Practical Neutron Experiments

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Outline

• Optimising sample size
• Sample shape - transmission of neutrons
• Normalisation of data
• Background Subtraction
• Counting time
Sample preparation

Calculating and optimizing the size and shape of your sample is important in neutron scattering

- Need to know roughly how long you need to count for
- Need to minimise multiple scattering
- Need to minimise self attenuation

In order to calculate and correct for these, the sample should ideally be a regular shape
Scattering unit

Most commonly this is the sample formula.
e.g. sample of polythene (monomer is ethylene)

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_{coh}$</th>
<th>$\sigma_{inc}$</th>
<th>$\sigma_{abs}^*$</th>
<th>$M_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>5.55</td>
<td>0.001</td>
<td>0.3326</td>
<td>12.011</td>
</tr>
<tr>
<td>H</td>
<td>1.7568</td>
<td>80.26</td>
<td>0.0035</td>
<td>1.0079</td>
</tr>
<tr>
<td>C_2H_4</td>
<td>18.1272</td>
<td>321.042</td>
<td>0.6792</td>
<td>28.0536</td>
</tr>
</tbody>
</table>

Can also choose atoms, unit cells, etc. Must be self consistent

Cross sections found in tables, e.g. V F Sears, Neutron News 3 1992

* Remember that $\sigma_{abs}$ is wavelength dependent
Sample size

Number of scattering units in the sample (mass, \(m\)) is given by

\[ N_s = \frac{m}{M_f} N_A \]

where \(M_f\) is the mass of the formula unit and \(N_A=6.022 \times 10^{23} \text{ mol}^{-1}\) is Avogadro’s Number. Similarly the number density is given by

\[ n_s = \frac{\rho}{M_f} N_A \]

where \(\rho\) is the density of the sample.

If we express \(N_A\) in units of \(x \times 10^{24} \text{ mol}^{-1}\) then we get the number density in units of \(x \times 10^{24} \text{ cm}^{-3} = (\text{barn.cm})^{-1}\) which is useful later on. i.e.

\[ n_s = \frac{\rho}{1.661 M_f} \quad \text{(with } \rho \text{ in g cm}^{-3} \text{ and } M_f \text{ in g)} \]

e.g. Polythene: \(\rho = 0.93 \text{ g cm}^{-3}, M_f = 28.054 \text{ g}\)

so: \(n_s = 0.02 \text{ (barn.cm)}^{-1} = 2 \times 10^{22} \text{ cm}^{-3}\)
Transmission of neutrons

Assume thin slab of material with molecules having an absorption cross-section, $\sigma$

$n_s \times A \times dz$ molecules in the slab

so total absorbing area is $\sigma n_s A dz$

and fraction of neutrons absorbed is the total absorbing area divided by the total area, therefore

$$\frac{dI_z}{I_z} = -\sigma n_s dz$$

Integrating both sides: $\ln(I_z) = -\sigma n_s z + C$

So for thick slab of thickness $t$, we have $l_z = l_0$ at $z = 0$ and $l_z = l_1$ at $z = t$, and

$$\ln(I_1) - \ln(I_0) = (-\sigma n_s t + C') - (-\sigma n_s 0 + C') = -n_s \sigma t$$

Finally, exponentiating both sides, we find the expression for the neutron transmission

$$T = \frac{I_1}{I_0} = \exp(-n_s \sigma t)$$

Beer’s Law
**Optimal size**

Experience shows that a good neutron sample is one which scatters around 10% of the incoming beam. Much less than this, the count-rate is too small. Much more, *self-attenuation* and especially *multiple scattering* become problematic.

To calculate the thickness of a 10% scattering sample we write

\[ T = \frac{I_1}{I_0} = \exp(-n_s \sigma t) = 0.9 \]

\[ \Rightarrow n_s \sigma t = 0.1 \]

In this case the cross-section we need is the total scattering cross-section, \( \sigma_T \)

e.g. Polythene: \( \sigma_T = 339.167 \text{ barns} \), \( n_s = 0.02 \text{ (barn.cm)}^{-1} \)

so: \( t_{10\%} = \frac{0.1}{(339.167 \times 0.02)} \)

\[ = 0.015 \text{ cm} \]

So only around 0.1 mm thickness of polythene scatters 10% of the beam. This indicates the strength of the scattering of neutrons from hydrogen.
Strong absorbers

There are a few “boagie” materials which prove challenging for neutron experiments due to their massive absorption cross-sections

