

Neutron Spin Echo Spectroscopy

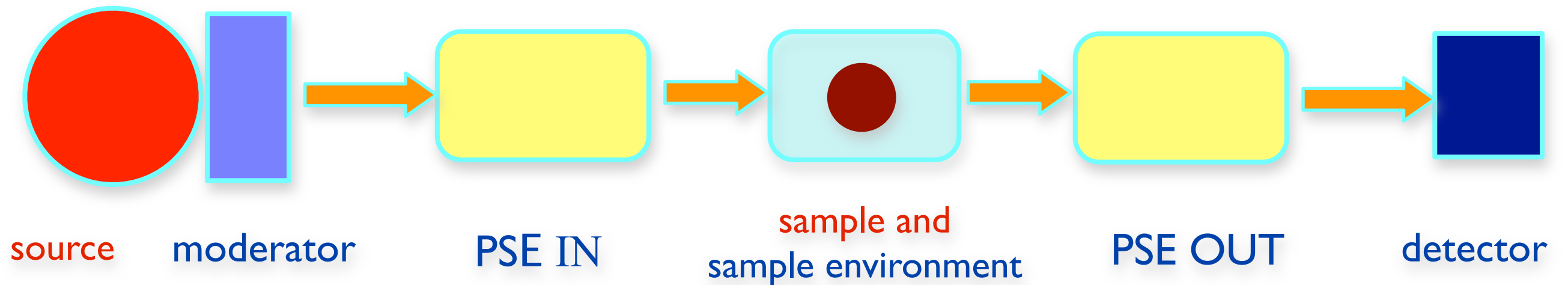
Catherine Pappas

Helmholtz Zentrum Berlin for Materials and Energy,

including slides and animations from

R. Gähler, R. Cywinski, W. Bouwman

from the source to the detector



Neutron flux

$$\varphi = \Phi \eta \frac{dE d\Omega}{4\pi}$$

source flux
distribution

intensity
losses

field of neutron
instrumentation

definition of the beam : Q, E and polarisation

Neutron Spin Echo

why ?

very high resolution without intensity losses

how ?

use the transverse components of
beam polarization

Larmor precession

Neutron Spin Echo

- I: polarized neutrons -Larmor precession
- II: NSE : classical
- III: NSE : semi-classical
- IV: movies
- V: quantum mechanical description ??
- VI: examples
- VII: NSE and structure

Neutron Spin Echo

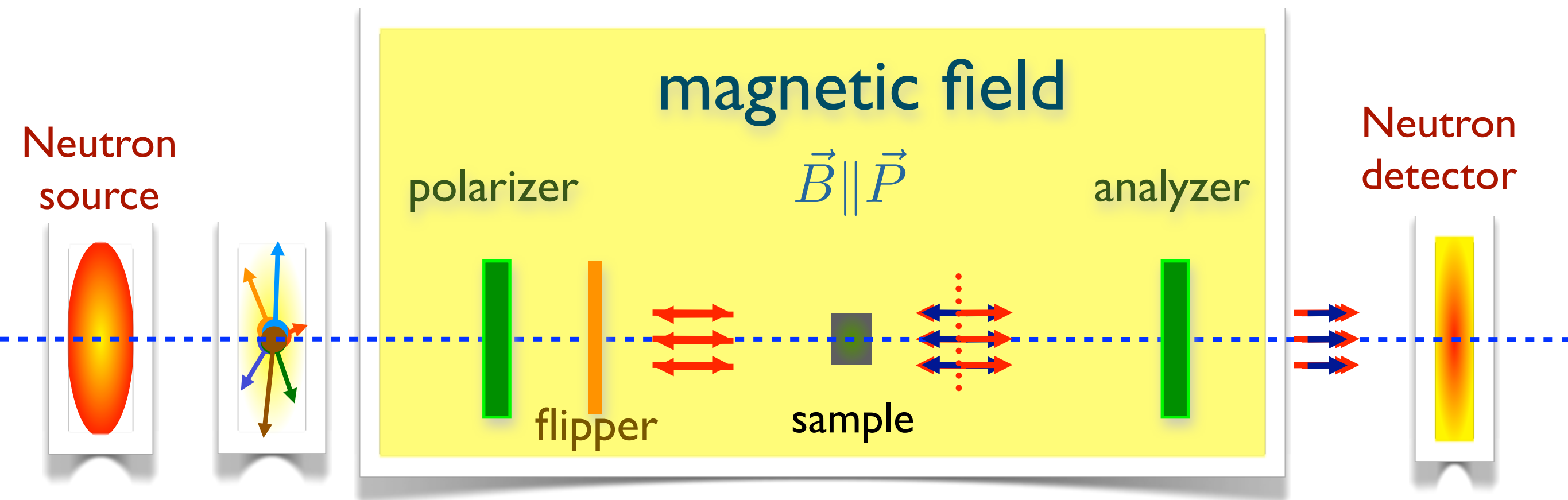
- I: polarized neutrons - Larmor precession
- II: NSE : classical description
- III: NSE : quantum mechanical description
- IV: movies
- V: NSE and coherence
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Polarized Neutrons

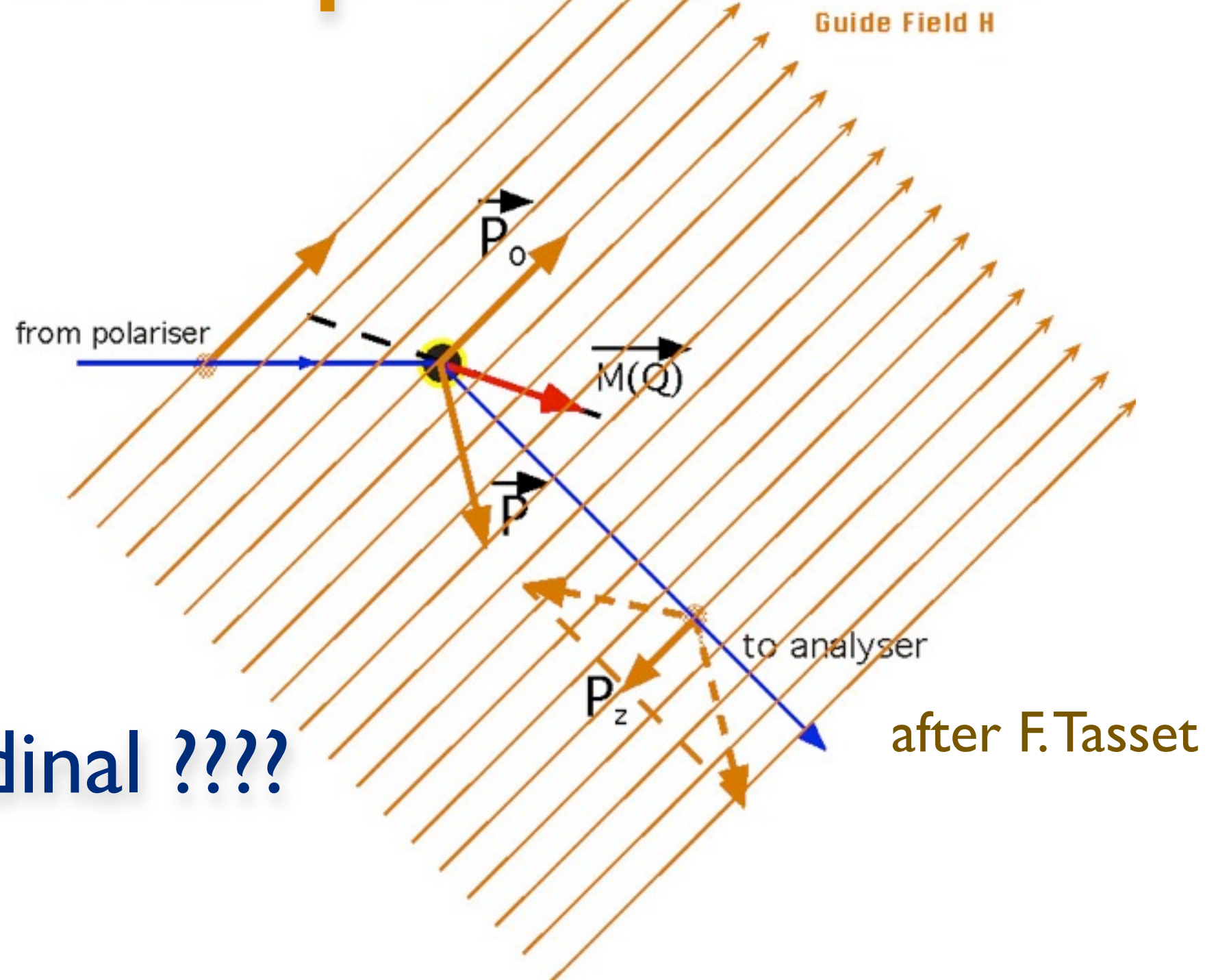
⊙ polarizer

⊙ analyzer

magnetic field (guide - precession)



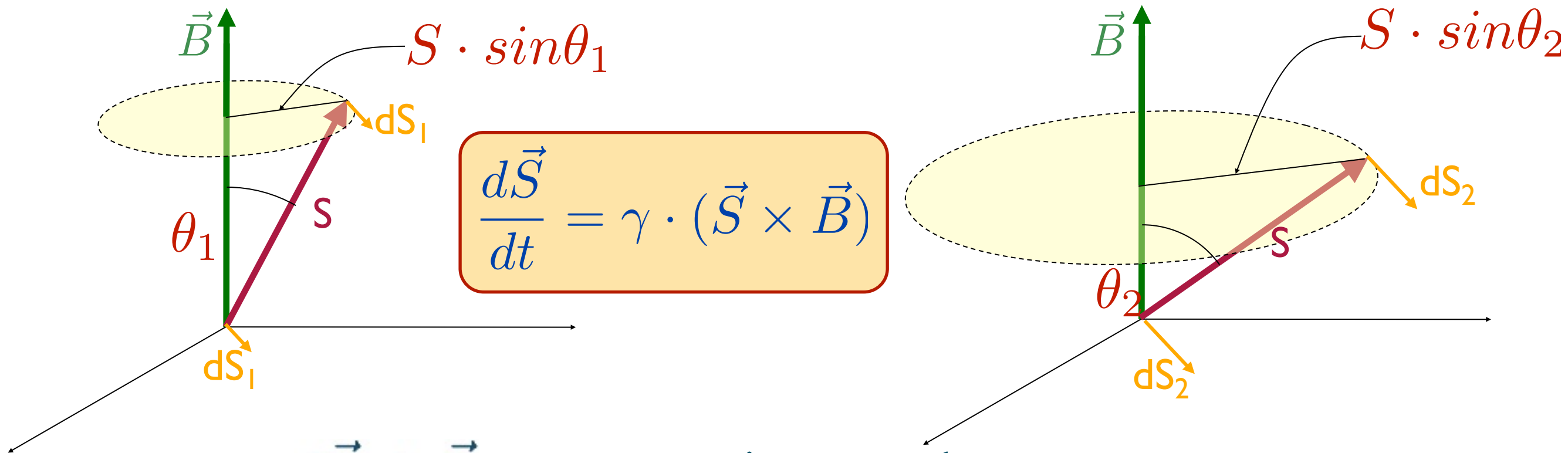
Longitudinal polarization analysis



Why longitudinal ????

because we apply a magnetic field and measure the projection of the polarization vector along this field

Larmor Precession



$d\vec{S} \perp \vec{B} \Rightarrow$ precession around B
 $d\vec{S} \perp \vec{S} \Rightarrow$ precession frequency is constant;

in both cases $dS \propto S \sin\theta$

during dt, the angular change of $S \sin\theta$ around B is constant:

$\Rightarrow$ the precession ‘Larmor’ frequency ω_L does not depend on θ

“gyromagnetic ratio” of neutrons $\gamma = 2.9 \text{ kHz/G}$

???? Precession ????

relation spin - magnetic moment

nucleons

$$\mu_N = \frac{e\hbar}{2m_p}$$

e is the elementary charge,
 $\hbar$ is the reduced Planck's constant,
 m_p is the **proton** rest mass

The values of nuclear magneton

SI $5.050 \times 10^{-27} \text{ J}\cdot\text{T}^{-1}$

CGS $5.050 \times 10^{-24} \text{ Erg}\cdot\text{Oe}^{-1}$

electrons

$$\mu_B = \frac{e\hbar}{2m_e}$$

e is the elementary charge,
 $\hbar$ is the reduced Planck's constant,
 m_e is the **electron** rest mass

