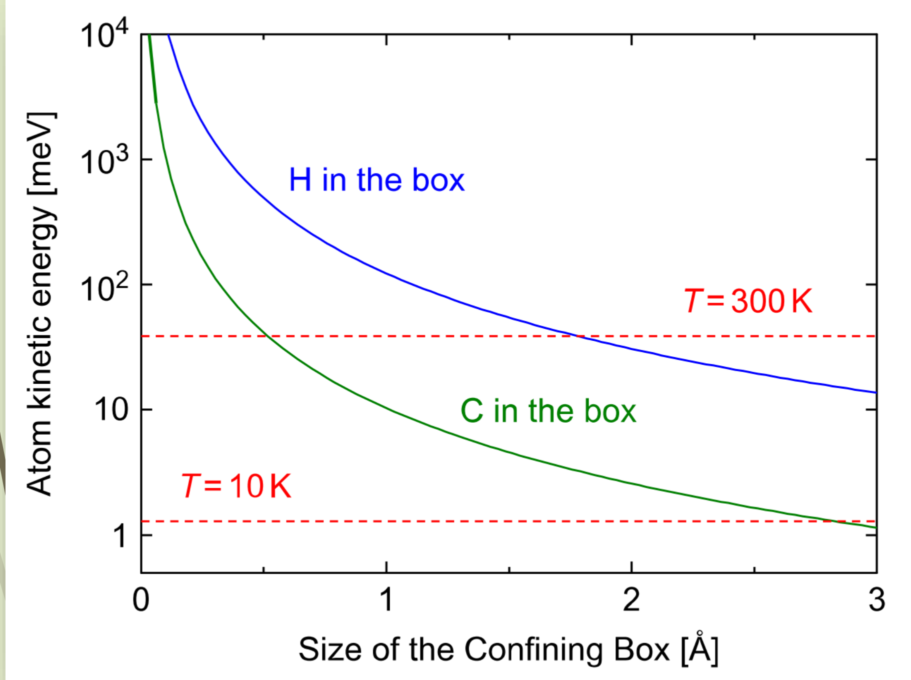
$$\frac{\hbar^2\sigma^2}{2M} = 3\int \frac{\hbar\omega}{4}\, g_M(\omega)\, coth\left(\frac{\hbar\omega}{2k_BT}\right)$$