<table>
<thead>
<tr>
<th>element</th>
<th>$\sigma_{\text{abs}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gd</td>
<td>49700 b</td>
</tr>
<tr>
<td>B</td>
<td>767 b</td>
</tr>
<tr>
<td>Cd</td>
<td>2520 b</td>
</tr>
<tr>
<td>Li</td>
<td>70 b</td>
</tr>
<tr>
<td>Ir</td>
<td>425 b</td>
</tr>
<tr>
<td>Dy</td>
<td>994 b</td>
</tr>
<tr>
<td>Sm</td>
<td>5922 b</td>
</tr>
<tr>
<td>$^3$He</td>
<td>5333 b</td>
</tr>
</tbody>
</table>

The numbers here are given for “thermal neutrons” ($\lambda = 1.8$ Å)

In general at low energies, the absorption cross-section is linear with wavelength

$$\sigma_{\text{abs}}(\lambda) = \frac{\lambda \sigma_{\text{abs}}^{\lambda=1.8}}{1.8}$$

Sometimes the high absorption can be avoided by choice of isotope - e.g. $^{160}$Gd and $^{11}$B have low absorption

Many of these elements are used in neutron detectors
Strong absorbers

Big absorbers are very useful in neutron shielding design

e.g. How much Cd is needed to absorb 99.9% of thermal neutrons?
\[ n_s = 0.046 \text{ (barn.cm)}^{-1}, \sigma_{\text{abs}} = 2520 \text{ b} \]
\[ t = -\ln(T) / n_s \sigma_{\text{abs}} = -\ln(0.001) / 115.9 = 0.6 \text{ mm} \]

For absorbing samples the 10% rule is generally not used

e.g Mn metal: \( \sigma_T = 2.15 \text{ b}, n_s = 0.08 \text{ (barn.cm)}^{-1}, \sigma_{\text{abs}} = 13.3 \text{ b} \)
so: \( t_{10\%} = 6 \text{ mm} \)

Suppose we’re doing an experiment on IN6 with a wavelength of 5.1 Å
\[ \sigma_{\text{abs}}(5.1\text{Å}) = 5.1 \times 13.3 / 1.8 = 37.7 \text{ barns} \]
so the fraction of neutrons absorbed in a 10% scattering sample is
\[ 1 - \exp(-n_s \sigma t) = 1 - \exp(-1.8) = 0.84 \]
i.e. 84% of the incoming & scattered neutrons are absorbed

So in this case we need a compromise solution
Optimised sample size

Assuming uniform sample shape (e.g. slab for SANS or cylinder for diffraction) then we can make the rough approximation that the neutrons all traverse a similar path through the sample.

Then we can write the fraction of neutrons scattered as

$$\Sigma = T (1 - \exp[-n_s \sigma_T t]) = \exp[-n_s \sigma_{abs} t] - \exp[-n_s (\sigma_{abs} + \sigma_T) t]$$

To maximise this, we differentiate and set to zero to find

$$\frac{d\Sigma}{dt} = n_s (\sigma_{abs} + \sigma_T) \exp[-n_s (\sigma_{abs} + \sigma_T) t] - n_s \sigma_{abs} \exp[-n_s \sigma_{abs} t] = 0$$

$$\Rightarrow \exp[-n_s \sigma_T t] = \frac{\sigma_{abs}}{\sigma_{abs} + \sigma_T}$$

$$\Rightarrow t = \frac{\ln(\sigma_{abs} + \sigma_T) - \ln(\sigma_{abs})}{n_s \sigma_T}$$
So taking the previous example of Mn metal at 5.1 Å, the optimum thickness is

\[ t = \frac{\ln(37.7 + 2.15) - \ln(37.7)}{0.08 \times 2.15} \approx 3.2 \text{ mm} \]

At this thickness the fraction of scattered neutrons is around 2%.

This occurs at a sample transmission of 38% - which is roughly 1/e.

More difficult to optimise sample size for broad wavelength bands - choose lowest useful wavelength in band and optimise there.
In order to calculate the attenuation of neutrons through a sample as a function of scattering angle, we need to know the shape of the sample, and average over all possible paths through the sample.