The values of Bohr magneton

SI $9.274 \times 10^{-24} \text{ J}\cdot\text{T}^{-1}$

CGS $9.274 \times 10^{-21} \text{ Erg}\cdot\text{Oe}^{-1}$

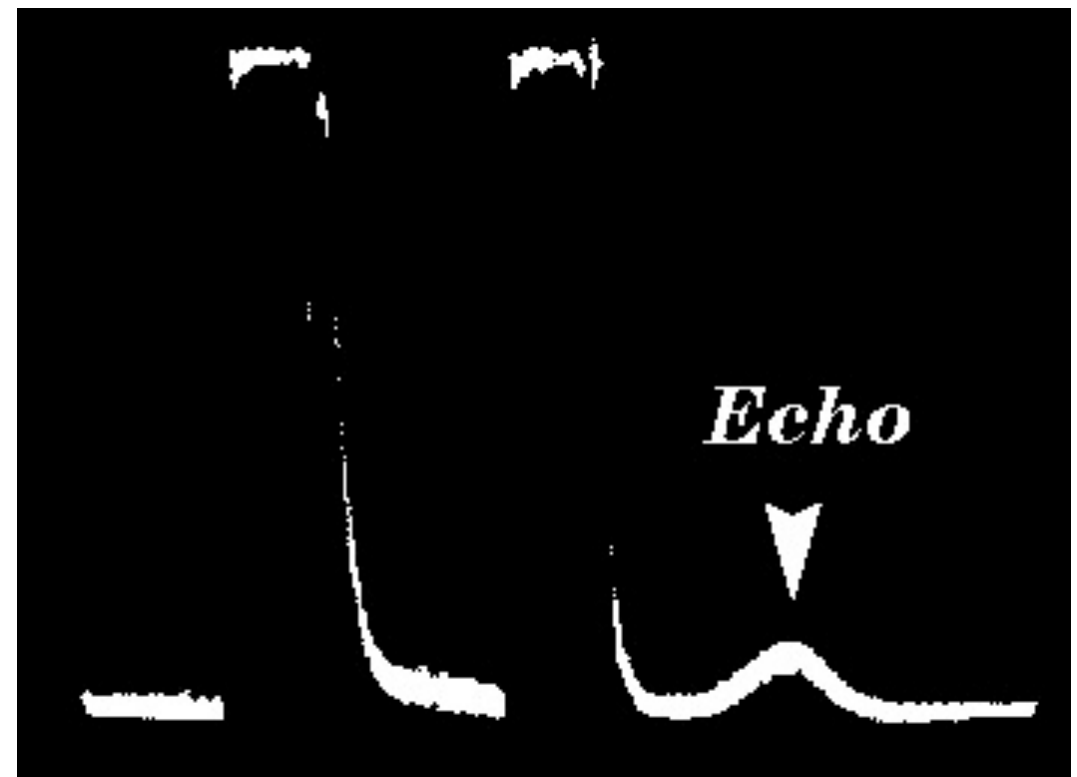
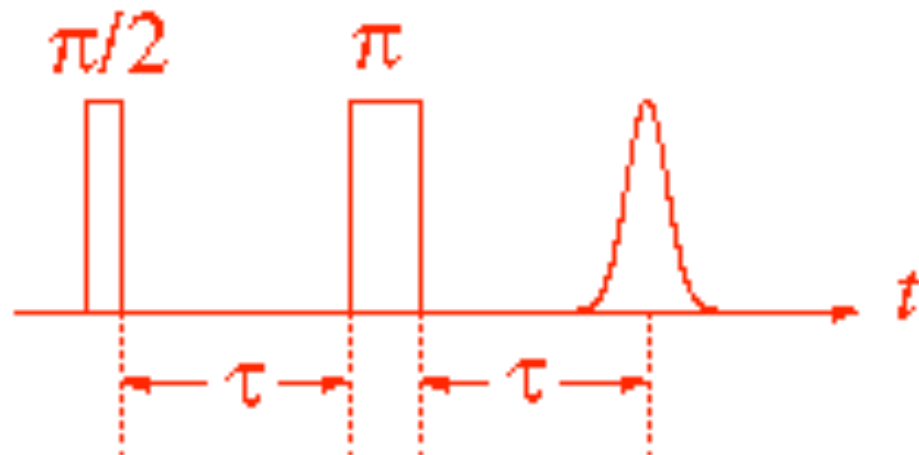
ratio ~ 1800

Larmor Precession

NMR spin echo

Erwin Hahn 1950

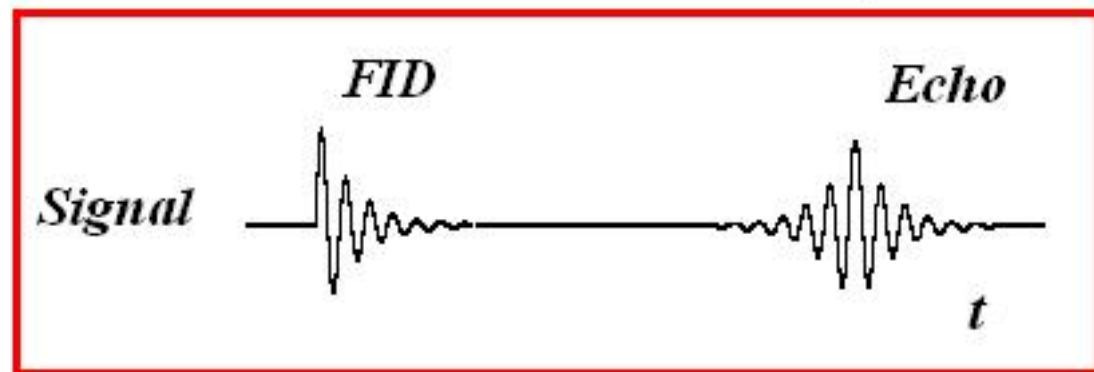
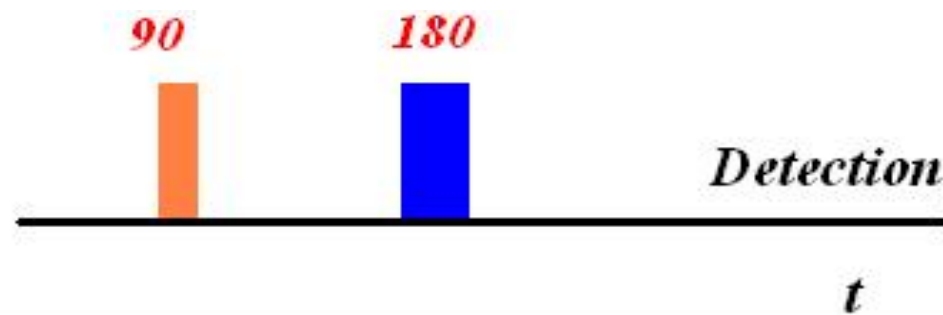
$$\frac{d\vec{\mu}}{dt} = -\gamma \vec{\mu} \times \vec{H} = \vec{\mu} \times \vec{\omega}_L$$



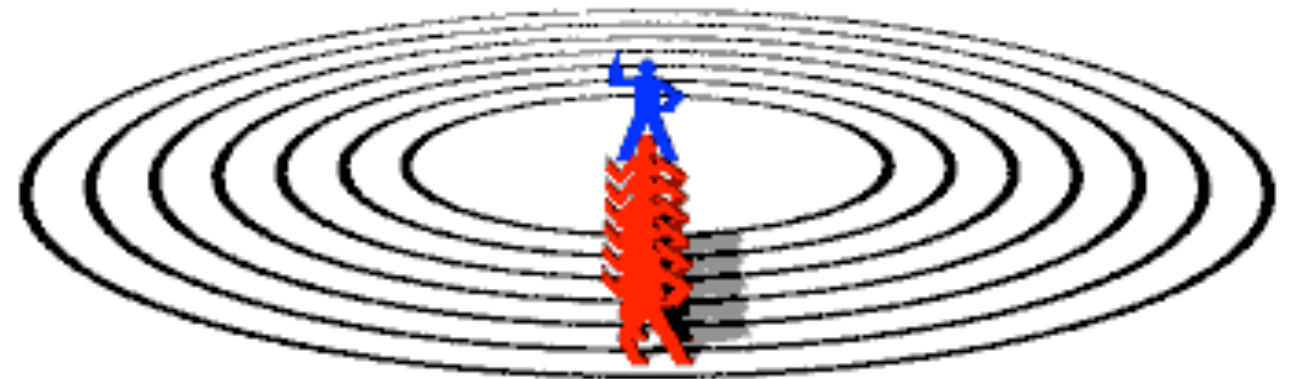
Larmor Precession

NMR spin echo

Erwin Hahn 1950



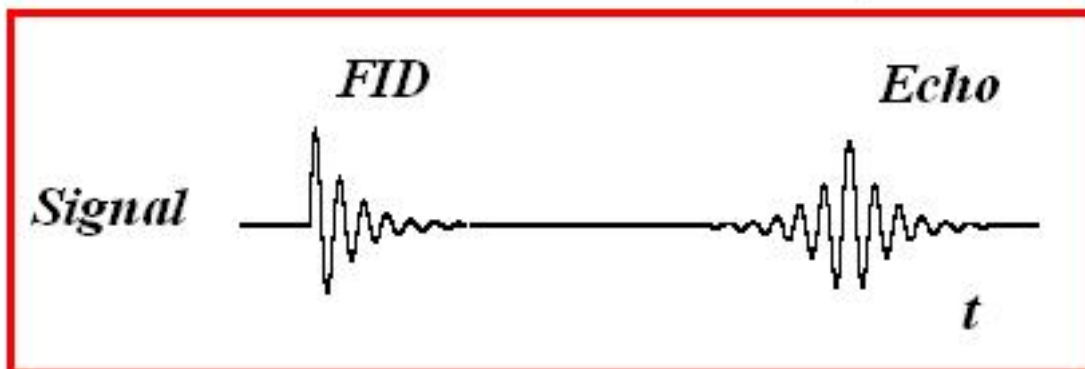
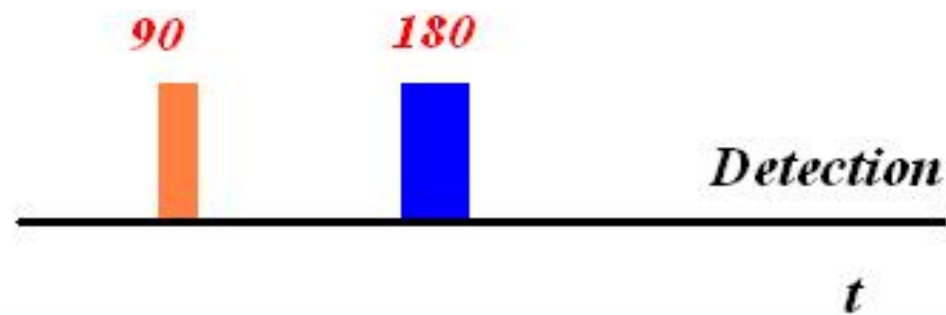
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Larmor Precession

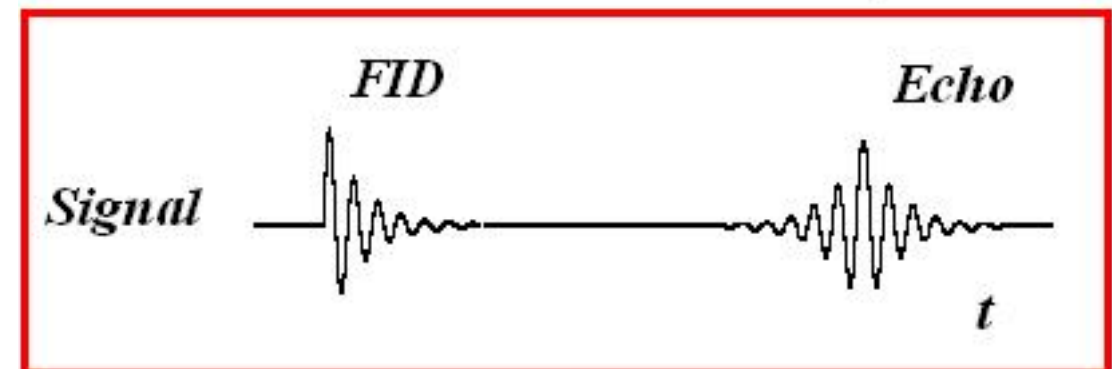
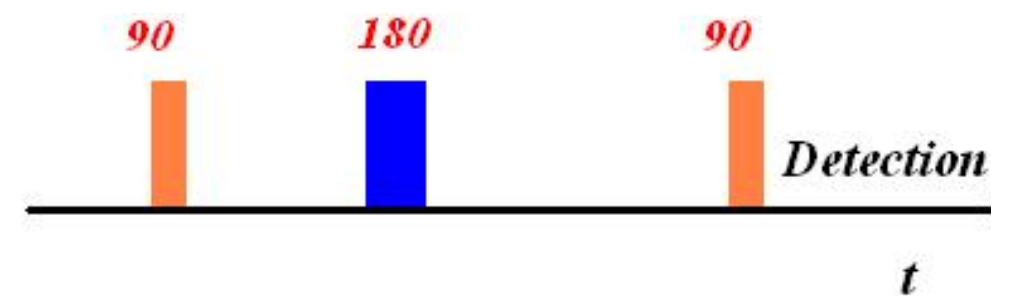
NMR spin echo

