

Small Angle Scattering (SAS)

• what is SAS & what can it measure?

- how is it measured?
- sample considerations
 data analysis





Reference Texts

- The SANS Toolbox, B. Hammouda, NIST (available as pdf: <u>http://www.ncnr.nist.gov/staff/hammouda/the_SANS_toolbox.pdf</u>)
- Structure Analysis by SAXS & SANS, L.A. Fegin & D.I. Svergun (1987) (available as pdf: <u>http://www.embl-</u> <u>hamburg.de/biosaxs/reprints/feigin_svergun_1987.pdf</u>)
- Small Angle X-ray Scattering, eds O. Glatter & O. Kratky (1982) (available as pdf: <u>http://physchem.kfunigraz.ac.at/sm/Software.htm</u>)

What is SAS?

 Coherent, elastic scattering of radiation at small angles – close to the straight-through beam



Size Range Comparisons



Scattering from Large Structures

- Neutron/X-ray wavelength $\lambda \approx$ space between atoms in crystal
 - \Rightarrow bounce off layers of atoms like light off a mirror
 - ⇒ see diffraction peaks at high angles (correspond to atomic positions)
- BUT for larger objects, sees <u>average</u> structure



- large structures scatter at small angles
- ⇒ for techniques using small angles use material properties rather than atomic properties

SAS Instruments



- Neutrons/X-rays must be parallel to each other; "collimated"
- Slit defines shape of beam (circle, square, slit)
- Distance from sample to detector & wavelength determines size range measured
 - Fof wide simultaneous Q range, lower flux
 - Reactor smaller Q range, higher flux at short sample-detector distances

Scattering Patterns: From detector to 1D

Real space

- eg diblock copolymers
- Circular 1D average
 - take average over ring





SAS Measurements

• Observed intensity:

 $J(\lambda,\theta) = J_0(\lambda) \Delta \Omega \eta(\lambda) T V I(Q)$

where:

- $JO(\lambda) = flux$ incident on sample
- $\Delta\Omega$ = angle covered by detector
- η = detector efficiency
- T = sample transmission
- V = volume of sample in beam

Can measure all of these

- \Rightarrow Used to correct
- data during data reduction
- I(Q) = differential cross section

 \Rightarrow contains information about sample

Sample Scattering

- Measured intensity due to:
 - Scattering from sample
 - > Scattering from container, slits, air etc
 - Stray X-rays and electronic noise



Need to measure more than just sample scattering...

Extra measurements

Empty cell



Source

Scattering from:

- 1) empty cell
- 2) windows & collimation slits
- 3) air scattering
- Minimize air in beam path
- Carefully choose cell & window materials
- Measure an empty cell

Blocked beam



Source

- 1) Detector dark current
- 2) Stray radiation
- 3) Cosmic radiation

Detector efficiency



Why? Sensitivity of each pixel is slightly different (~ 1%)

- Use isotropic
 - scattering material (Plexiglass or water) or "flood" source
- Measure a blocked beam

Standards - Intensity

- Y-axis in "counts"
 - Need to convert to absolute intensity
- Intensity standards:
 - > water
 - glassy carbon
 - direct beam + attenuator (if flux is known)
 - standard polymer sample
- Scattering cross section (intensity) is known
- Measure intensity of standard under same conditions as sample
- Compare measured and known intensities
- Calculate "scale factor" to multiply data

Scattered Intensity

 observed scattered intensity is Fourier Transform of real-space shapes

$$I(Q) = N_p V_p^2 (\rho_p - \rho_s)^2 F(Q) S(Q) + B$$

where: N_p = number of particles

 V_p = volume of particle

- ρ = scattering length density (of particle/solvent)
- B = background
- F(Q) = form factor
- S(Q) = structure factor
- Sample considerations...

Sample thickness

- Affects transmission (total intensity)
- Also affects shape of curve \Rightarrow hard to analyse
- More problematic for high flux, strongly scattering samples
- Aim for ~70% transmission (X-rays/D₂O solutions, >~50% H₂O solutions)



Concentration

 $I(Q) \propto N_p V_p$

- Big particles scatter more (can hide small ones)
- Higher concentration = more signal

BUT

Consider detector limits!

Don't burn out your detector...