**e.g. (infinite) slab shaped sample**

Incoming flux $\varphi$ attenuated along $L_1$

$$\phi = \phi_0 \exp(-N\sigma L_1)$$

$$= \phi_0 \exp\left(-\frac{N\sigma x}{\sin \gamma}\right)$$

The outgoing beam is attenuated along $L_2$

$$T_2 = \exp\left(-N\sigma \frac{t-x}{\sin(\gamma - 2\theta)}\right)$$

(assuming diffraction condition $k_i = k_f$)
Sample shape - slab

Outgoing beam proportional to cross-section into solid angle $\Delta \Omega$, flux, $n_s$, thickness of slab $\Delta t$ and is attenuated along $L_2$

$$\Delta N = \phi n_s \Delta t \left( \frac{d\sigma}{d\Omega} \right) T_2 \Delta \Omega$$

$$= \phi_0 n_s \exp \left( - \frac{n_s \sigma x}{\sin \gamma} \right) \frac{dx}{\sin \gamma} \left( \frac{d\sigma}{d\Omega} \right) \exp \left( - \frac{n_s \sigma(t-x)}{\sin(\gamma-2\theta)} \right) \Delta \Omega$$

So total neutrons scattered is found by integrating above wrt. $x$ between $x=0$ and $x=t$

$$N = \phi_0 n_s \left( \frac{d\sigma}{d\Omega} \right) \Delta \Omega \frac{1}{\sin \gamma n_s \sigma (\csc \gamma - \csc(\gamma-2\theta))} \left\{ \exp \left( - \frac{n_s \sigma t}{\sin(\gamma-2\theta)} \right) - \exp \left( - \frac{n_s \sigma t}{\sin \gamma} \right) \right\}$$

In order to get the attenuation factor we need to know the number of counts in the limit of zero absorption, $N_0$ and divide

Finally we get

$$T = \frac{1}{n_s \sigma t (\csc \gamma - \csc(\gamma-2\theta))} \left\{ \exp \left( - \frac{n_s \sigma t}{\sin(\gamma-2\theta)} \right) - \exp \left( - \frac{n_s \sigma t}{\sin \gamma} \right) \right\}$$

This is one of the very few solvable cases for transmission as a function of $2\theta$
Sample shape - slab

Transmission plotted for slab normal to incident beam (e.g. SANS)

Thickness of sample = 1 cm

Slab geometries for samples are also commonly used in Neutron spin-echo experiments

Beyond 90° - in reflection geometry - another (similar) expression exists

Sample shape - cylinder

With the advent of large detector arrays which commonly surround the sample (in the equatorial plane) cylindrical sample geometries have become more common. Sometimes an annular cylinder is used to fill beam, but minimise absorption.

In these cases there is no analytic expression for the transmission as a function of angle and the attenuation must be calculated numerically (i.e. lookup tables or Monte-Carlo algorithms are commonly used).

In the case of a solid cylinder or sphere with \( n_s \sigma \leq 1 \) a good approximation (better than 0.5% accuracy) is given by the expression

\[
T = \exp \left\{ -(a_1 + b_1 \sin^2 \theta) n_s \sigma R - (a_2 + b_2 \sin^2 \theta) (n_s \sigma R)^2 \right\}
\]

where \( R \) is the radius of the cylinder/sphere, and the coefficients, \( a_1, b_1, a_2 \) and \( b_2 \) have the values

<table>
<thead>
<tr>
<th></th>
<th>( a_1 )</th>
<th>( b_1 )</th>
<th>( a_2 )</th>
<th>( b_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>cylinder</td>
<td>1.7133</td>
<td>-0.0368</td>
<td>-0.0927</td>
<td>-0.3750</td>
</tr>
<tr>
<td>sphere</td>
<td>1.5108</td>
<td>-0.0315</td>
<td>-0.0951</td>
<td>-0.2898</td>
</tr>
</tbody>
</table>

Sample shape - cylinder

Much flatter transmission as a function of angle - so friendly for wide-angle instruments.

Generally transmission is assumed to be constant as a function of angle in these cases.

Radius = 1 cm
Sample Holders

Choice of material for sample container / holder is crucial - and depends strongly on type of neutron experiment. Should be made as thin as possible to reduce mass in beam (typically 0.1 mm - 1 mm thickness)

<table>
<thead>
<tr>
<th></th>
<th>Material</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Powder diff.</td>
<td>vanadium</td>
<td>no Bragg peaks, large $\sigma_{inc}$ some absorption</td>
</tr>
<tr>
<td>Inelastic scattering</td>
<td>Al / Cu</td>
<td>low-ish scattering cross-sections, low incoherent scattering (Cu for low T)</td>
</tr>
<tr>
<td>Polarized neutrons</td>
<td>Al / Cu</td>
<td></td>
</tr>
<tr>
<td>Liquids / glasses</td>
<td>vanadium / TiZr</td>
<td>no Bragg peaks TiZr is a “null-matrix”</td>
</tr>
<tr>
<td>SANS</td>
<td>Quartz</td>
<td>very low small angle scattering</td>
</tr>
<tr>
<td>Single crystals</td>
<td>Small pin (Al)</td>
<td>sample holds own shape</td>
</tr>
</tbody>
</table>
Data Normalisation