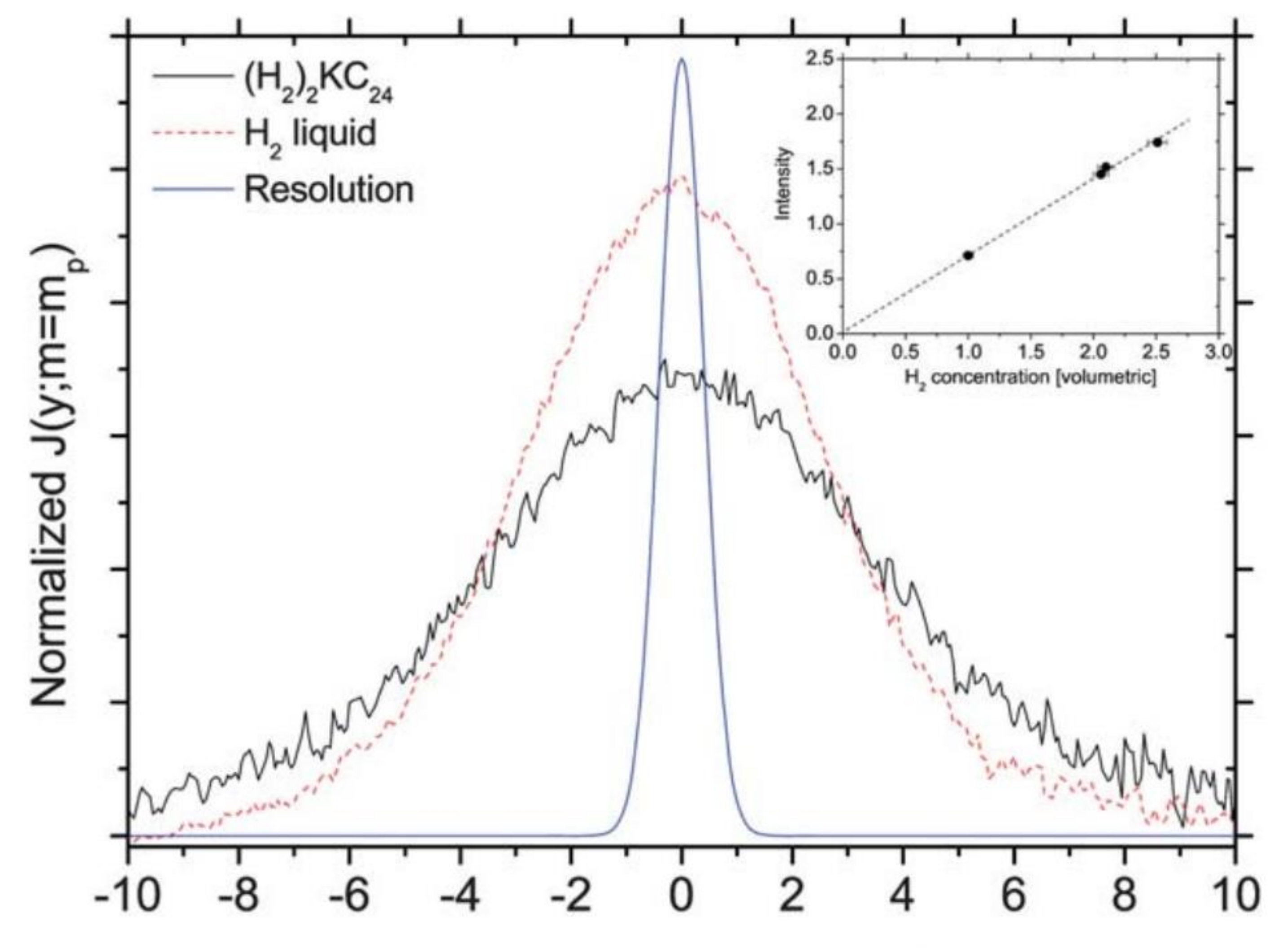
# Multi Phonon Expansion towards the IA





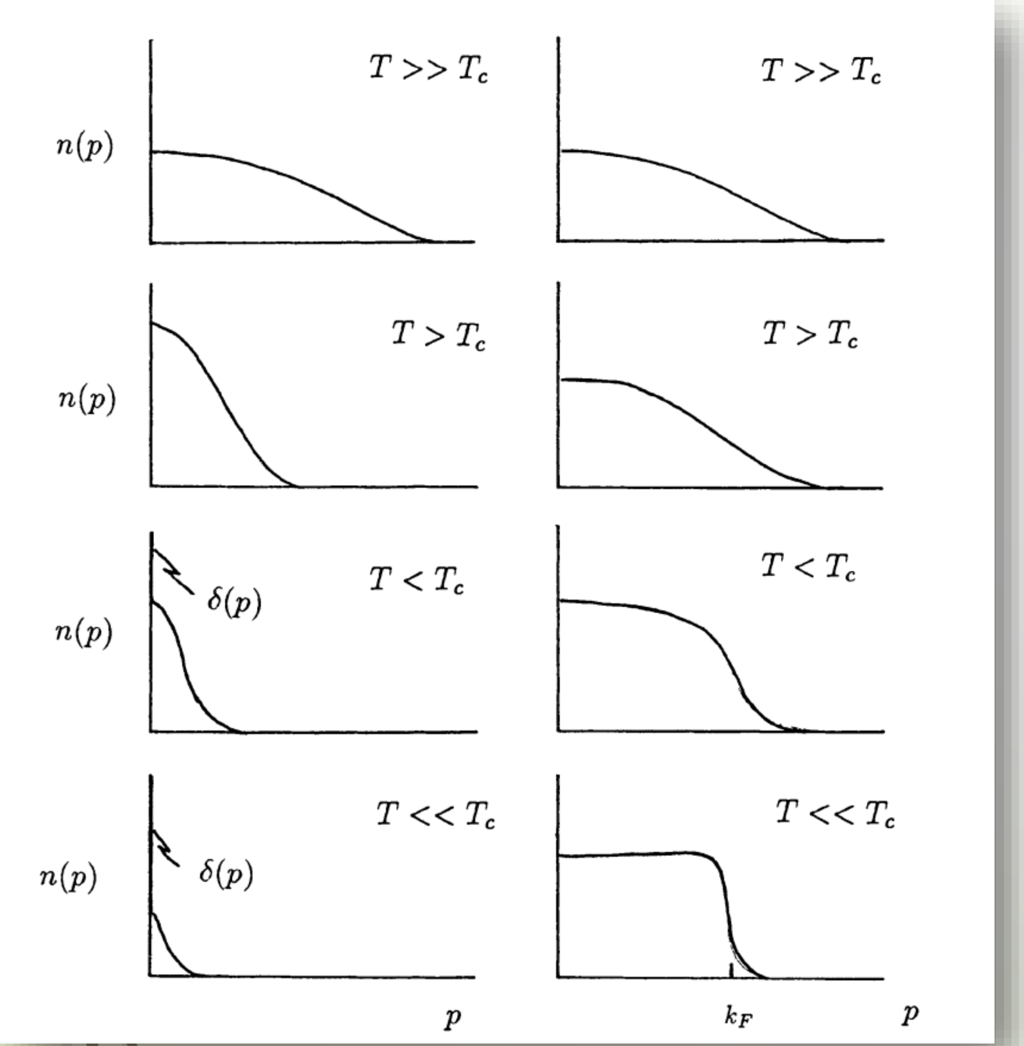
# Effect of confinement





*M Krzystyniak, MA Adams, A Lovell, NT Skipper, SM Bennington, J Mayers, and F Fernandez-Alonso; Faraday Discussions 151 (2011)*

# Effect of statistics: Bose/Einstein vs Fermi/Dirac

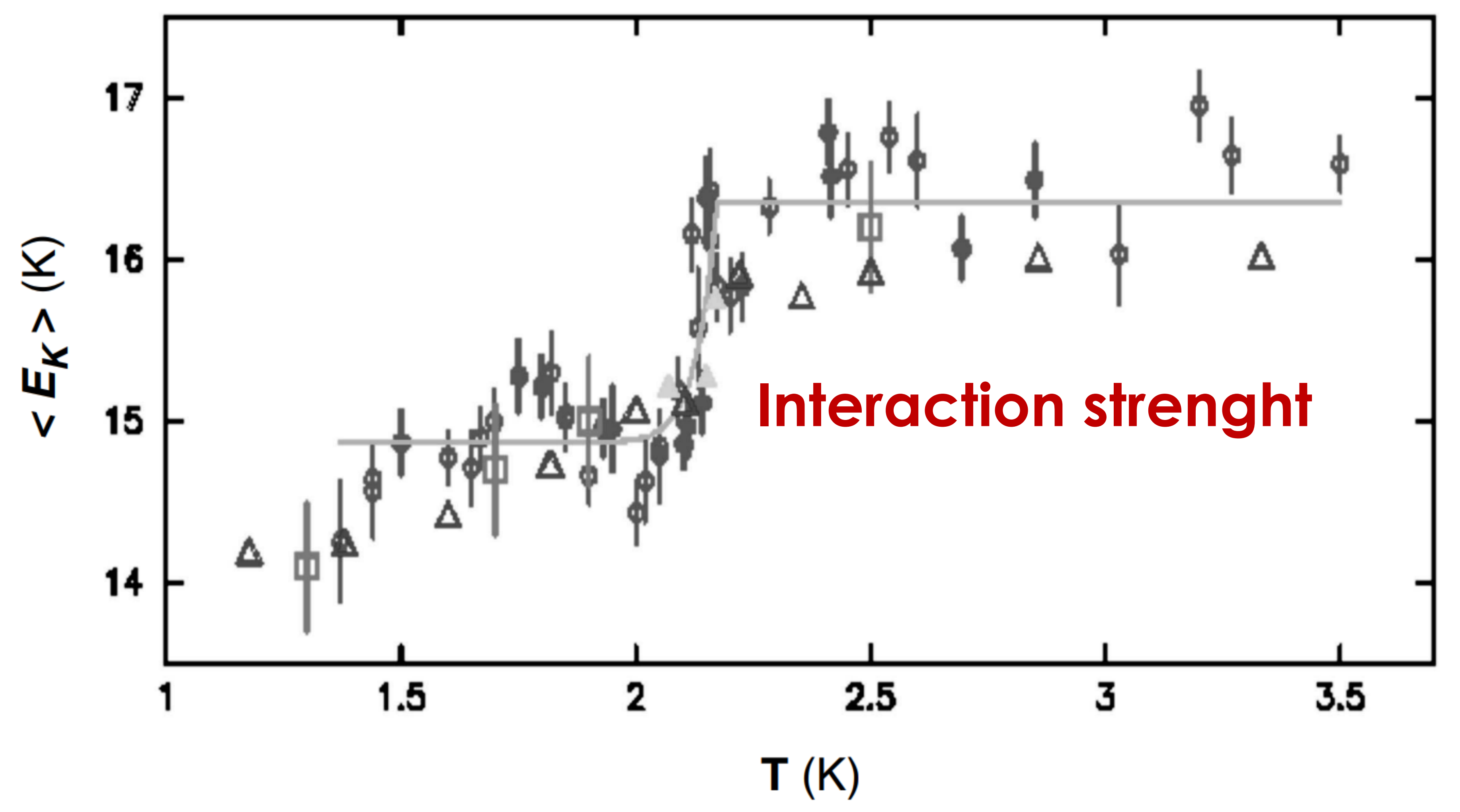


At low temperature, a gas of identical Bosons tends to condensate in the minimum energy state (p=0).