High concentration can complicate analysis

especially for charged particles (see later)

- Minimum concentration for neutrons/lab X-ray source: ~10mg/ml
 - > watch out for highly coloured solutions eg nanoparticles

Neutrons/X-rays & "Contrast"

- Neutrons more penetrating than X-rays (interact less with matter)
- Interaction of neutrons with nuclei depends on isotope
- Interaction of X-rays just depends on number of electrons
- b = scattering length (units Å or cm, normally)
- Scattered intensity measured depends on which isotopes are in sample for neutrons, only on elements for X-rays



Scattering from Large Structures



 \Rightarrow for distances > ~5 molecules, see only average density

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Q = 2\pi/d
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• so can use material properties for Q < ~0.6 Å⁻¹

Scattering Length Density

- scattering from an object/material depends on how many electrons or nuclei there are in a unit volume
- use scattering length density, Nb, to calculate scattering from molecules:

$$Nb = \frac{N_A \cdot \rho}{MW} \sum_i b_i$$

Units of Nb: cm⁻²

$$= N \sum_{i} b_{i}$$

where: b_i = scattering length for element, cm (for X-rays b = $2.81 \times 10^{-13} \times no.$ of e⁻ in atom) ρ = density of compound, g cm⁻³ N_A = Avogadro's number, mol⁻¹ MW = molecular weight, g mol⁻¹ N = number density of atoms in material, cm⁻³

NB/ if feeling lazy see: www.ncnr.nist.gov/resources/sldcalc.html

Important Scattering Length Densities

Contrast & Contrast Matching

- Both tubes contain pyrex fibers + borosilicate beads + solvent.
- (A) solvent refractive index matched to pyrex fibres
- (B) solvent index different from both beads & fibers scattering from fibers dominates



Similarly, there must be a difference between object and surrounding to measure scattering

$$I(Q) \propto (\rho_p - \rho_s)^2$$

Babinet's Principle





These two structures give the same scattering

$$I(Q) \propto (\rho_p - \rho_s)^2$$

- Contrast is relative
- Loss of phase information i.e.: is $\rho_1 > \rho_2$?
- Very important in multi-phase systems
 - Solve by use of multiple contrasts using SANS! (for X-rays = anomalous scattering)

Scattering ~ "Contrast"

- objects and solvent have different scattering length densities (SLD)
- Intensity ∝ SLD difference between solvent & particle
- in water for neutrons can manipulate solvent ρ by using mixture of H₂O and D₂O
- When solvent and object have same SLD they are said to be "contrast matched"

Example: silica spheres in water



30% $\mathrm{D_2O}$ in $\mathrm{H_2O}$



59% $\mathrm{D_2O}$ in $\mathrm{H_2O}$



95% D₂O in H₂O

Predicting Contrast Match Point

- By calculating the SLD can predict %D₂O where the scattering signal will be zero
- BUT if have exchangeable hydrogens in the structure the SLD will vary with %D₂O



Neutron "Contrast" Series

 intensity of scattering depends on difference between particle and solution.

$$I \propto (Nb_{particle} - Nb_{solution})^2$$

- measure scattering at a series of solution contrasts
- extrapolate scattering to Q = 0 and measure I₀



Contrast Match Point





- Place where line cuts zero is where the solution has the same scattering length density as the particle
 ⇒ contrast matched
- Can use this to find the density of the particle

Neutron "Contrast" for Complex Objects

 contrast matching allows us to "remove" scattering from parts of an object





"shell-contrast" \Rightarrow see only core

"core-contrast" \Rightarrow see only shell

Solvent matching for COC2-actin assembly

- cardiac myosin binding protein C (COC2) has extended modular structure
- Mixing COC2 with G- actin solutions results in a dramatic increase in scattering signal due to formation of a large, rod-shaped assembly





Whitten, Jeffries, Harris, Trewhella (2008) Proc Natl Acad Sci USA 105, 18360-18365

Scattered Intensity

• For concentrated solutions:

$$I(Q) = N_p V_p^2 (\rho_p - \rho_s)^2 F(Q) S(Q) + B$$

- where: N_p = number of particles
 - V_p = volume of particle
 - ρ = scattering length density (of particle/solvent)
 - B = background
 - F(Q) = form factor
 - S(Q) = structure factor
- Form Factor = scattering from within same particle \Rightarrow depends on particle shape

Structure Factor = scattering from different particles ⇒ depends on interactions between particles

Solution of particles



SolutionMotif*LatticeI(c,Q)(protein, micelle, nanoparticle)S(c,Q)F(0,Q)

Form factor of the particle

Structure factor of the particle

c = concentration

SAS Data Analysis

• Simple but not very accurate:

- Porod slopes
- Guinier analysis
- More helpful, but more complex:
 - fitting models to data
- Most complex (need more data):
 - fitting protein structures
 - > monte carlo/simulated annealing methods

Scattering from Independent Particles

- Scattered intensity per unit volume of sample
 - arises from spatial distribution of regions with different scattering length density

$$I(q) = \frac{d\sigma}{d\Omega} = \frac{1}{V} \left| \int_{V} \rho(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}} d\mathbf{r} \right|^{2}$$

For identical particles:

$$V_{p}, \rho_{p}$$

 V, ρ_{s}

$$I(q) = \frac{N}{V} (\rho_p - \rho_s)^2 V_p^2 \left| \frac{1}{V_p} \left| \int_{particle} e^{iq.r} dr \right|^2 \right|$$