The total number of neutrons counted during a measurement will also depend on uninteresting factors, such as:

- the counting time
- the solid angle coverage of the detectors
- the efficiency of the detectors

To correct for counting time we can either:

1) Divide the measured counts by the counts measured in an incident beam monitor
2) Divide the measured counts by the counting time

It’s generally better to use a monitor since the flux may not be constant over time (e.g. ISIS)

Monitor counters are generally low efficiency (1 in $10^4$) detectors made from:
- Low pressure $^3$He
- $^{235}$U foils (fission chambers)
- Low density scintillator materials
Data Normalisation

The solid angle and detector efficiency can be corrected by dividing the measured counts by a measurement of vanadium

<table>
<thead>
<tr>
<th>Isotope</th>
<th>abund. %</th>
<th>$\sigma_{\text{coh}}$</th>
<th>$\sigma_{\text{inc}}$</th>
<th>$\sigma_{\text{abs}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{50}\text{V}$</td>
<td>0.25</td>
<td>7.3</td>
<td>0.5</td>
<td>60</td>
</tr>
<tr>
<td>$^{51}\text{V}$</td>
<td>99.75</td>
<td>0.0203</td>
<td>5.07</td>
<td>4.9</td>
</tr>
</tbody>
</table>

Due to the tiny coherent cross-section, and the abundance of the $^{51}\text{V}$ isotope V is approximated to be a purely incoherent scatterer

Therefore a vanadium count in a particular detector is proportional to the efficiency and solid angle coverage of that detector
Data Normalisation

The solid angle and detector efficiency can be corrected by dividing the measured counts by a measurement of vanadium.
Data Normalisation

On a neutron time of flight diffractometer, the vanadium measurement additionally gives the flux profile as a function of wavelength. So division of the data by a vanadium measurement corrects also for the incident flux profile.

\[
\text{Counts per micro-second} \Rightarrow \text{Differential cross-section / arb. units}
\]

\(\frac{\text{Counts per micro-second}}{\text{Differential cross-section}}\)
Absolute Normalisation

Since the spin incoherent cross-section of vanadium is well known, division of the data by vanadium can also be used to set the neutron data on an absolute scale of barns/ster./scattering unit.

For vanadium, $\sigma_{\text{inc}} = 5.07$

Therefore,

$$\left( \frac{d\sigma}{d\Omega} \right)_V = \frac{5.08}{4\pi}$$

The number of counts from the vanadium per second per solid angle is then (after transmission correction)

$$N_v = \phi n_s(V) t_V \frac{5.08}{4\pi} \Delta \Omega$$

So dividing the sample counts by the vanadium counts we get

$$\frac{N_S}{N_V} = \frac{n_s(S) t_S}{n_s(V) t_V} \frac{\frac{d\sigma}{d\Omega}}{\frac{5.08}{4\pi}}$$

and therefore

$$\left( \frac{d\sigma}{d\Omega} \right)_S = \frac{N_S}{N_V} \frac{n_s(V)}{n_s(S)} t_V \frac{5.08}{t_S 4\pi}$$

can substitute ratio of volumes here for non-slabs.
Background subtraction

- Source
- Single crystal monochromator
- Windows
- Sample env.
- Collimator
- Detectors
- Air
Case Study: IN4 (ILL)

1) Reactor off

Background measurements

1) Reactor off background

The reactor off background is presented as a function of scattering angle in figure 1.

Figure 1: Reactor off background for the top (blue), middle (red) and bottom (green) rows of detectors in the wide angle bank.

It is interesting to note that the top row of detectors is significantly noisier than the bottom and middle rows.

The mean value of the reactor off background is found to be:

0.066 counts min$^{-1}$ detector$^{-1}$ (in 2005: 0.0834 counts min$^{-1}$ detector$^{-1}$)

Although it is clear from the picture that there are several much noisier detectors, with the median value much lower than the mean (0.044 counts min$^{-1}$ detector$^{-1}$).

2) Reactor on, OT closed

This background was measured in the two extreme flight-chamber positions FCP 140 and FCP 370. Since the OT was closed, we expect this background to be sourced by neutrons in the reactor hall. Measurements of neutron background in the casemate (see appendix A) show that with the OT closed, the neutron background in the IN4 casemate is very low, with no discernible fast neutron contribution coming from the H1/H2 guide. By contrast, the neutron flux outside the IN4 detector tank - in the experimental zone is considerably higher (appendix B). It is therefore likely that this contribution to the measured background is from the reactor hall.