Erwin Hahn 1950

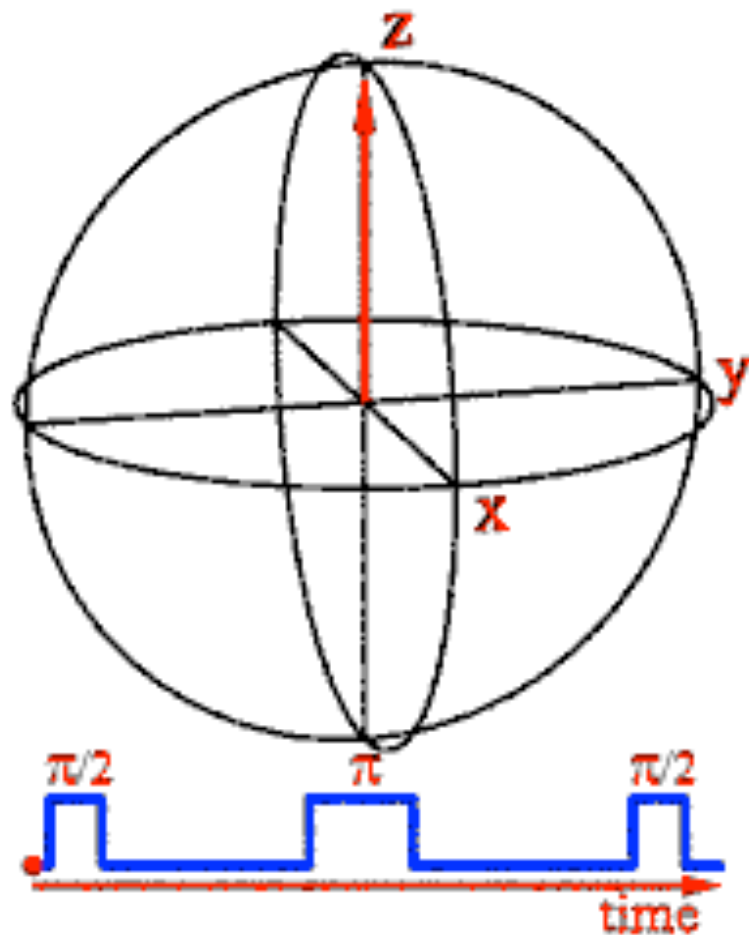


Neutron spin echo

Ferenc Mezei 1972

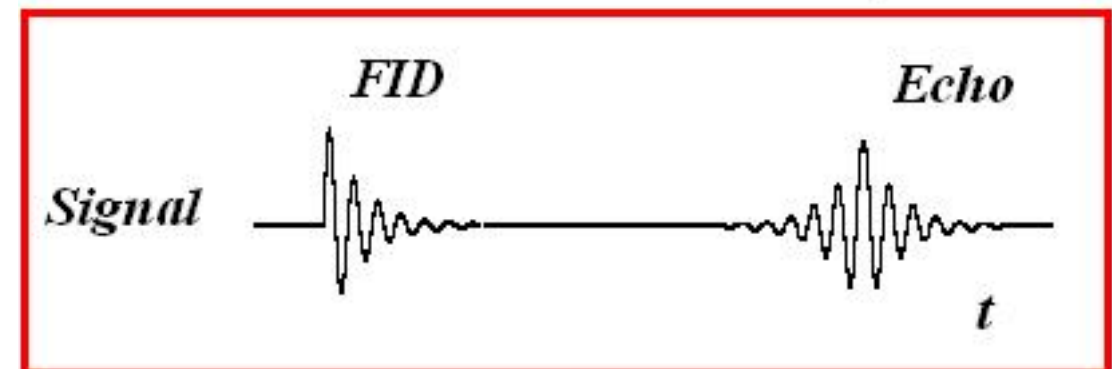
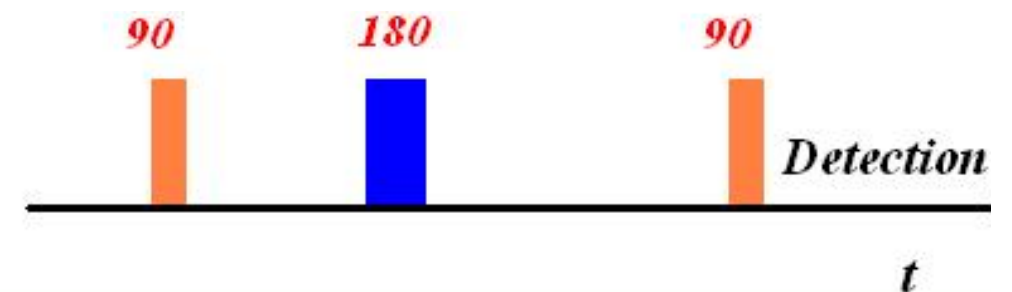


Larmor Precession



Neutron spin echo

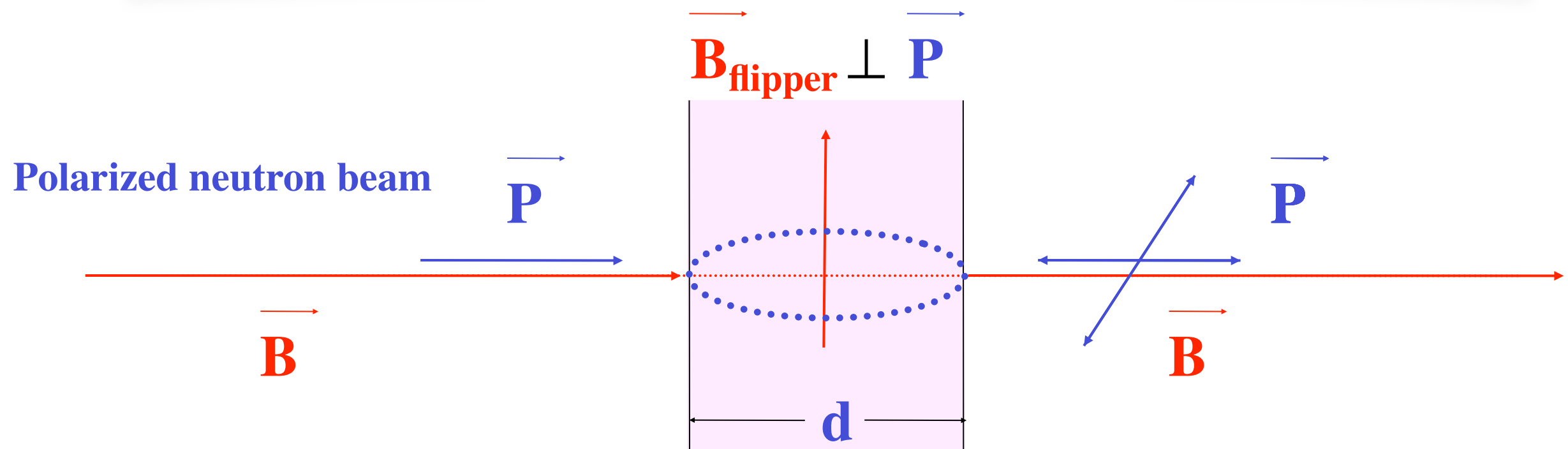
Ferenc Mezei 1972



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Larmor precession and flippers



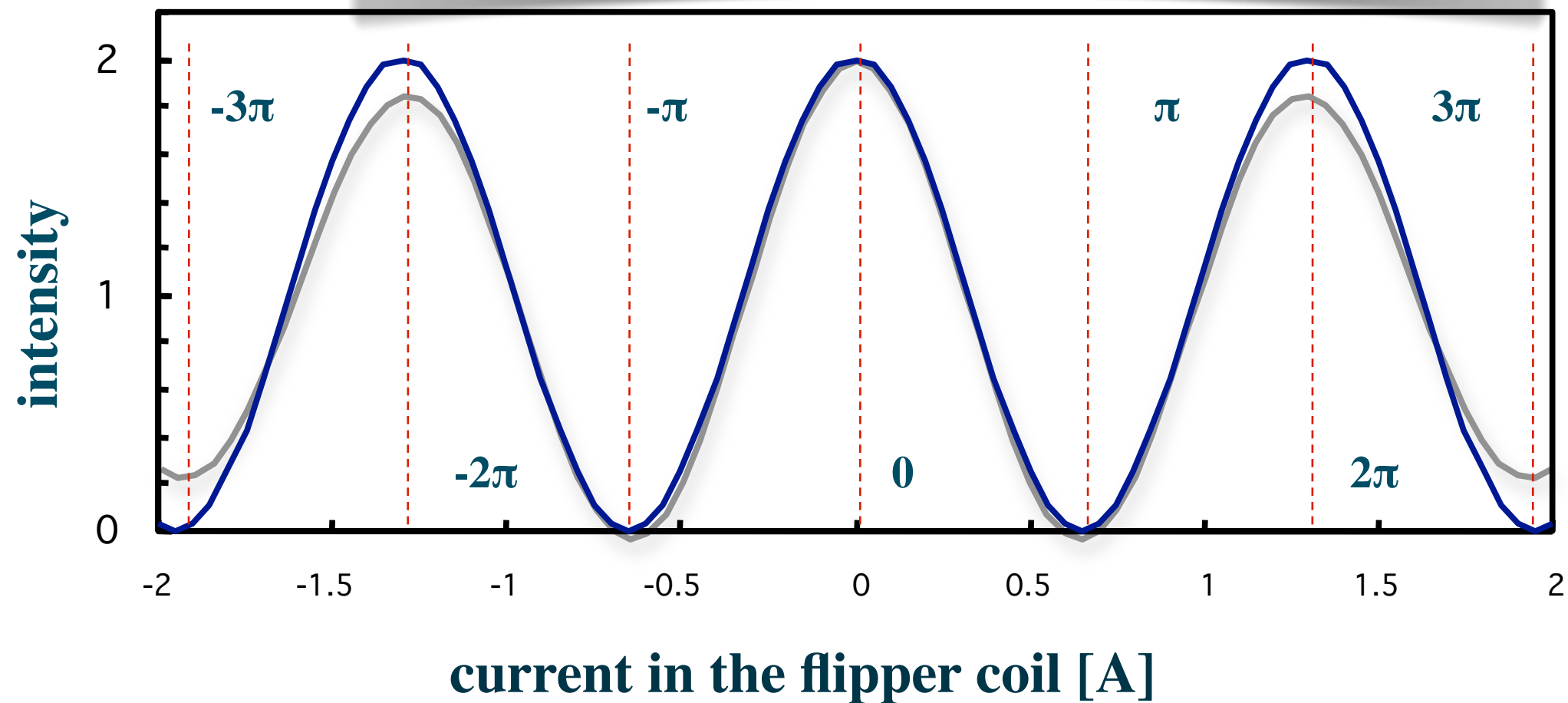
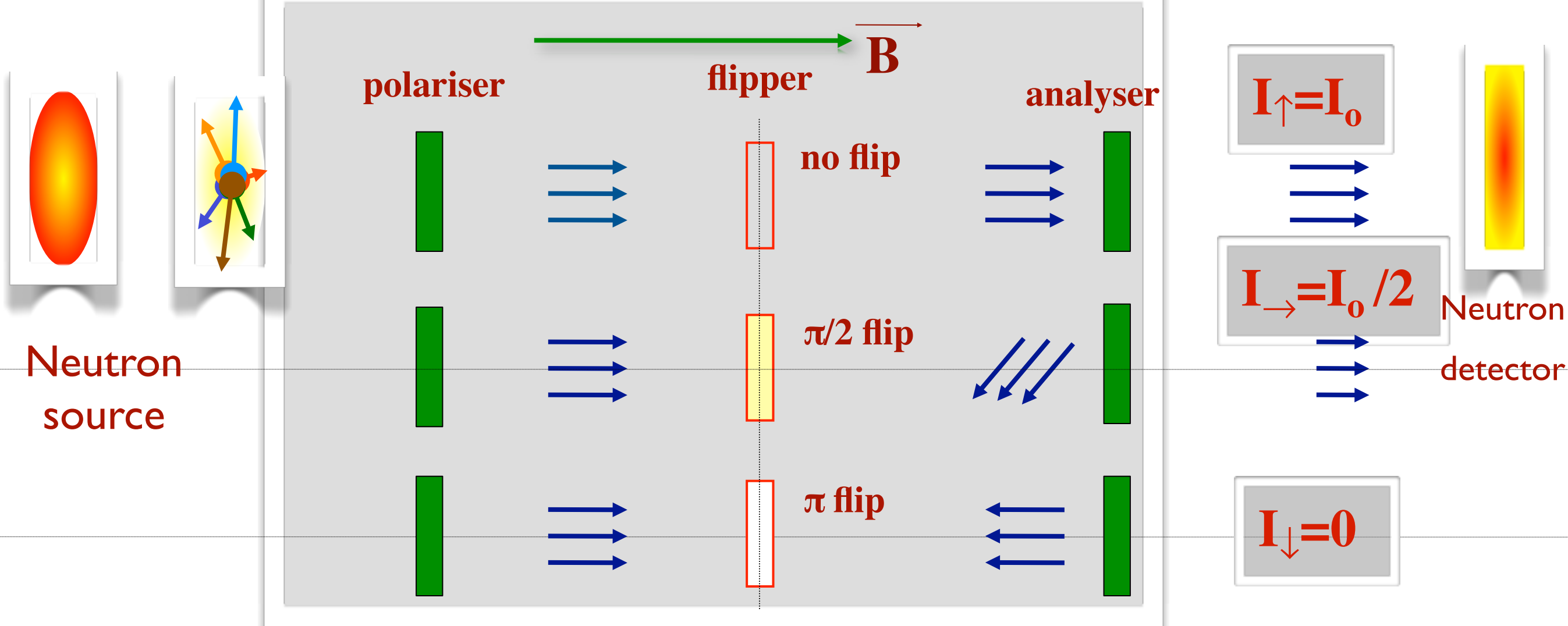
Precession angle