Particle form factor, F(Q)

Dilute Randomly Ordered Uniform Particles

scattering from independent particles:

$$I(q) = \frac{N}{V} (\rho_p - \rho_s)^2 V_p^2 \left(\frac{1}{V_p} \left| \int_{particle} e^{i\boldsymbol{q}.\boldsymbol{r}} \, d\boldsymbol{r} \right|^2 \right)$$

• Assume: i) system is isotropic, then $\langle e^{-iqr} \rangle = \frac{\sin(qr)}{qr}$ ii) no long range order, so no correlations between two widely separated particles

$$I(q) = I_e(q)(\rho_p - \rho_s)^2 V_p \int_0^\infty \gamma(r) \frac{\sin(qr)}{qr} 4\pi r^2 dr$$

 $\gamma(r)$ = correlation function within particle

 $P(r)=4\pi r^2\gamma(r)$ is the probability of finding two points in the particle separated by r

Porod's Law

- Start with form factor: $F(q) = \frac{1}{V_n} \int_0^\infty \gamma(r) \frac{\sin(qr)}{qr} 4\pi r^2 dr$
- Now consider radial pair correlation function for sphere, with sharp edges, radius R:

$$\gamma(r) = 1 - \frac{3}{4} \left(\frac{r}{R}\right) + \frac{1}{16} \left(\frac{r}{R}\right)^3$$
$$F(qR) = \frac{1}{V_p} \int_0^\infty \left[1 - \frac{3}{4} \left(\frac{r}{R}\right) + \frac{1}{16} \left(\frac{r}{R}\right)^3\right] \frac{\sin(qr)}{qr} 4\pi r^2 dr$$



Integrate by parts three times: $F(qR) \approx \frac{3}{2R^3} \frac{S_p}{V_p} \frac{1}{q^4}$

At high scattering angles, for any system with sharp, smooth surfaces: $I(Q) \propto \frac{1}{q^4}$

Porod Scattering

- Slope at high q the same
- But point where slope changes depends on particle dimensions



Fractal Systems

Fractals are systems that are self-similar as you change scale



Diffusion-limited aggregation in 3 dimensions (Paul Bourke, http://local.wasp.uwa.edu.au/~pbourke/fractals/dla3d/)

For a Mass Fractal the number of particles within a sphere radius
 R is proportional to R^D where D = fractal dimension

• Thus:

 $4\pi R^2 \gamma(R) dR$ = number of particles between distance R and R+dR

 $= cR^{D-1}dR$

Fractal Systems Continued...

• So for a Mass Fractal:

$$F(Q) = \int dR \ e^{iQR} \gamma(R) = \frac{2\pi}{Q} \int dR \ Rsin(QR) \ \left(\frac{c}{4\pi}\right) R^{D-3}$$
$$= \frac{c}{2} \frac{1}{Q^D} \int dx \ x^{D-2} \ sinx = \frac{constant}{Q^D}$$
Paul Bourke
• For a fractal surface can show that $F(Q) = \frac{constant}{Q^{6-D}}$ (this reduces to the Porod Law for smooth surfaces of dimension 2)
First stages of Koch (triangle) surface (Robert Dickau)



The SANS Toolbox. Boualem Hammouda, NIST

http://www.ncnr.nist.gov/staff/hammouda/the_SANS_toolbox.pdf

Porod Slopes & Structures



Form Factors

• Form factors are the sum of scattering from every point inside a particle $\sum \sum |\sin(\theta r_u)|$

$$F(Q) = \sum_{I} \sum_{J} \left\langle \frac{\sin(Qr_{IJ})}{Qr_{IJ}} \right\rangle_{orientations}$$

- Simplify to the integral
- Scattering pattern calculated from the Fourier transform of the



- real-space density distribution
- Pattern for most shapes must be solved analytically
- Some simple shapes can be solved directly

Simple Analysis - Guinier Approximation

• Assume particle is a sphere in dilute solution $F(Q) = \left[\frac{3(\sin(QR_p) - QR_p\cos(QR_p))}{(QR_p)^3}\right]^2$ $R_p = \text{radius of sphere}$

Measure scattering at very low angles so

$$R_g Q \lesssim 1$$

• Use mathematical expansion of F(Q)

$$F(Q) = 1 - \left(\frac{Q^2 R_g^2}{3}\right) + O()^5 \dots$$

• Write in logarithmic form \Rightarrow



Guinier Plots

• at low concentrations <u>and</u> small values of Q, can write intensity as:

$$I(Q) = I(0) \exp\left(\frac{-R_g^2 Q^2}{3}\right)$$

- so plot of ln(I) against Q² will have slope = $-R_g^2$
- <u>only</u> valid for $R_g Q \leq 1$

Radius of Gyration – depends on particle shape

• Sphere
$$R_g^2 = \frac{3}{5}R^2$$

What do we mean by "R_g"?