Background from electronic noise in detectors, cosmic rays or perhaps natural radioactivity
Case Study: IN4 (ILL)

2) Reactor on: beam shutter closed

Background from “Neutron gas” in experimental hall
Case Study: IN4 (ILL)

3) Reactor on: shutter open, nothing in beam

Scattering from windows, beam dumps and fast neutrons (thermalised in shielding). Can be time dependent on tof instruments.
Case Study: IN4 (ILL)

4) Reactor on: shutter open, cryostat in beam

Scattering from cryostat tails, sample can (red). Will be time dependent on tof instruments.
Background subtraction

Measured counts from the sample is the sum of three terms

\[
N_{\text{meas}} = N_{\text{sam}} + N_{\text{bg}_A} + N_{\text{bg}_B}
\]

\[
N_{\text{sam}} = \text{neutrons scattered from sample - attenuated}
\]

\[
N_{\text{bg}_A} = \text{background neutrons - attenuated}
\]

\[
N_{\text{bg}_B} = \text{background neutrons - unattenuated}
\]

Need to be able to separate the attenuated and unattenuated backgrounds for an accurate background subtraction
If we measure an empty sample holder we are measuring

\[ N_{\text{empty}} = \left( \frac{N_{\text{bg}_A}}{T} \right) + N_{\text{bg}_B} \]

where the \( bg_A \) term is now no longer attenuated, so must be divided by the sample transmission factor.
If we measure a completely absorbing sample (such as cadmium) we are measuring

\[ N_{Cd} = N_{bg_B} \]
Background subtraction

Taking these together we have

\[ N_{\text{meas}} = N_{\text{sam}} + N_{\text{bg}_A} + N_{\text{bg}_B} \]
\[ N_{\text{empty}} = \left( \frac{N_{\text{bg}_A}}{T} \right) + N_{\text{bg}_B} \]
\[ N_{\text{Cd}} = N_{\text{bg}_B} \]

Combining these we get for the counts from the sample

\[ N_{\text{sam}} = (N_{\text{meas}} - N_{\text{Cd}}) - T(N_{\text{empty}} - N_{\text{Cd}}) \]

But to correct for sample attenuation, we need to divide by the transmission factor, therefore

\[ N_{\text{sam\_corr}} = \frac{(N_{\text{meas}} - N_{\text{Cd}})}{T} - (N_{\text{empty}} - N_{\text{Cd}}) \]

Things to note

- If the sample transmission is close to 1, then cadmium measurement not required
- \( N_{\text{Cd}} \) is often close to zero (e.g. SANS) - but still checked
beta-Mn (!) measured on IN4
Counting time

We need then to do 4 measurements to produce a corrected data set. (not counting experimental verification of the sample transmission)
- sample
- vanadium
- empty cell
- cadmium (or equivalent)

So the question now is how do we divide our time between these measurements?

This is an eternal question.... and opinion is divided.

The error bars in the measurements are given by Poisson Statistics \( \delta(N) = \sqrt{N} \)

and therefore the fractional error is \( \frac{\delta(N)}{N} = \frac{1}{\sqrt{N}} \)

so in order to improve the fractional error by a factor of \( x \), we need to count a factor of \( x^2 \) more neutrons
Counting time

e.g. After 20 minutes we’ve counted 1000 in a detector. What’s the fractional error bar?

\[
\frac{\delta(N)}{N} = \frac{1}{\sqrt{1000}} = 0.032
\]

How long do we need to count to get the error bar below 1%?

\[
\frac{1}{\sqrt{N_{1\%}}} = 0.01
\]

\[\Rightarrow N_{1\%} = 10000\]

the count rate is 50 counts per minute, so we need to count for 200 minutes.

NB In order to improve the error bars from around 3% to 1% we need to count 10x longer!

Neutron fluxes are low, so it’s important not to over-count our measurements. However - if we decide we need a certain level of error bar, we have to be prepared to “sit it out”
Counting time

The last question is - how much time do we have to “waste” measuring $N_{\text{empty}}$, $N_{\text{Cd}}$ and $N_{V}$?

Often $N_{V}$ is already done by the instrument scientist (assuming sample size and shape doesn’t alter very much)

If the backgrounds are close to zero - we don’t need to count them at all
If they are large - they should be counted carefully.

The general “rule of thumb” is that the times spent on measuring sample and background runs should be in proportion to their count rates

$$\frac{T_{\text{meas}}}{T_{\text{empty}}} \sim \frac{N_{\text{meas}}}{N_{\text{empty}}}$$

for the Cadmium run, the count time is modified by the transmission

$$\frac{T_{\text{Cd}}}{T_{\text{meas}}} \sim \left( \frac{T}{1 - T} \right) \frac{N_{\text{Cd}}}{N_{\text{meas}}}$$