On the other hamd, a gas of identical fermions tends to occupy all available states (exclusion principle) up to a Fermi energy state.

*Momentum Distributions. R. N. Silver and P. E. Sokol Eds. Springer 1988*

# Effect of statistics: $^4$He (Bosons) below $T_c$

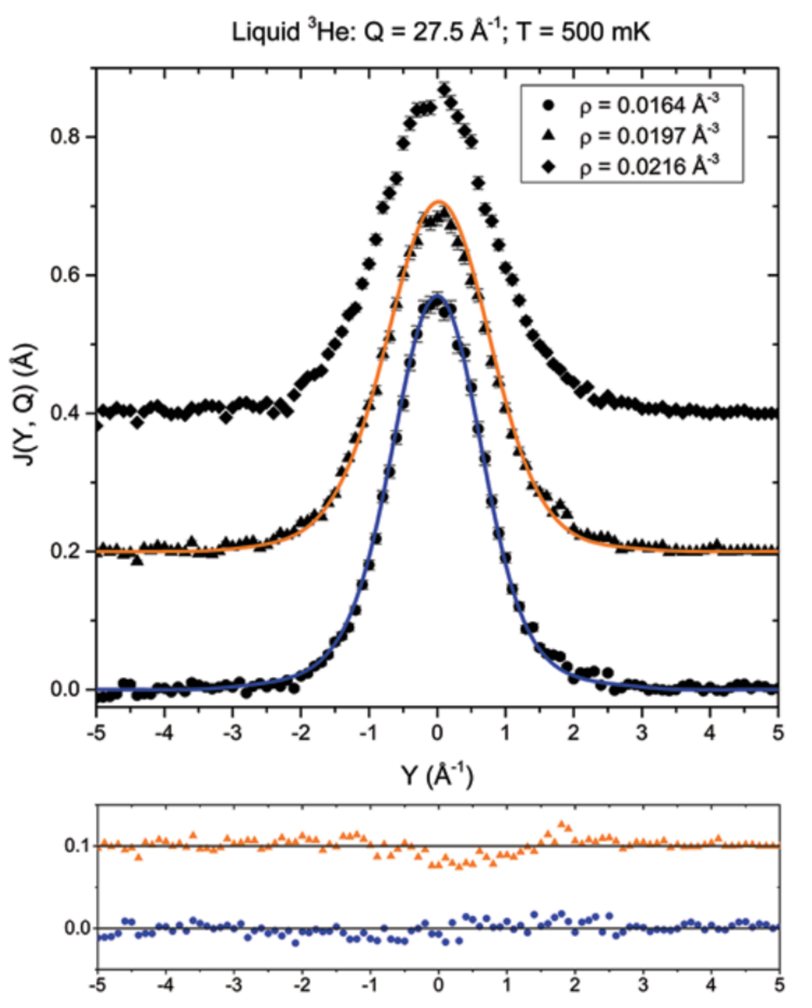


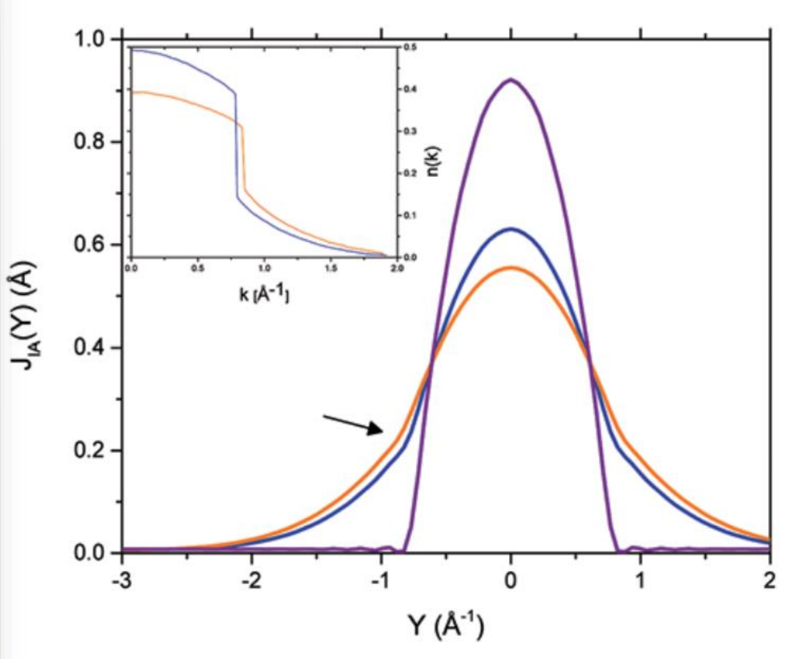
Interaction strenght

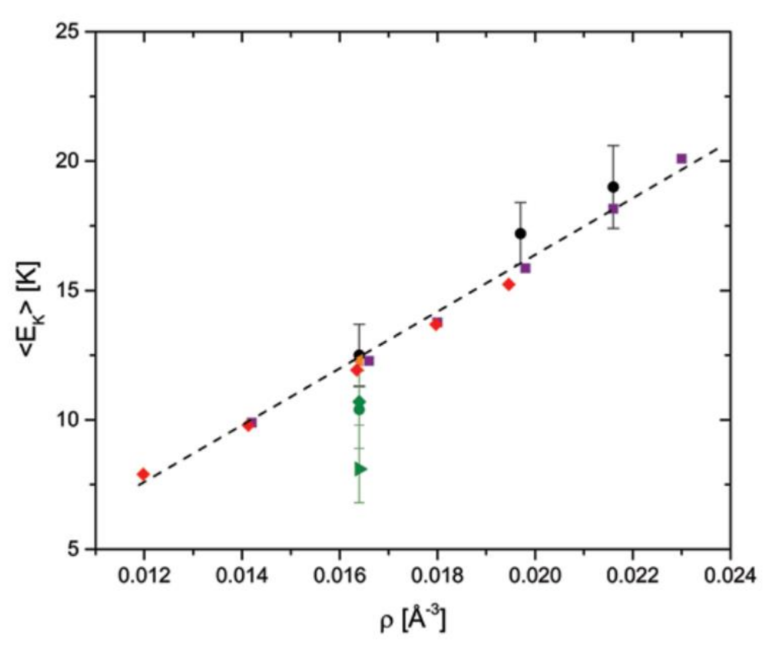
C Andreani, D Colognesi, J Mayers, G F Reiter, and R Senesi; Advances in Physics, (2005)