Radius of gyration:

R_g² is the average squared distance of the scatterers from the centre of the object



$$R_g^2 = (1^2 + 1^2 + 1^2 + 2^2 + 2^2 + 3^2)/6 = 20/6$$

 $R_g = \sqrt{3.333} = 1.82$

Radius of Gyration – depends on particle shape

• Sphere $R_g^2 = \frac{3}{5}R^2$ • Ellipse $R_g^2 = \frac{a^2 + b^2}{4}$ • Cylinder $R_g^2 = \frac{R^2}{2} + \frac{h}{12}$

Guinier Plot Example

Polymerised surfactant micelles

Large Scale Structures, ISIS Annual Report, 1999-2000 http://www.isis.rl.ac.uk/isis2000/science/laraescale.htm



Check validity: $R_g \times Q_{max} = 11.6 \times 0.095 = 1.1$ OK

More Complex: Fitting Scattering

 observed scattered intensity is Fourier Transform of real-space shapes

$$I(Q) = N_p V_p^2 (\rho_p - \rho_s)^2 F(Q) S(Q) + B$$

where: N_p = number of particles V_p = volume of particle ρ = scattering length density (of particle/solvent) B = background F(Q) = form factor S(Q) = structure factor

Form Factor = scattering from within same particle
⇒ depends on particle shape
Structure Factor = scattering from different particles
⇒ depends on interactions between particles

Form Factors

- depend on shape of particle
- for dilute solutions S(Q) = 1 and so $I(Q) \propto F(Q)$



Polydispersity

- "smears out" sharp features in pattern
- "smearing" can also be due to poor Q resolution or beam shape (correct for this during data reduction)



Au Nanorods

Fitted to charged cylinders

- Radius 80Å
- Length 190Å
- Polydispersity 0.29





Structure Factors

- for dilute solutions S(Q) = 1
- particle interactions will affect the way they are distributed in space ⇒ changes scattering
- for charged spheres:



Average distance between nearest neighbours relatively constant

= "correlation distance"

Position of first maximum related to correlation distance



Concentration effects





Figure 1: Cross-section for several different volume fractions of PS spheres in glycerol vs. QR.

Figure 2: Measured and model structure factors, S(Q), (circles and dashed lines, respectively) vs. QR for PS spheres in glycerol.

Small Angle X-ray Scattering Study of a Hard-Sphere Suspension: Concentrated Polystyrene Latex Spheres in Glycerol

L. B. Lurio¹, D. Lumma¹, A. R. Sandy¹, M. A. Borthwick¹, P. Falus¹, S. G. J. Mochrie¹, J. F. Pelletier², M. Sutton², Lynne Regan³, A. Malik⁴ and G. B. Stephenson⁴

Combining F(Q) & S(Q)

- In most cases when fitting will need to include both form and structure factor
- Can tell by taking concentration series
 - if shape of scattering doesn't change when sample is diluted then S(Q) = 1



- Polymer-lipid discs
- Normalised for concentration

Combining F(Q) & S(Q)

Use computer programs to combine form factor and structure factor:



- Fit using ellipse + structure factor for charged objects which repel each other ⇒ many parameters!
- Use three contrasts to help pin down shape and size accurately

Fourier Inversion Techniques

- Scattering from dilute, uniform, independent particles
- Assuming i) system is isotropic, then $\langle e^{-iQr} \rangle = \frac{\sin(Qr)}{Qr}$

ii) no long range order, so no correlations between two widely separated particles

$$I(Q) = I_e(Q)(\rho_p - \rho_s)^2 V_p \int_0^\infty \gamma(r) \frac{\sin(Qr)}{Qr} 4\pi r^2 dr$$

 $\gamma(r)$ = correlation function

 $P(r)=4\pi r^2\gamma(r)$ is the probability of finding two points in the particle separated by r

 If can measure I(Q) over big enough range can take inverse Fourier transform to find P(r):

$$\mathsf{P}(\mathsf{r}) = 4\pi \mathsf{r} 2\gamma(\mathsf{r}) = \frac{2}{\pi} \int QI(Q) \sin(Qr) \, dQ$$

P(r) for Simple Shapes



Effects of Sample Alignment

- Scattering no longer circular
- Form areas of high intensity perpendicular to direction of alignment



Isotropic vs Nonisotropic Structures



Edler, Reynolds, Brown, Slawecki, White, J. Chem. Soc., Faraday Trans. **1998**, 94(9) 1287

Free SANS Fitting Software

DANSE SANSView software

- Designed for fitting neutron data but can also be used (with care) for X-ray data
- Includes reflectivity analysis
- Available from: <u>http://danse.chem.utk.edu/sansview.html</u>

OR library of other available software at:

http://www.small-angle.ac.uk/small-angle/Software.html