$$\phi = \omega_L t = -\gamma B t = -\gamma B d / v$$

$$\phi = 2916.4 \text{ Hz/G} * 2\pi * B [\text{G}] * d[\text{cm}] * \lambda[\text{\AA}] / 395600$$

For $\phi = 2\pi$

$$B d = 135.7 \text{ G.cm} / \lambda[\text{\AA}] = 1.357 \cdot 10^{-3} \text{ T.m} / \lambda[\text{nm}]$$

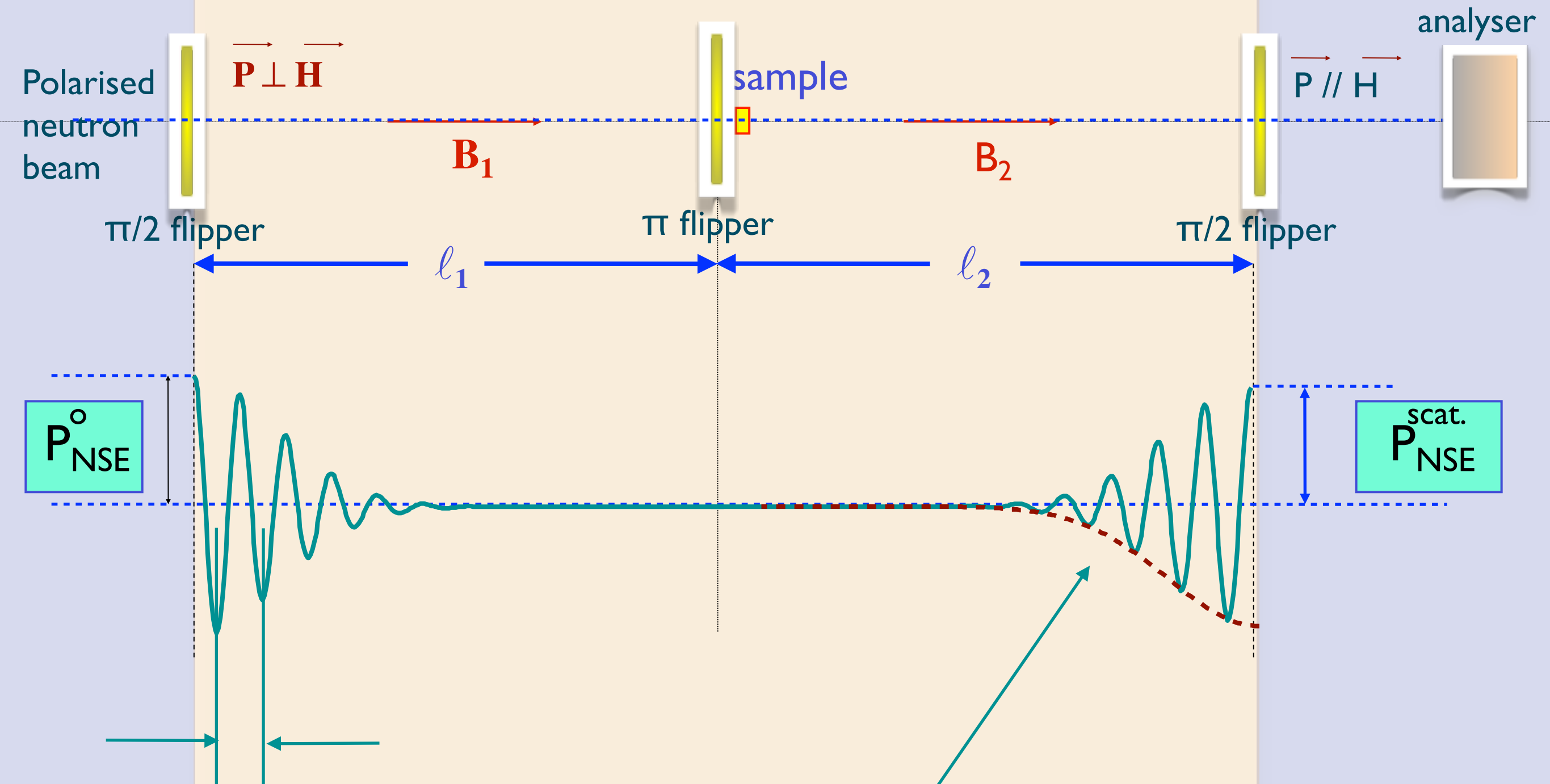


Beam Polarization

$$|\vec{P}| = \frac{I_+ - I_-}{I_+ + I_-}$$

$$\Delta\phi \propto \Delta\lambda$$

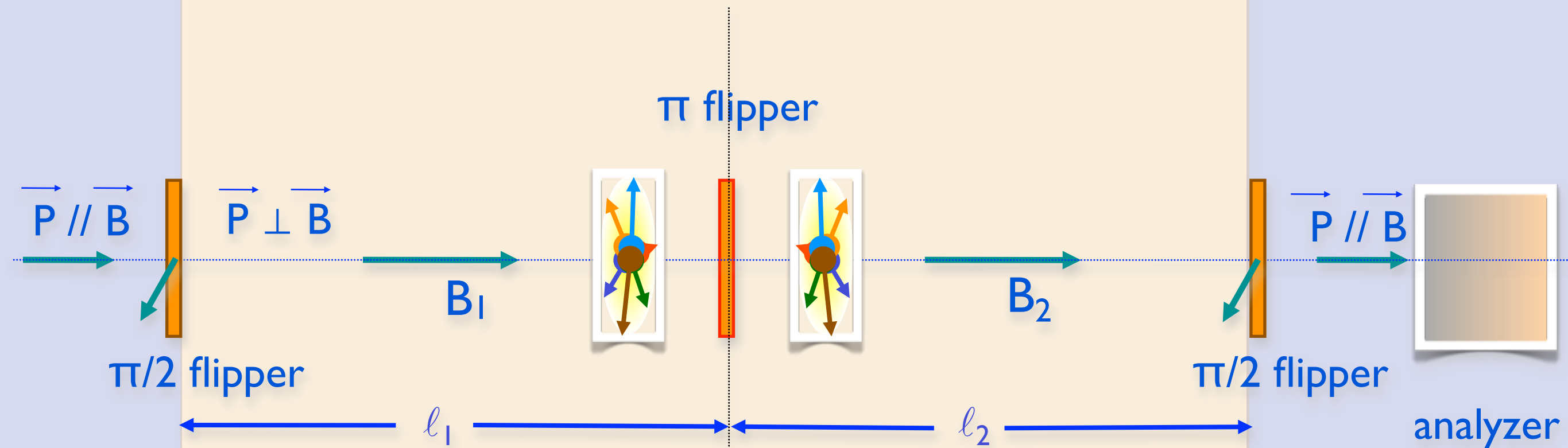
neutron spin echo



$$1.357 \cdot 10^{-3} T.m / \lambda [nm]$$

$$P_{//\vec{B}} = \langle \cos(\gamma \vec{B} \ell / v) \rangle = \int f(v) \cos(\gamma \vec{B} \ell / v) dv$$

neutron spin echo

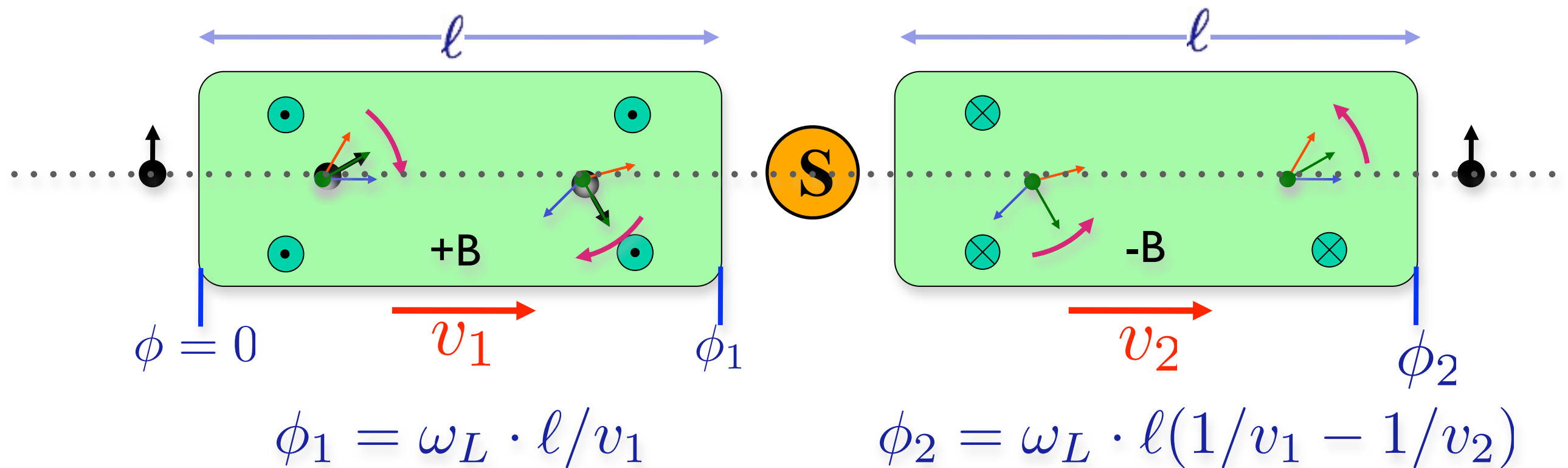


polarized
neutron beam

echo condition does not depend on the
wavelength

$$B_1 l_1 = B_2 l_2$$

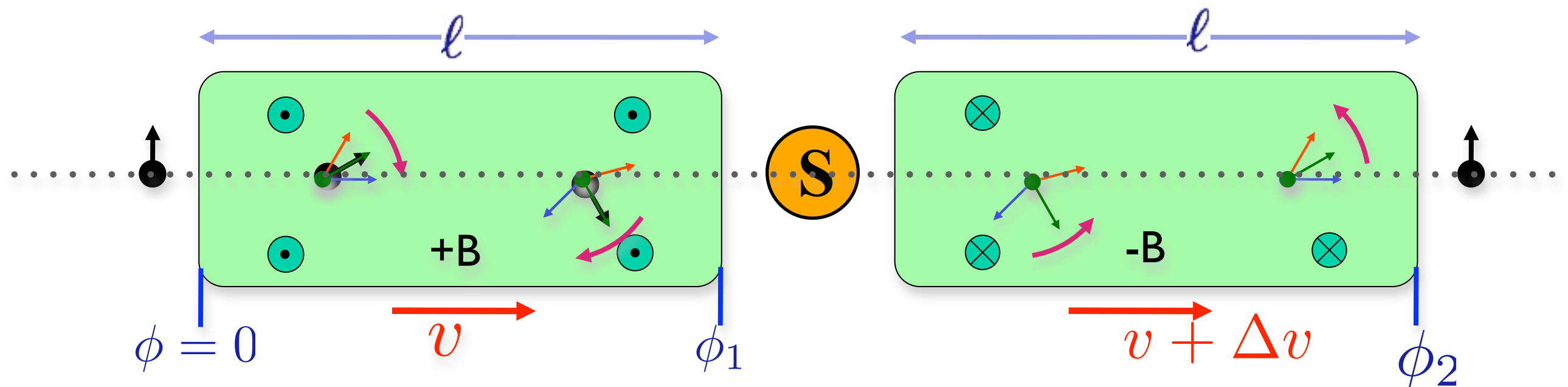
basic equations



for $v_1 = v$ and $v_2 = v + \Delta v \Rightarrow$

$$\phi_2 = \omega_L \ell [1/v - 1/(v + \Delta v)] \approx \omega_L \ell \Delta v / v^2 = \omega_L t \Delta v / v$$