# Effect of statistics: $^3$He (Fermions) below $T_c$

$$J(y) = 2\pi\hbar \int_{|\hbar y|}^{\infty} p\, n(p)\, dp$$

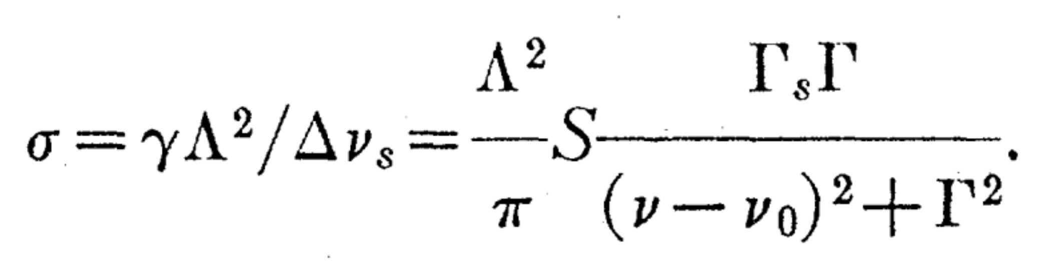




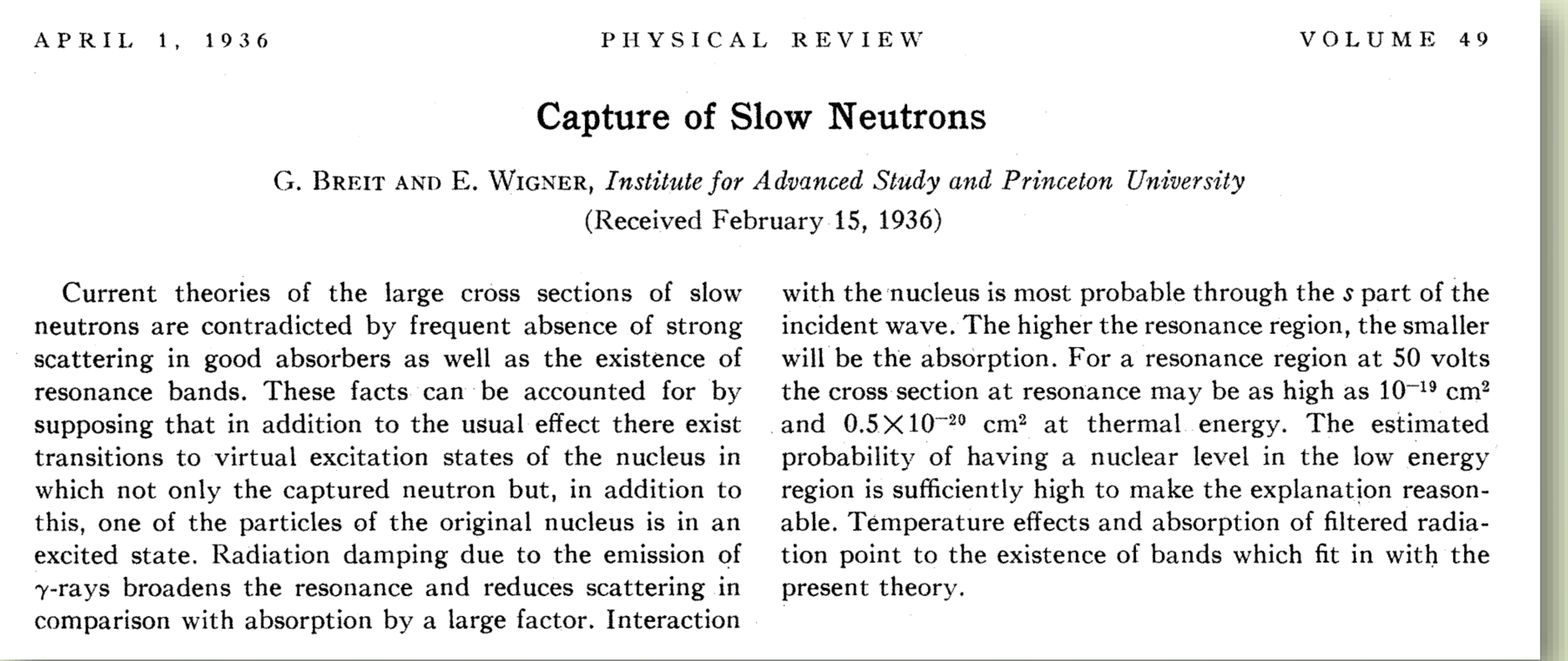


*M. S. Bryan et al*, EPL, 115 (2016) 66001 10.1209/0295-5075/115/66001

# How to select epithermal energies



$$\sigma=\gamma\Lambda^2/\Delta\nu_s=\frac{\Lambda^2}{\pi}S\frac{\Gamma_s\Gamma}{(\nu-\nu_0)^2+\Gamma^2}.$$



APRIL 1, 1936 PHYSICAL REVIEW VOLUME 49

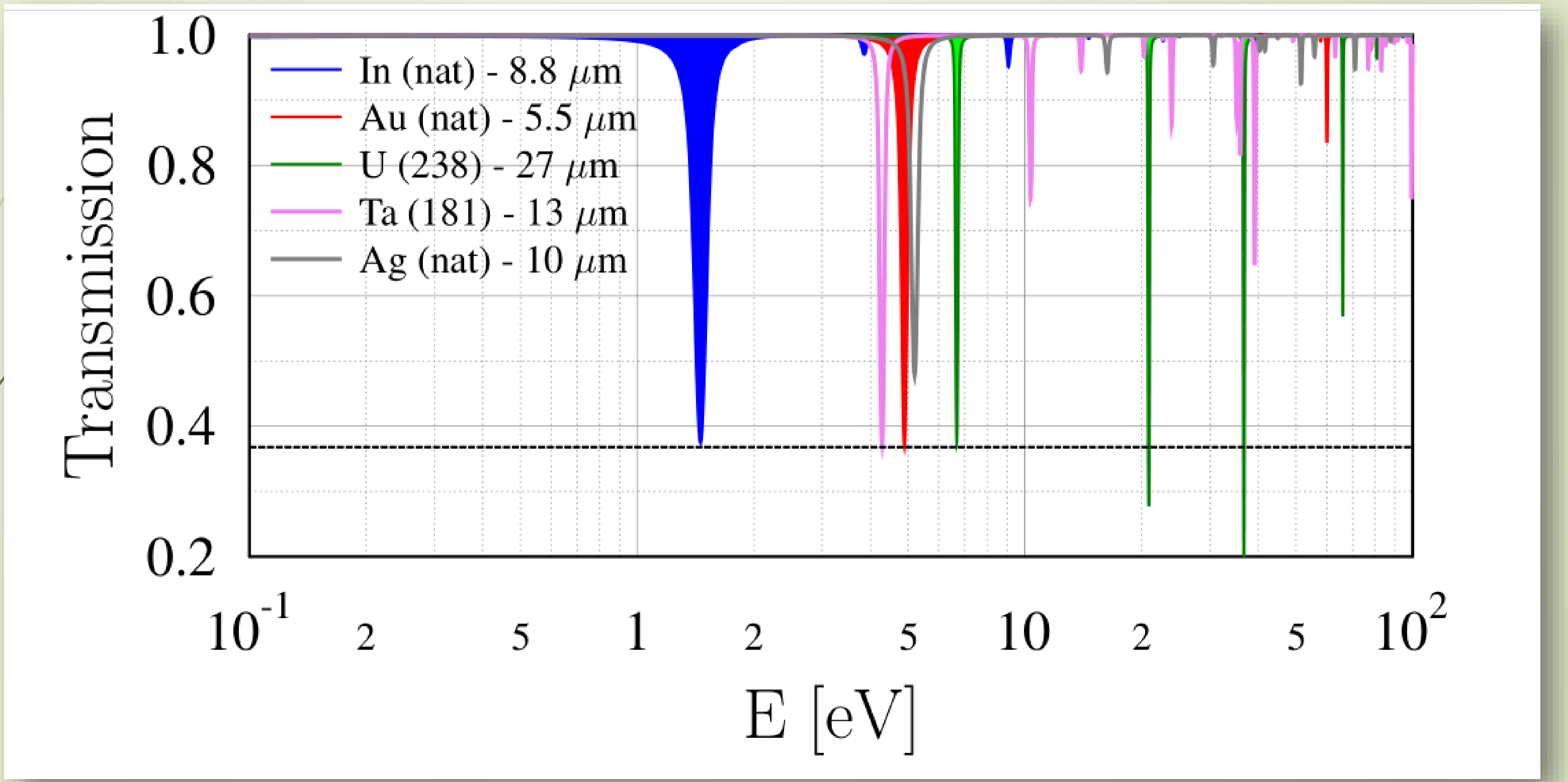
## Capture of Slow Neutrons

G. Breit and E. Wigner, *Institute for Advanced Study and Princeton University*

(Received February 15, 1936)

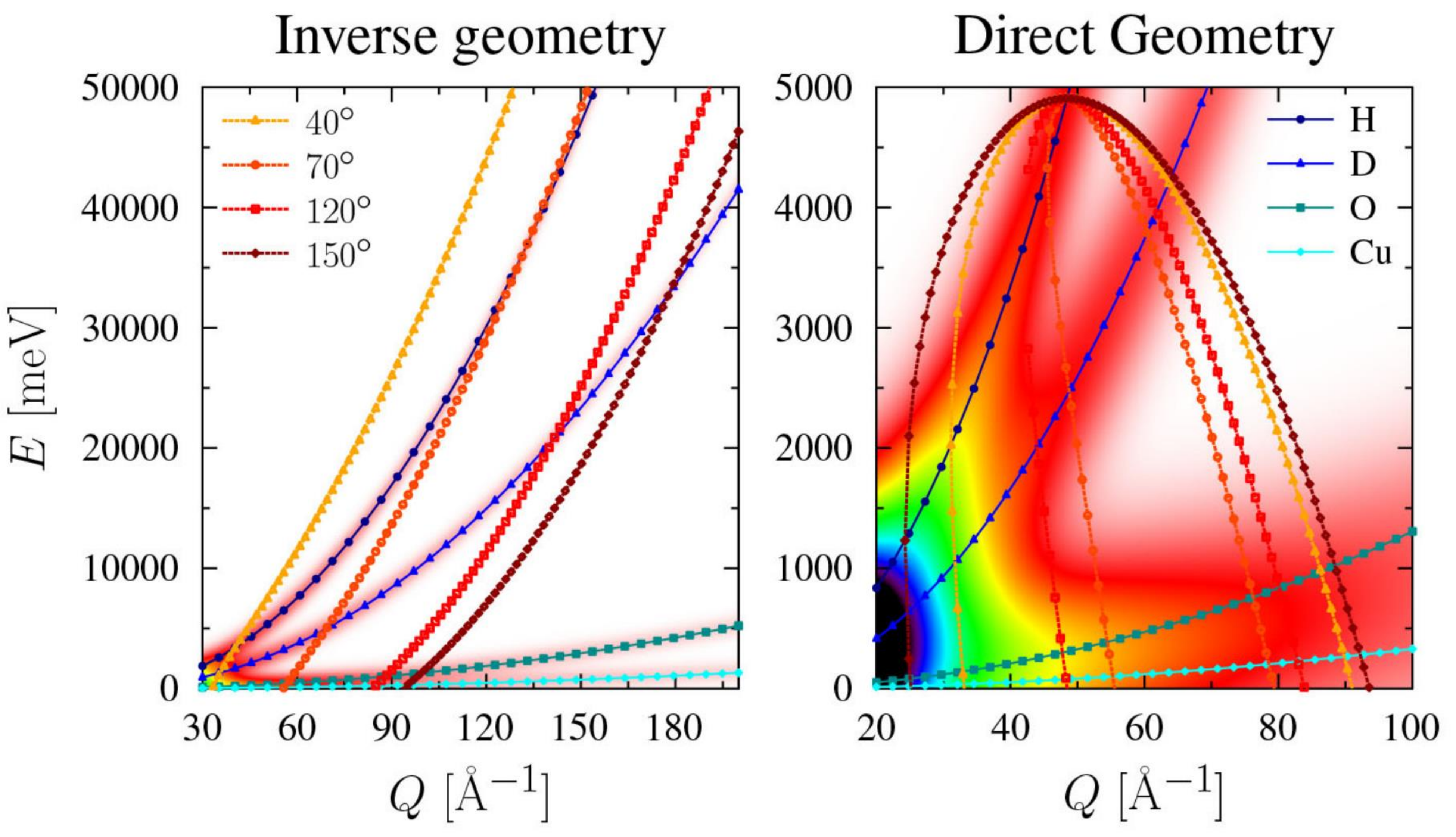
Current theories of the large cross sections of slow neutrons are contradicted by frequent absence of strong scattering in good absorbers as well as the existence of resonance bands. These facts can be accounted for by supposing that in addition to the usual effect there exist transitions to virtual excitation states of the nucleus in which not only the captured neutron but, in addition to this, one of the particles of the original nucleus is in an excited state. Radiation damping due to the emission of $\gamma$-rays broadens the resonance and reduces scattering in comparison with absorption by a large factor. Interaction with the nucleus is most probable through the *s* part of the incident wave. The higher the resonance region, the smaller will be the absorption. For a resonance region at 50 volts the cross section at resonance may be as high as $10^{-19}$ cm² and $0.5\times10^{-20}$ cm² at thermal energy. The estimated probability of having a nuclear level in the low energy region is sufficiently high to make the explanation reasonable. Temperature effects and absorption of filtered radiation point to the existence of bands which fit in with the present theory.