basic equations

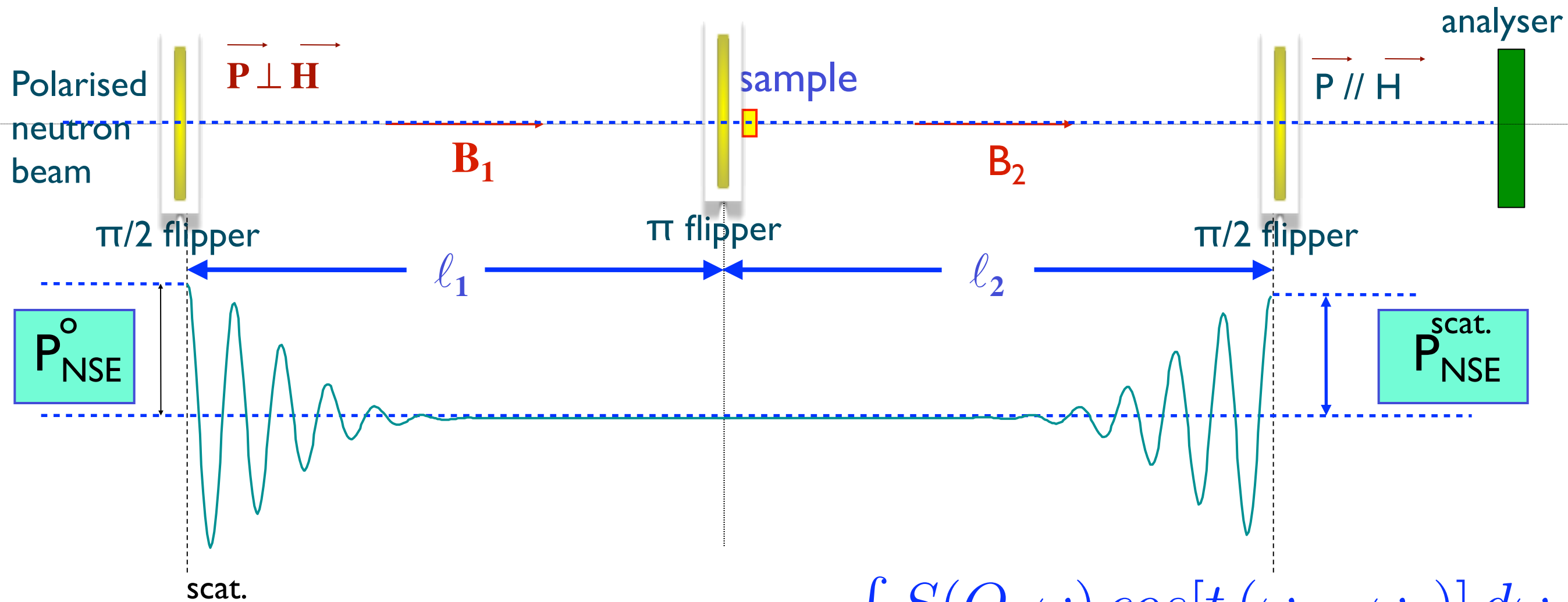


scattering theory : $\Delta v \rightarrow \omega$

$$\hbar \omega = m/2 \cdot (v_1^2 - v_2^2) = m/2 \cdot (v_1 + v_2)(v_1 - v_2) \approx m \cdot v \cdot \Delta v$$

$$\phi_2 - \phi_1 = \omega_L \ell [1 - v t_{NSE} / (v + \Delta v)] \approx \omega_L \ell \Delta v / v^2 = \omega_L t \Delta v / v$$

and $t_{NSE} = \omega_L \cdot \ell \cdot \hbar / (m v^3) = \omega_L \cdot t / (2 \omega_o)$



$$P_{NSE} = P_s \langle \cos(\phi - \langle \phi \rangle) \rangle = P_s \frac{\int S(Q, \omega) \cos[t(\omega - \omega_o)] d\omega}{\int S(Q, \omega) d\omega}$$

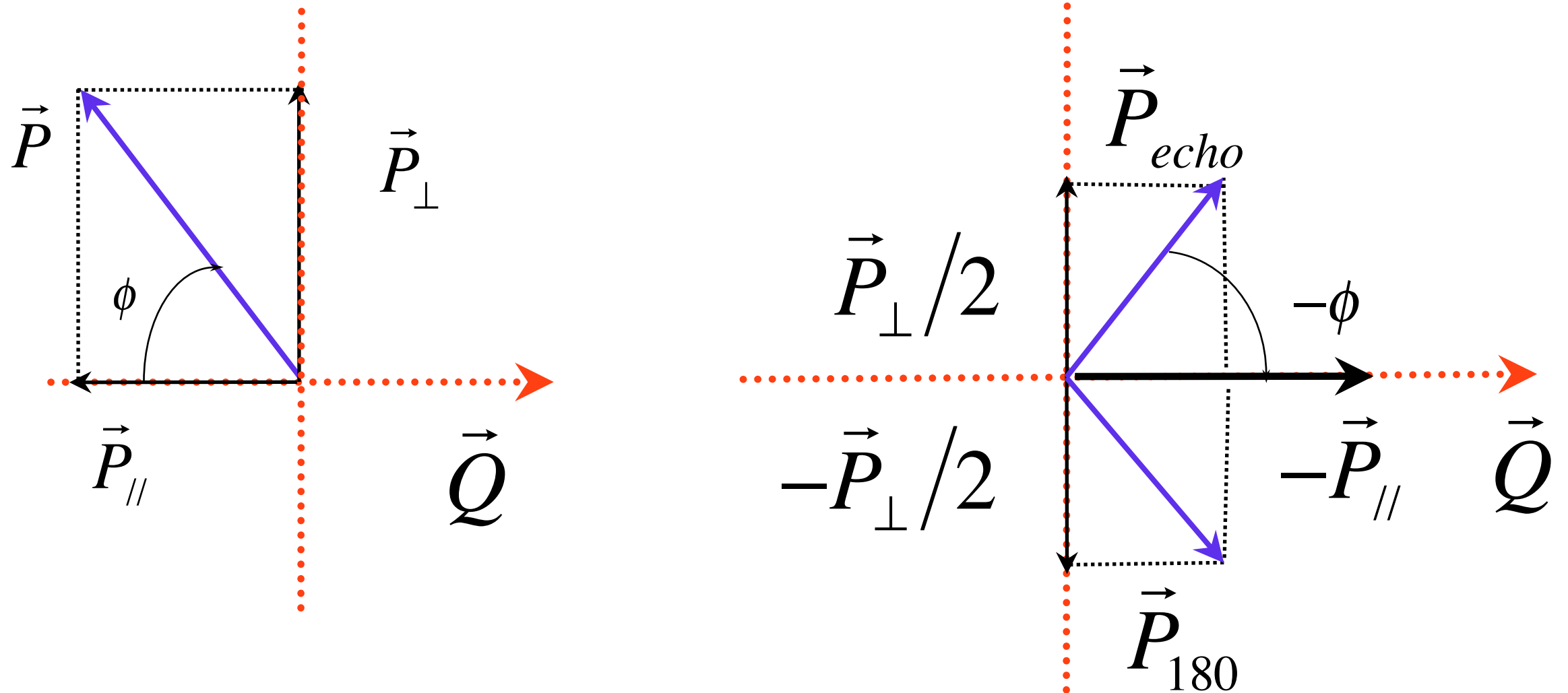
for quasi-elastic scattering $\omega_o = 0$

$$P_{NSE}^{scat}/P_s = \Re [S(Q, t)]/S(Q) = I(Q, t)$$

most generally $\phi - \langle \phi \rangle = f(\vec{q}, \omega) \propto S(\vec{Q}, t)$
locally

paramagnetic scattering :

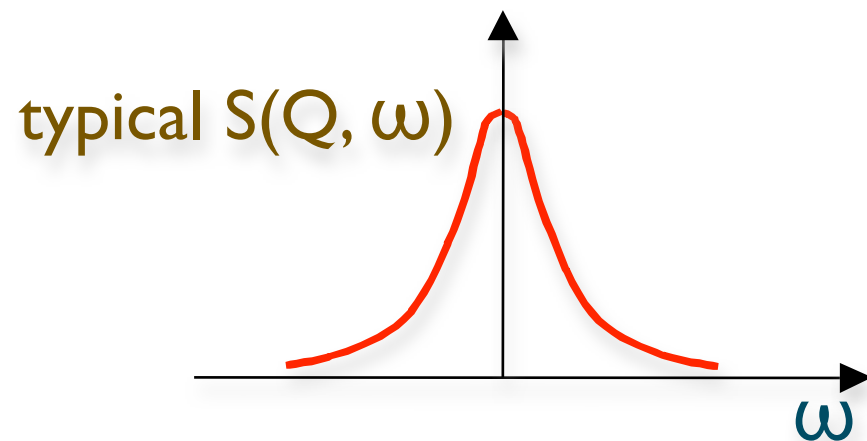
$$\vec{P}' = -\hat{Q} (\hat{Q} \cdot \vec{P})$$



the π flipper is the sample

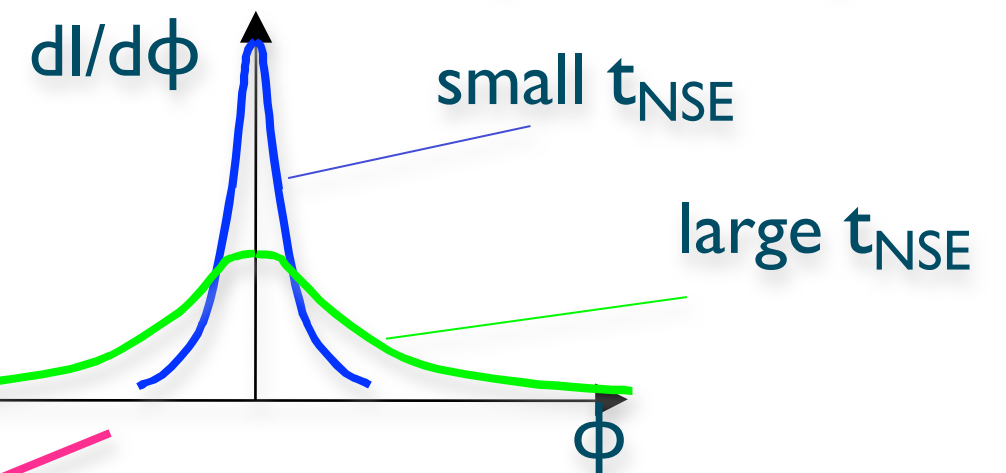
measuring principle

$S(Q, \omega)$: probability for a momentum change Q and an energy change ω upon scattering

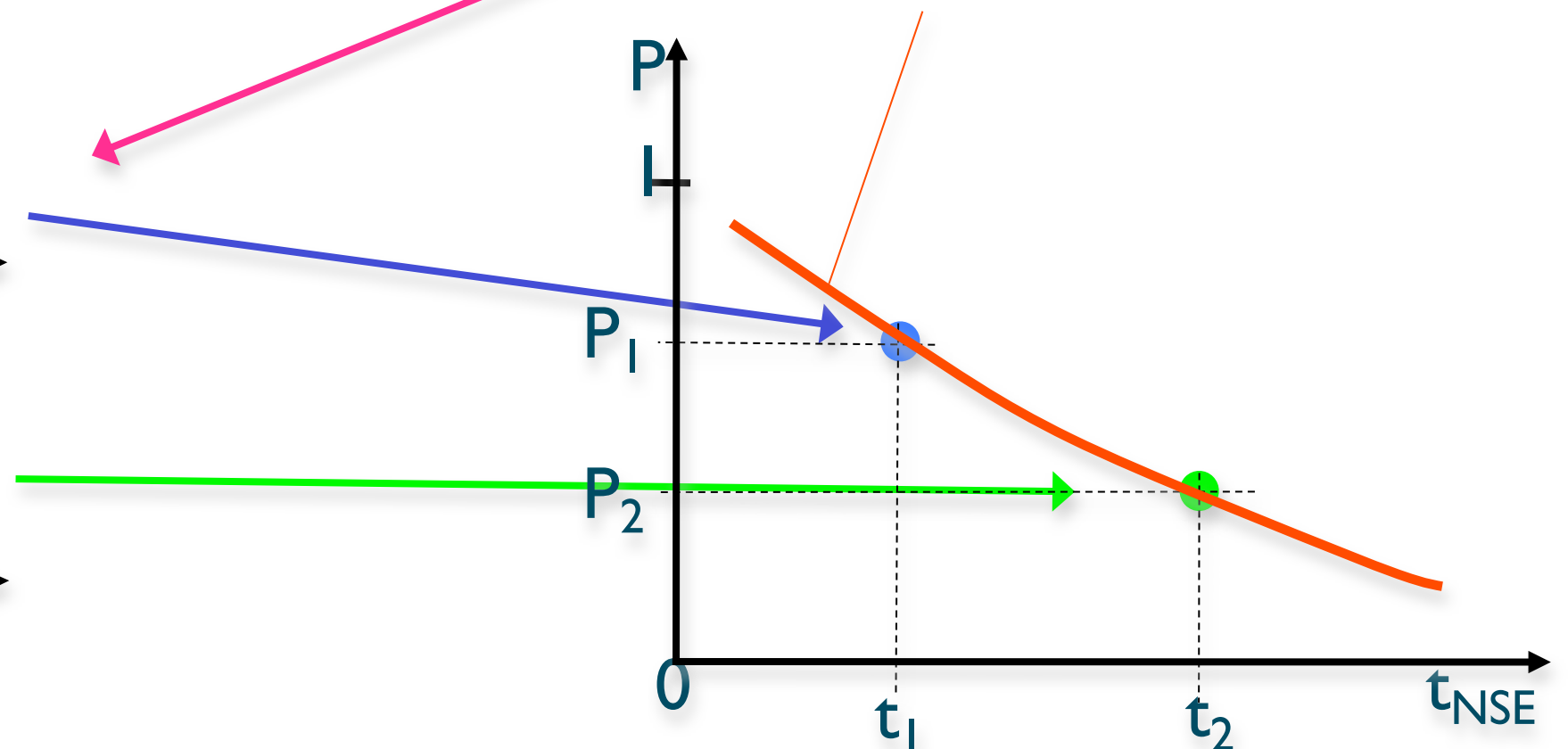
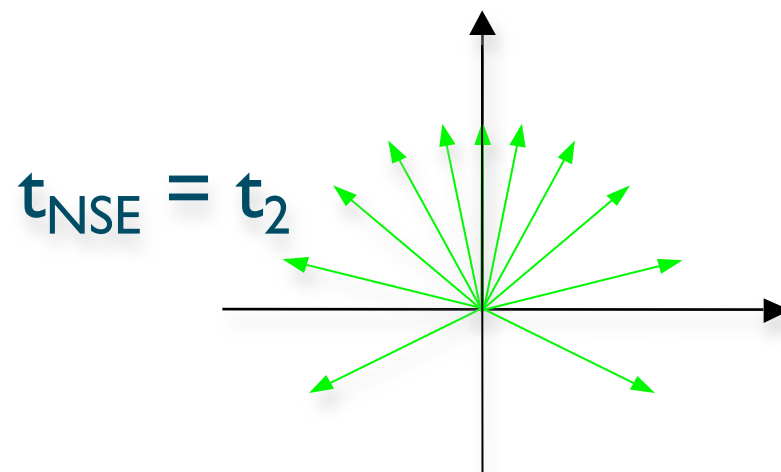
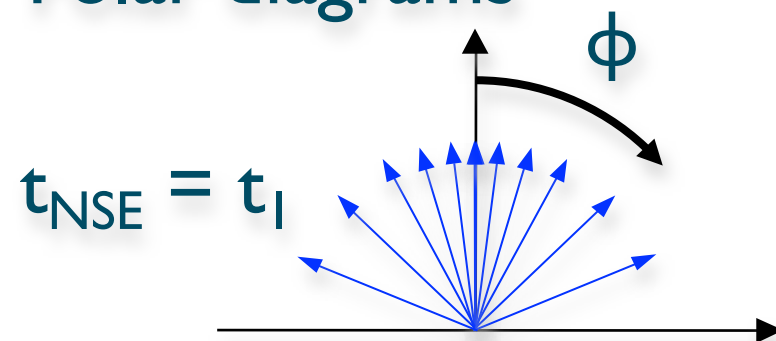


$$\phi = t_{\text{NSE}} \cdot \omega$$

distribution of spins at analyzer

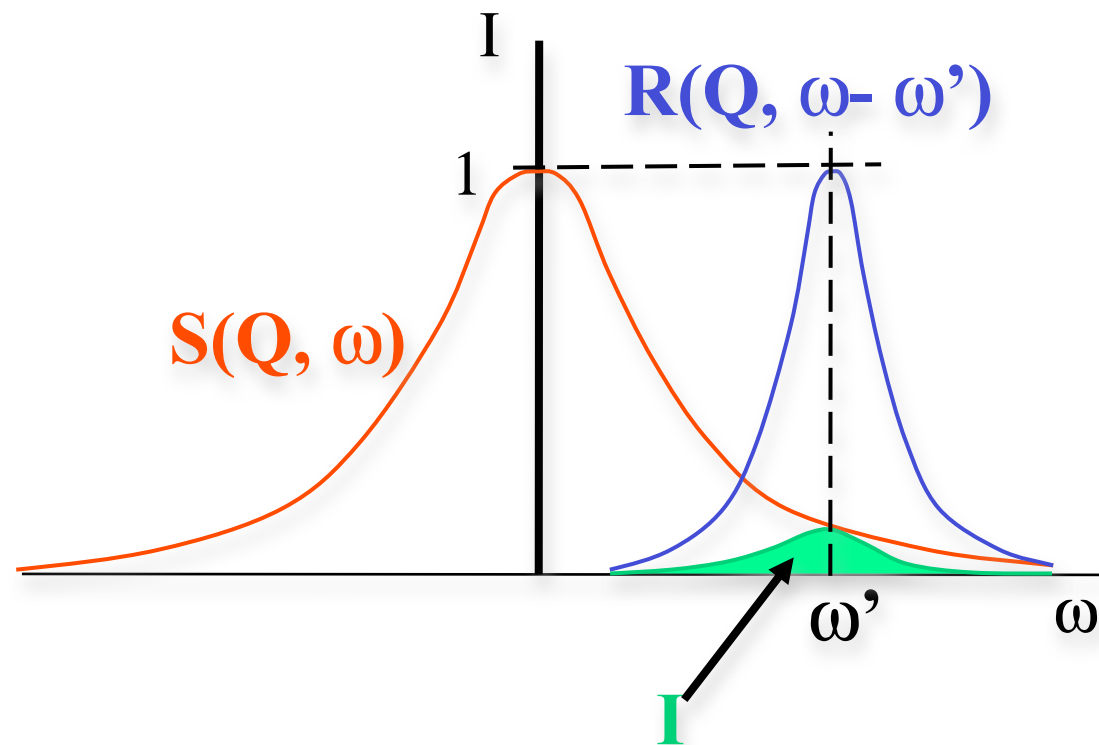


Polar diagrams



Fourier transform and resolution

Let $S(Q, \omega)$ be the scattering function and let $R(Q, \omega)$ be the resolution function.



If a spectrometer is set to ω' ,
then the norm. countrate is:

$$I(Q, \omega') = \int S(Q, \omega) R(Q, \omega - \omega') d\omega \\ = S \otimes R; \text{convolution};$$

The function $R(Q, \omega - \omega')$ should be
the same over the range,
where $S(Q, \omega)$ is significant;

If, like in spin echo, the Fourier Transform of I is the signal,
then the convolution of S and R can be written as

$$\text{FT}(I) = \text{FT}\{S \otimes R\} = \text{FT}\{S\} \cdot \text{FT}\{R\}$$

$$\text{For NSE: } \text{FT}(I) = \text{FT}\left\{ \frac{\gamma^2}{\gamma^2 + \omega^2} \otimes \frac{\gamma_o^2}{\gamma_o^2 + \omega^2} \right\} = e^{-\gamma t} e^{-\gamma_o t} = e^{-(\gamma + \gamma_o)t}$$

Subtleties of NSE

fields are mostly longitudinal;

adiabatic transitions at ends;

‘ $\pi/2$ coils’ to start or end precession;

‘ π coils’ to reverse effective field direction; (Hahn’s echoes; NMR-imaging);

‘Fresnel coils’ to compensate field inhomogeneities

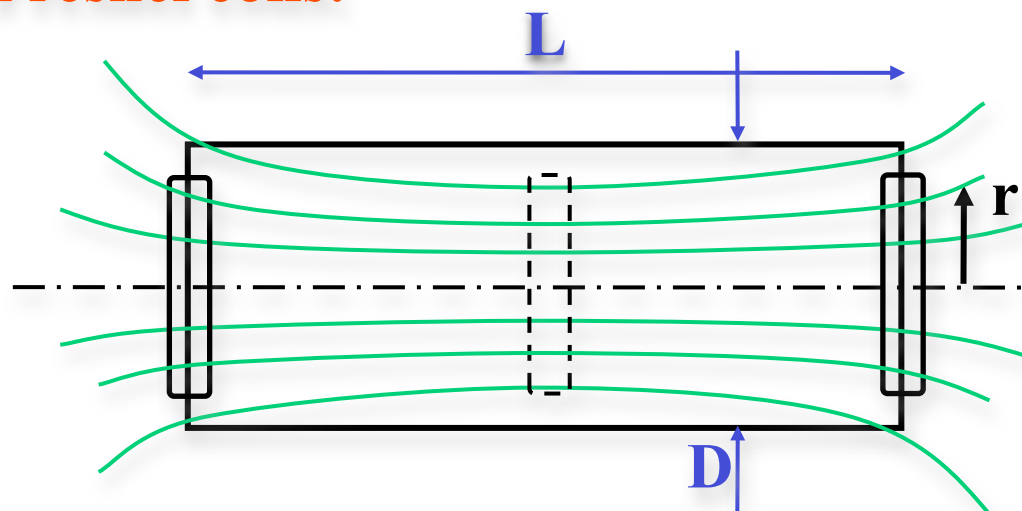
Adiabaticity parameter; at sample; at coils;

How to measure polarization?

Spin flip due to spin-incoherent (2/3)
or paramagnetic scattering (depends on $O(\vec{P}, \vec{Q})$)

Spin echo for ferromagnetic samples;

Fresnel coils:



Standard cylindrical coil:

$$\int B dl \approx B_0 L + B_0 r^2/2D;$$

Correction by current loops:

$$\int \mathbf{B}_F df \sim I;$$

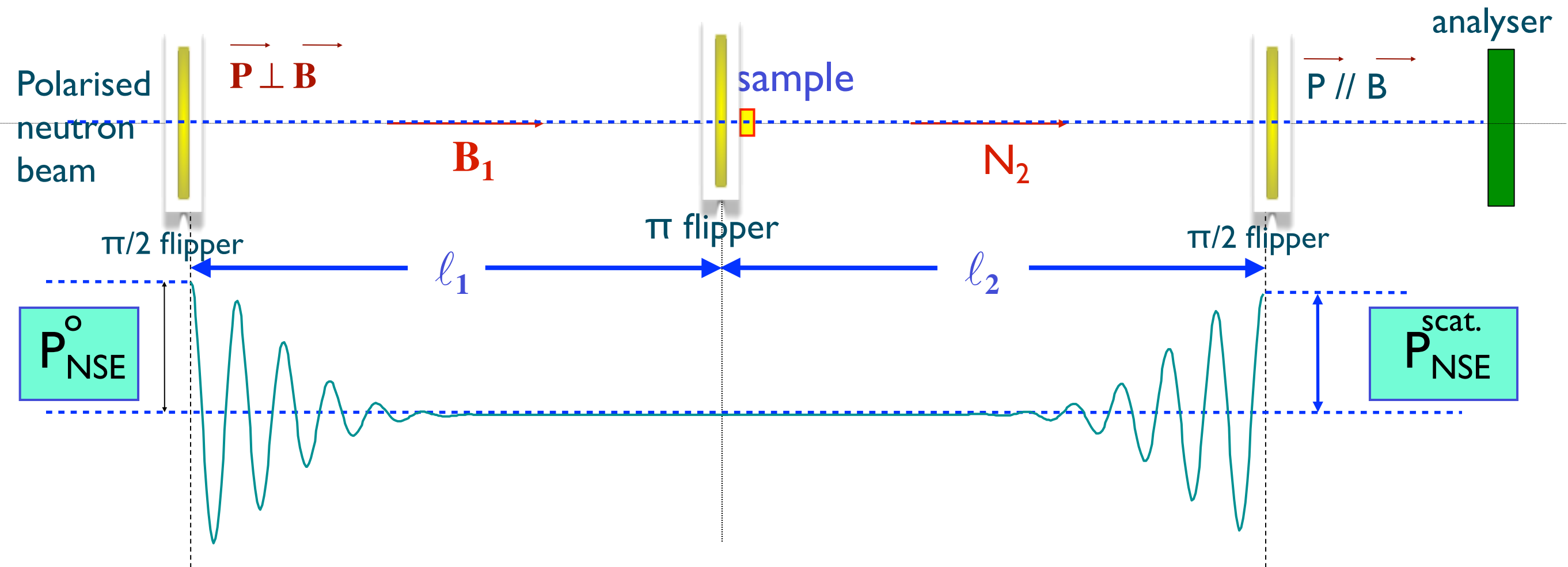
Current around loop

Fresnel coil



density of loops
increases with r^2 ;

Is the echo amplitude a polarization ???