# How to select epithermal energies

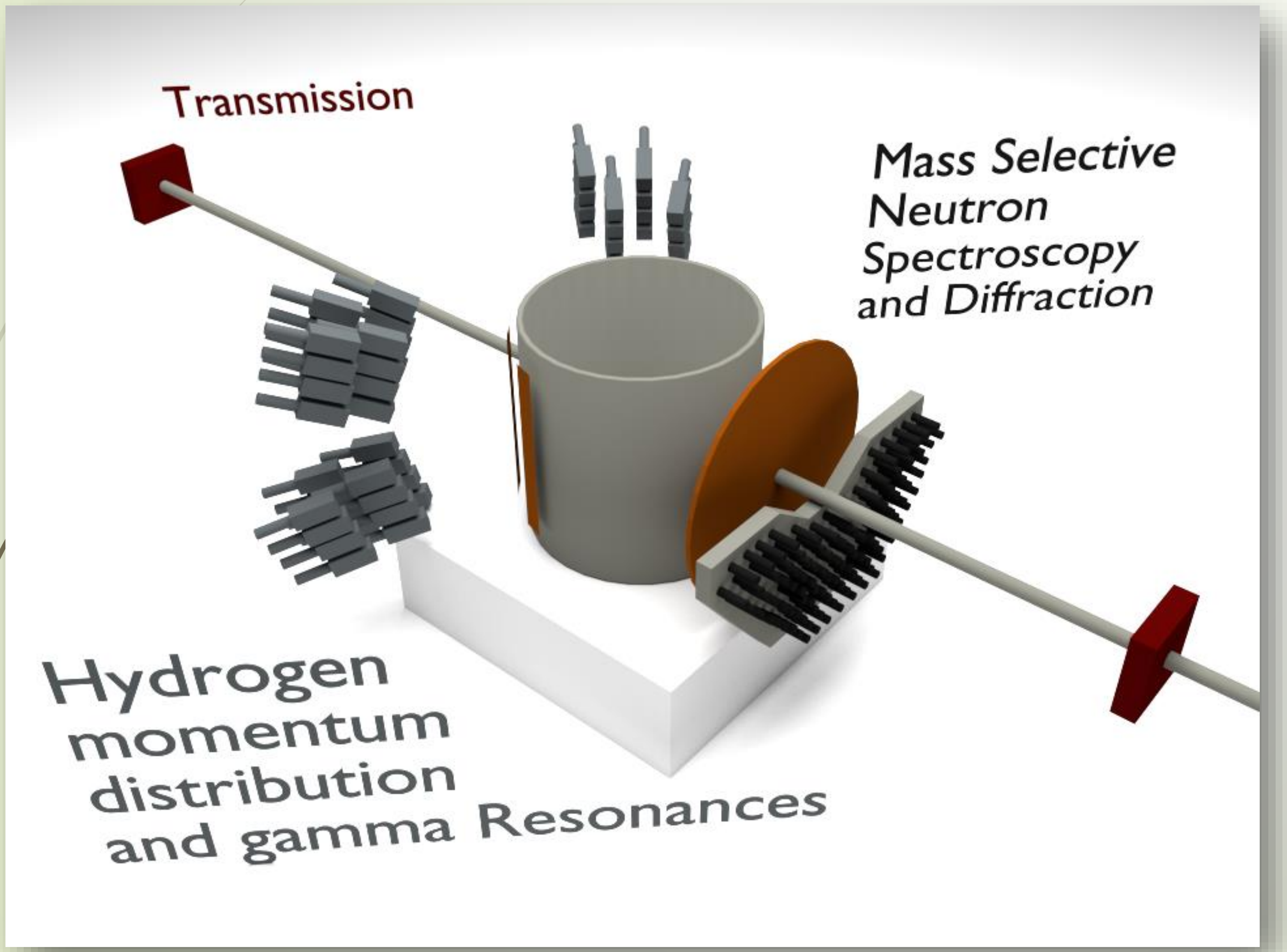


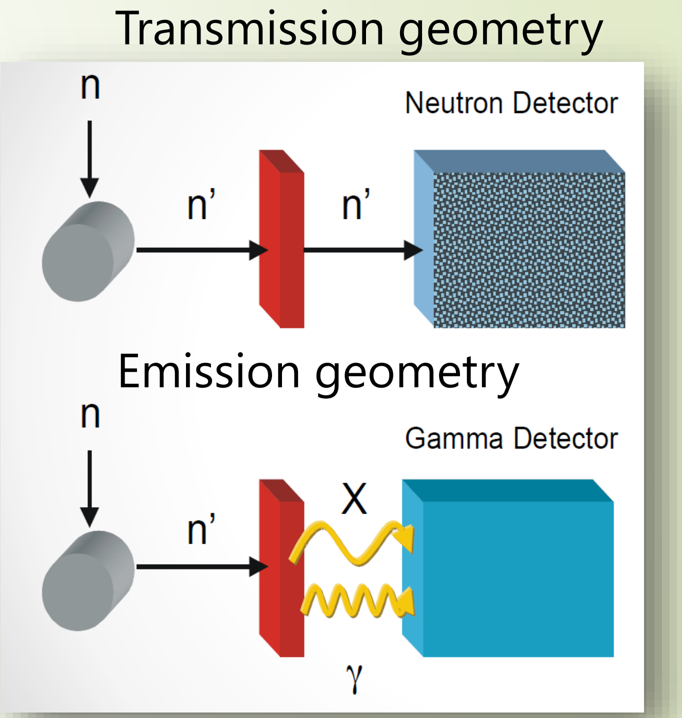
See also: RM Brugger, AD Taylor, CE Olsen, JA Goldstone, and AK Soper; Nucl Inst Meth Phys Res A 221 393 (1984)