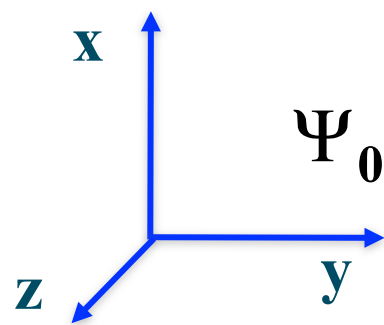


what is the polarization of the beam ???

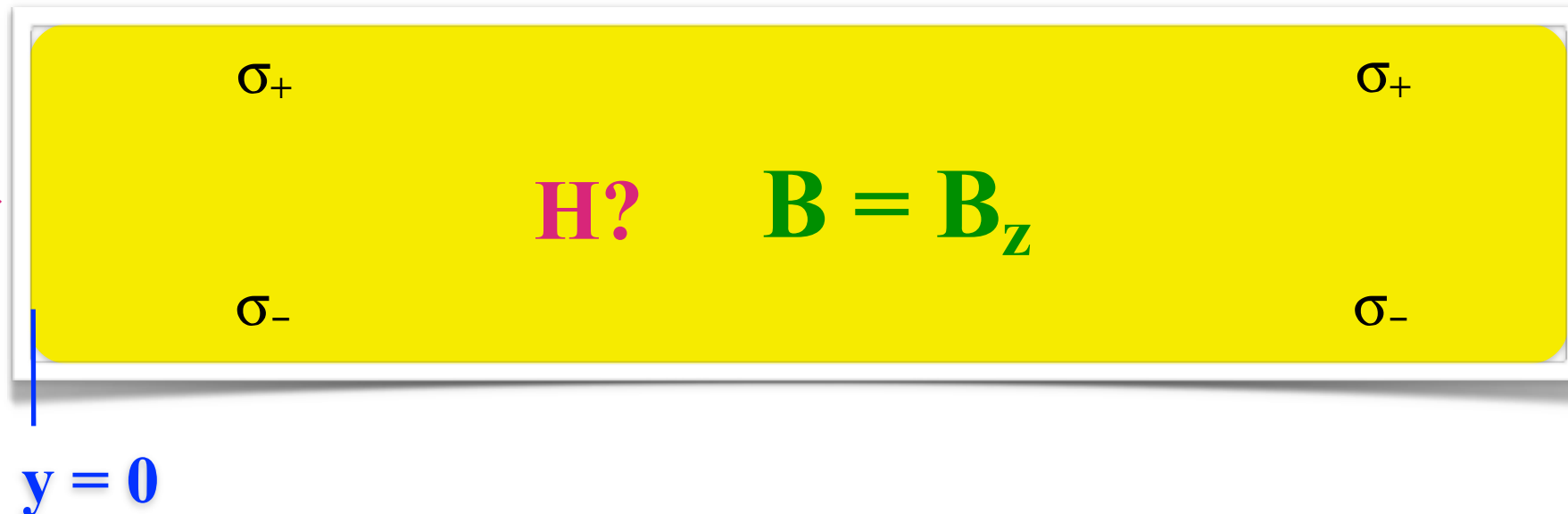
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Plane wave entering a static B-field



$$\Psi_0 = e^{i(k_0 y - \omega_0 t)} \Rightarrow$$



the 2 states Ψ_+ and Ψ_- have different kinetic energies $E_0 \pm \mu \cdot B$

Static case $[dB/dt = 0]$: no change in total energy ($\omega = \omega_0$) but change in k

$$\frac{\hbar^2 \mathbf{k}_{\pm}^2}{2m} = \frac{\hbar^2 \mathbf{k}_0^2}{2m} \pm \mu B; \Rightarrow \frac{\hbar^2}{2m} (\mathbf{k}_{\pm}^2 - \mathbf{k}_0^2) = \pm \mu B;$$

$$\mu \cdot B, E_{\text{kin}} \Rightarrow (\mathbf{k}_{\pm}^2 - \mathbf{k}_0^2) \approx (\mathbf{k}_{\pm} - \mathbf{k}_0) 2\mathbf{k}_0 = \Delta \mathbf{k}_{\pm} \cdot 2\mathbf{k}_0; \quad \Delta \mathbf{k}_{\pm} = \frac{2m}{\hbar^2} \frac{\mu B}{2\mathbf{k}_0} = \frac{\mu B}{\hbar \times \mathbf{v}}$$

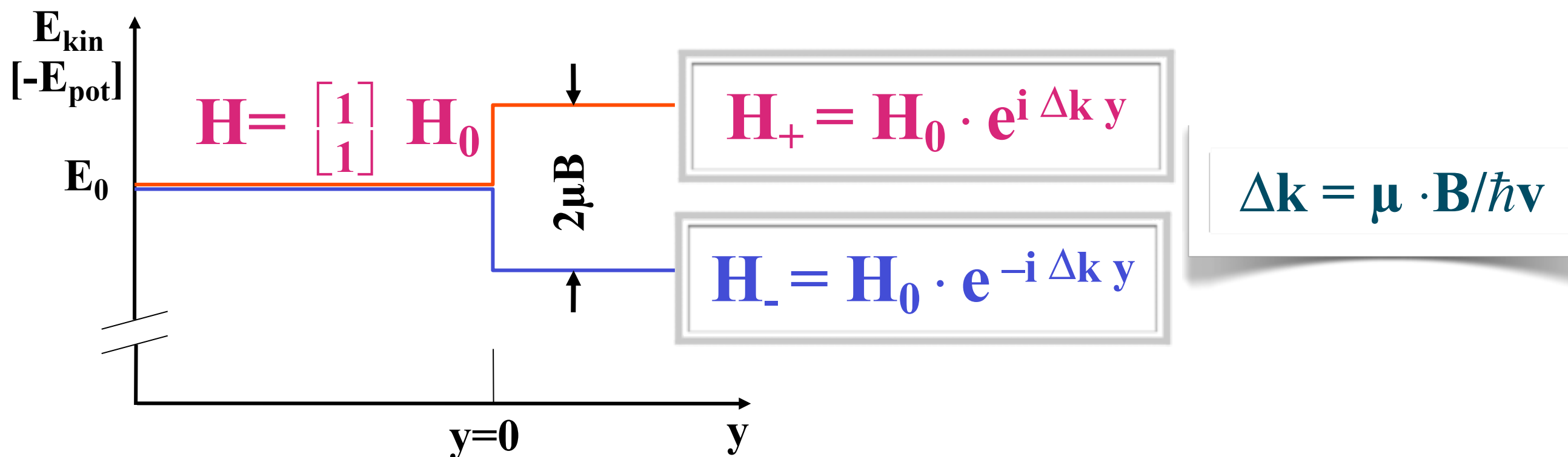
$\mathbf{v}$ is the classical neutron velocity

$$\Psi = \begin{vmatrix} \Psi_+ \\ \Psi_- \end{vmatrix} = \begin{vmatrix} e^{i(yk_+ - \omega_o t)} \\ e^{i(yk_- - \omega_o t)} \end{vmatrix} = e^{i(yk_o - \omega_o t)} \begin{vmatrix} e^{+iy\Delta k} \\ e^{-iy\Delta k} \end{vmatrix}$$

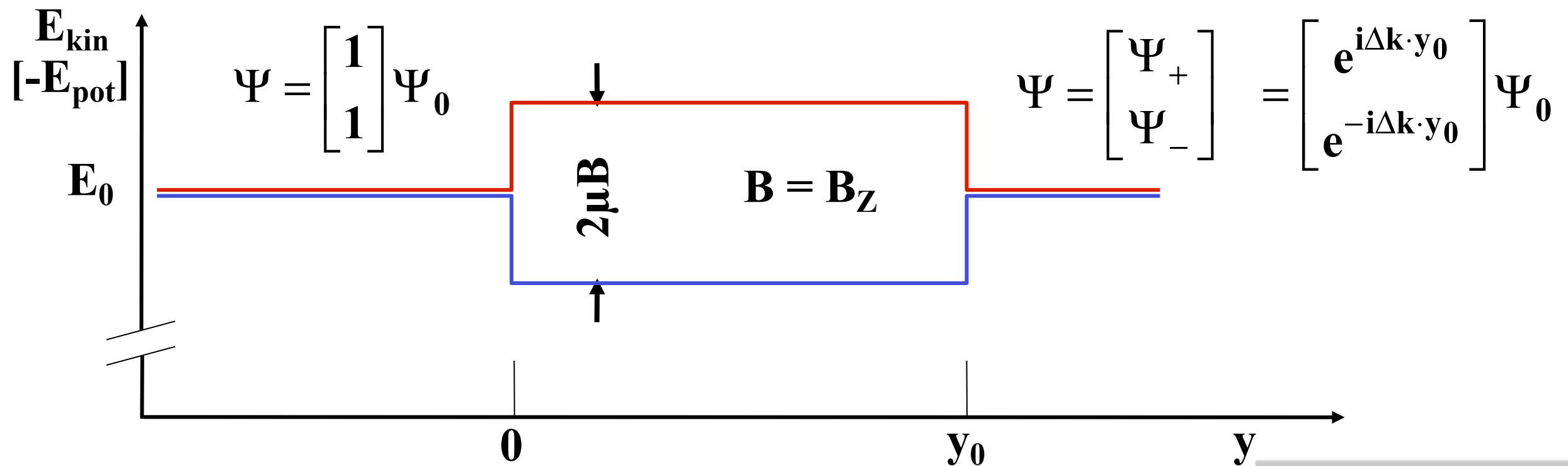
Both states have equal amplitudes, as the initial polarization is perpendicular to the axis of quantization (z-axis);

These amplitudes $\frac{1}{\sqrt{2}}$ are set to 1 here.