# Geometry and Impulse Approximation

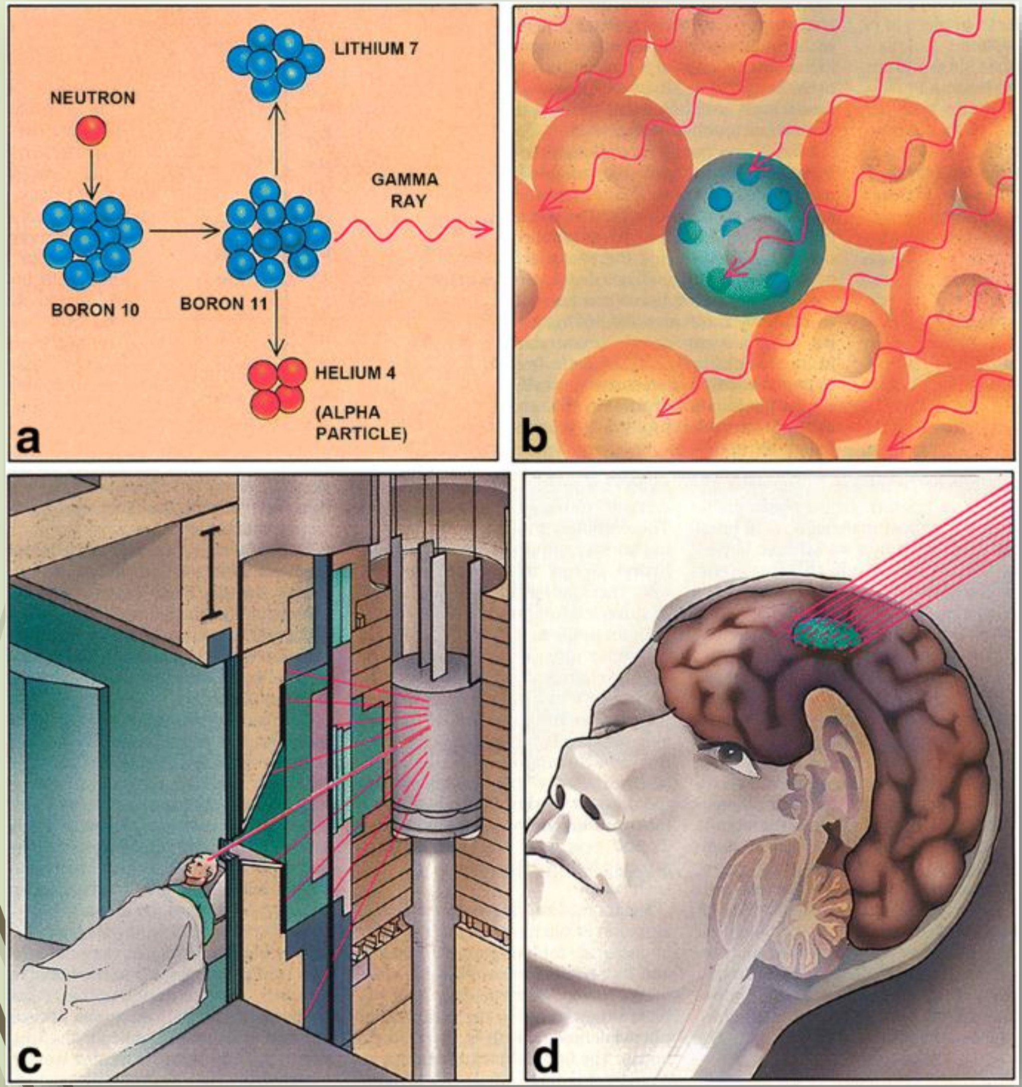


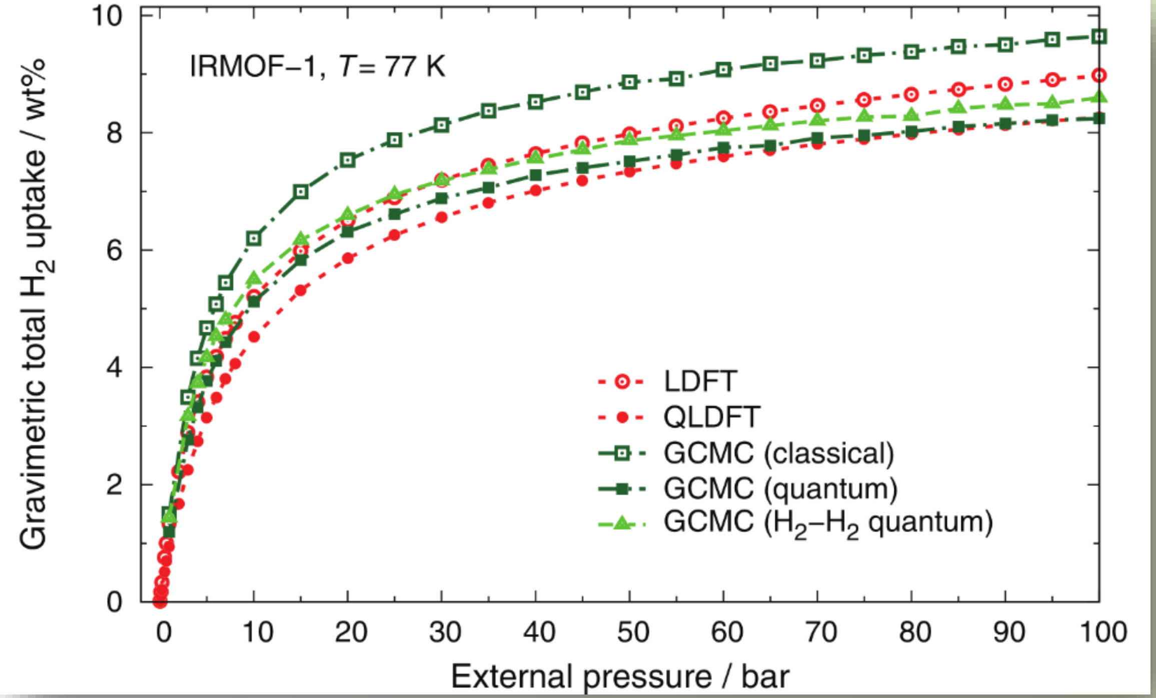
# The VESUVIO Spectrometer (ISIS)

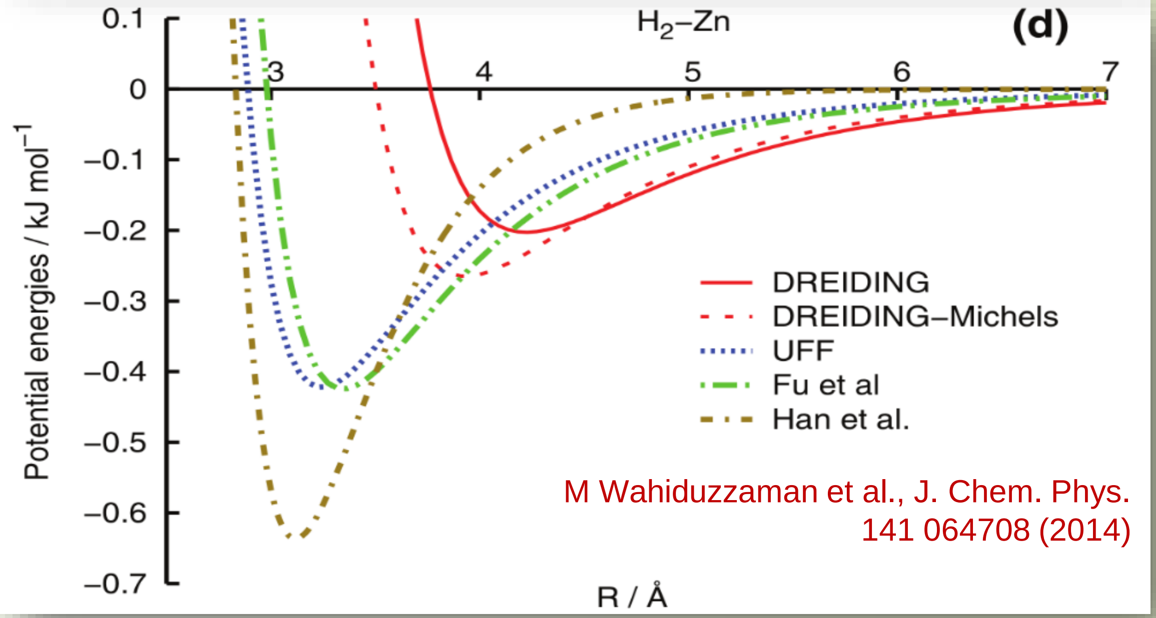




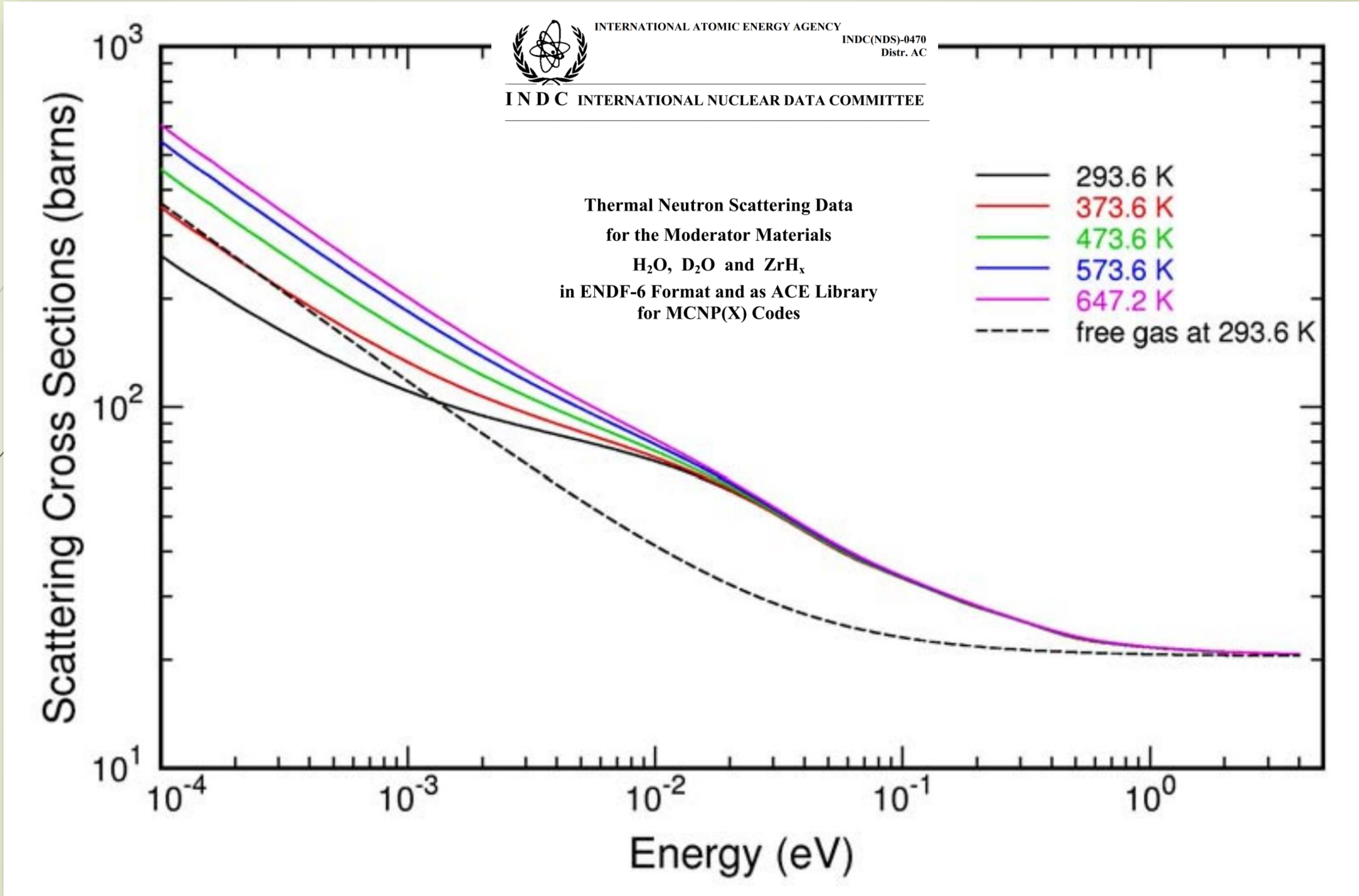
# Applications







M Wahiduzzaman et al., J. Chem. Phys. 141 064708 (2014)



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