Energy diagram:



in a magnetic field



Setting the polarizer to x-direction:

$$\Delta k = \mu \cdot B / \hbar v$$

$$t_0 = y_0 / v$$

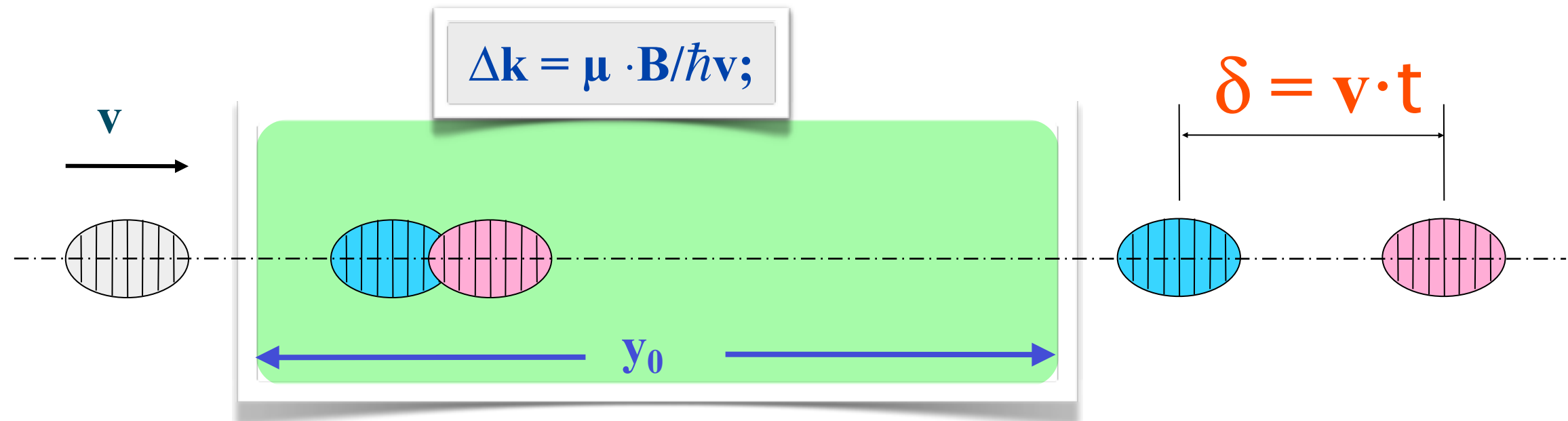
$$I_x = \frac{1}{\sqrt{2}} (\Psi_+ + \Psi_-) \times CC = \frac{1}{2} (e^{i\Delta k \cdot y_0} + e^{-i\Delta k \cdot y_0}) (e^{-i\Delta k \cdot y_0} + e^{i\Delta k \cdot y_0})$$

$$I_x = \frac{1}{2} (1 + 1 + e^{2i\Delta k \cdot y_0} + e^{-2i\Delta k \cdot y_0}) = 1 + \cos(2\Delta k \cdot y_0) = 1 + \cos\left(\frac{2\mu B}{\hbar v} \cdot y_0\right)$$

$$I_x = 1 + \cos(\omega_L t_0); \quad [I_y = 1 + \sin(\omega_L t_0)]; \quad \omega_L = 2\mu B / \hbar; \quad \text{Larmor precession!}$$

wavepackets instead of plane waves

wavepacket (bandwidth $\Delta\lambda$) of length Δy and lateral width $\Delta x = \Delta z$;
 $\Delta y \approx \lambda^2 / \Delta\lambda$; $\Delta x \approx \lambda / (2\pi\theta)$; θ = beam divergence; typ. values: $\Delta x, \Delta y \approx 100 \text{ \AA}$

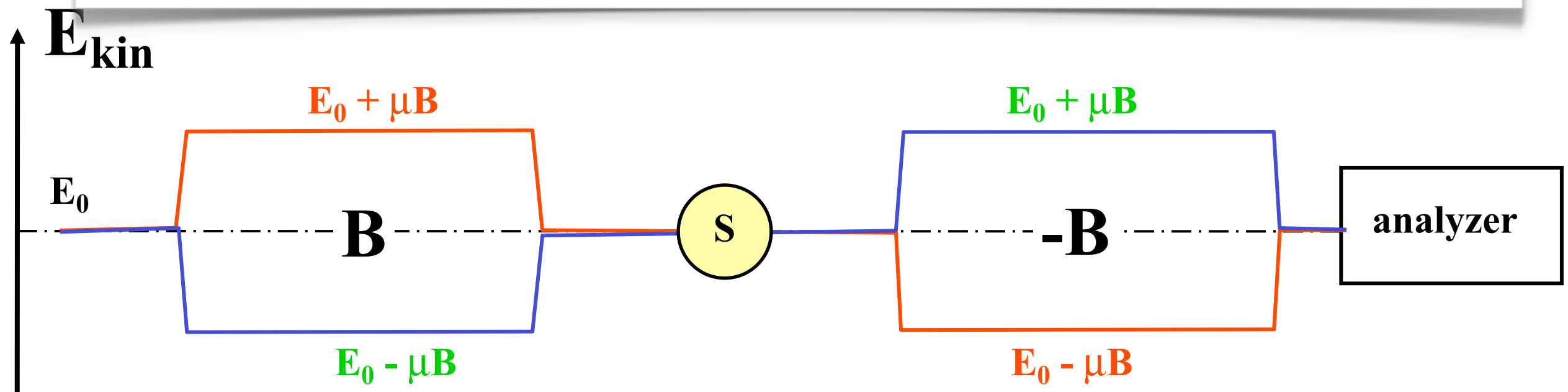


Time splitting $dt = t$ of the two wave packets, separated by the propagation through the field of length y_0 :

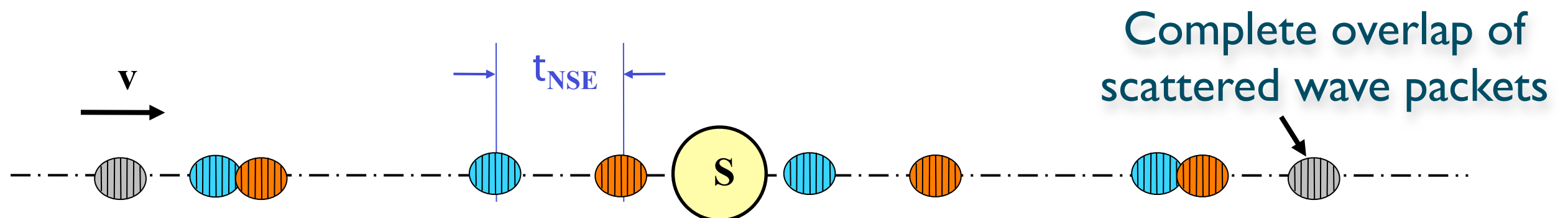
$$\frac{dE}{E} = \frac{2t}{t_0} \Rightarrow t = \frac{y_0}{2v} \cdot \frac{2\mu B}{\frac{1}{2}m v^2} = \frac{2\mu B \cdot y_0}{m v^3} = \frac{\omega_L \cdot t}{2\omega_0}$$

For fields of typ. 1 kG and length of m, t is in the ns range for cold neutrons;
 In Neutron Spin echo spectroscopy, t is the 'spin echo time';

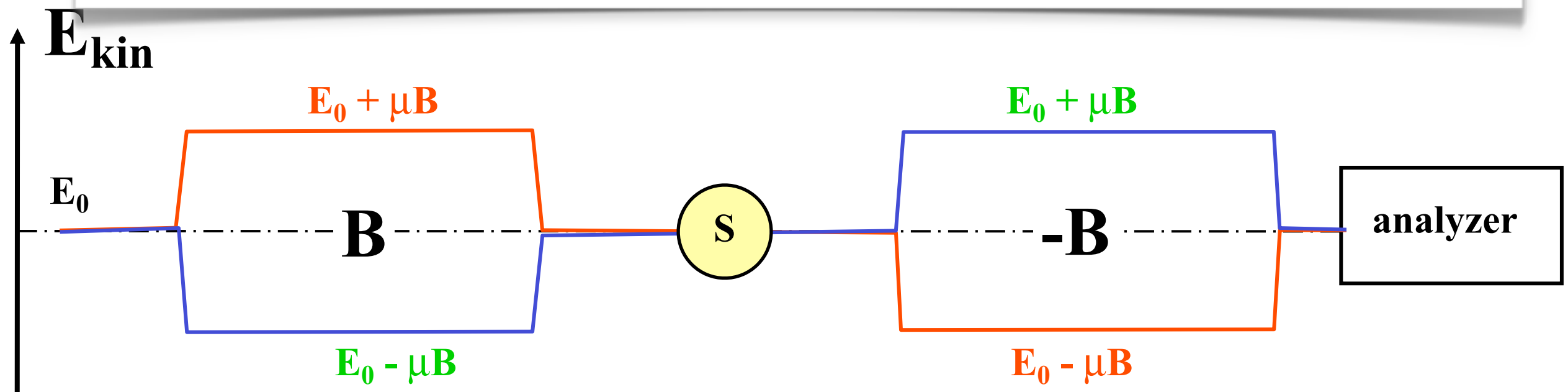
semi-classical description: measuring principle



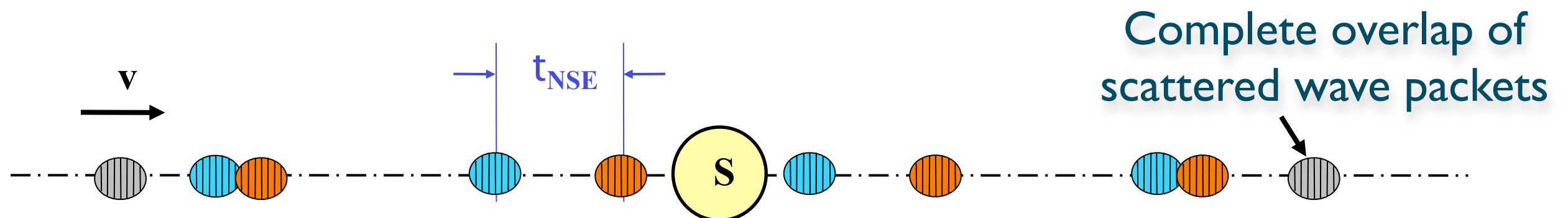
- ✿ The first field splits the wavepacket into two
- ✿ the second one overlaps them again;
- ✿ The analyser superposes both packets;



semi-classical description: measuring principle



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- ✿ the second one overlaps them again;
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S

S(Q, τ)

energy $E = h\nu$ [meV]

10^4
 10^2
 10^0
 10^{-2}
 10^{-4}
 10^{-6}
 10^{-8}

10^{-4}

10^{-3}

10^{-2}

10^{-1}

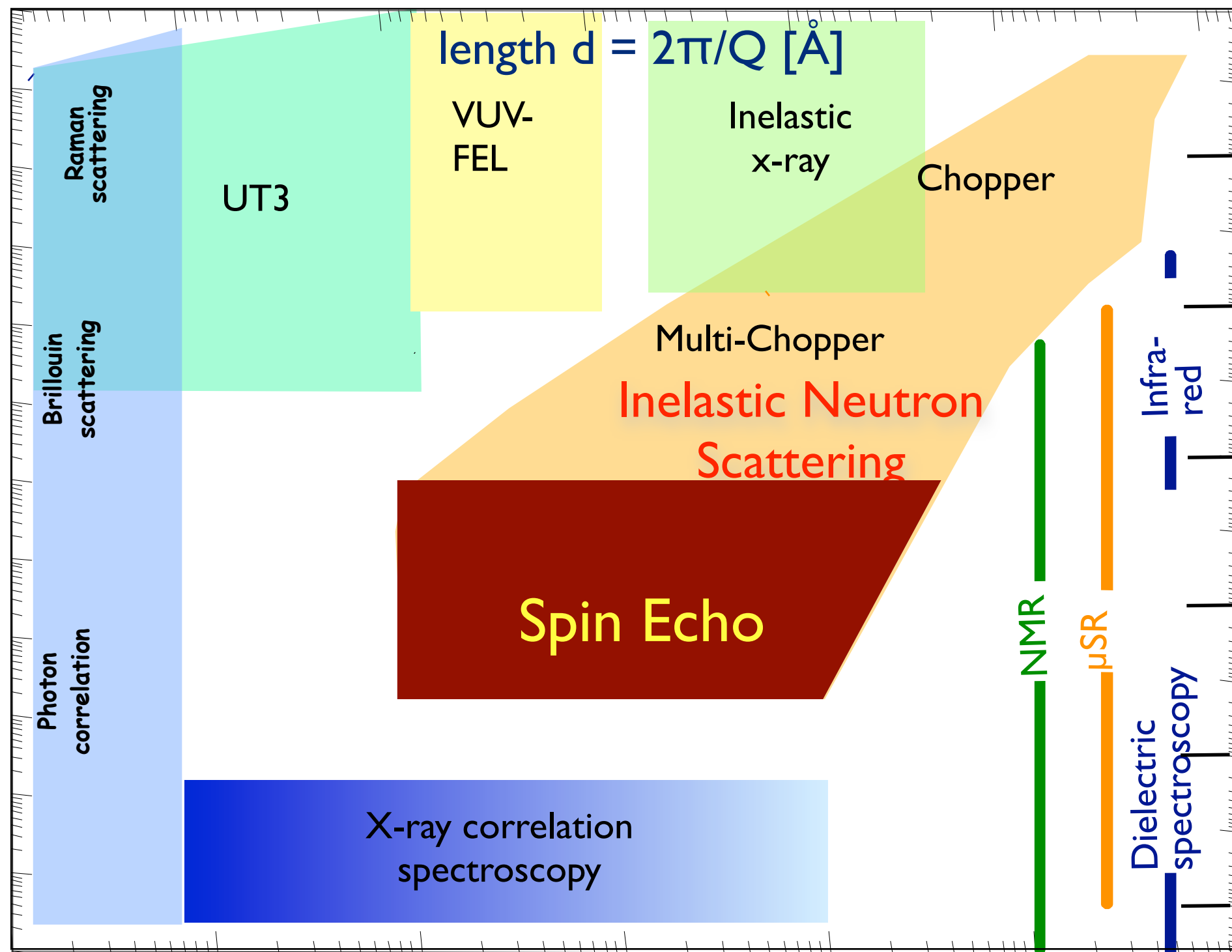
10^0

10^1

10^2

scattering vector Q [$\text{\AA}^{-1}$]

source: ESS



time τ [ps]

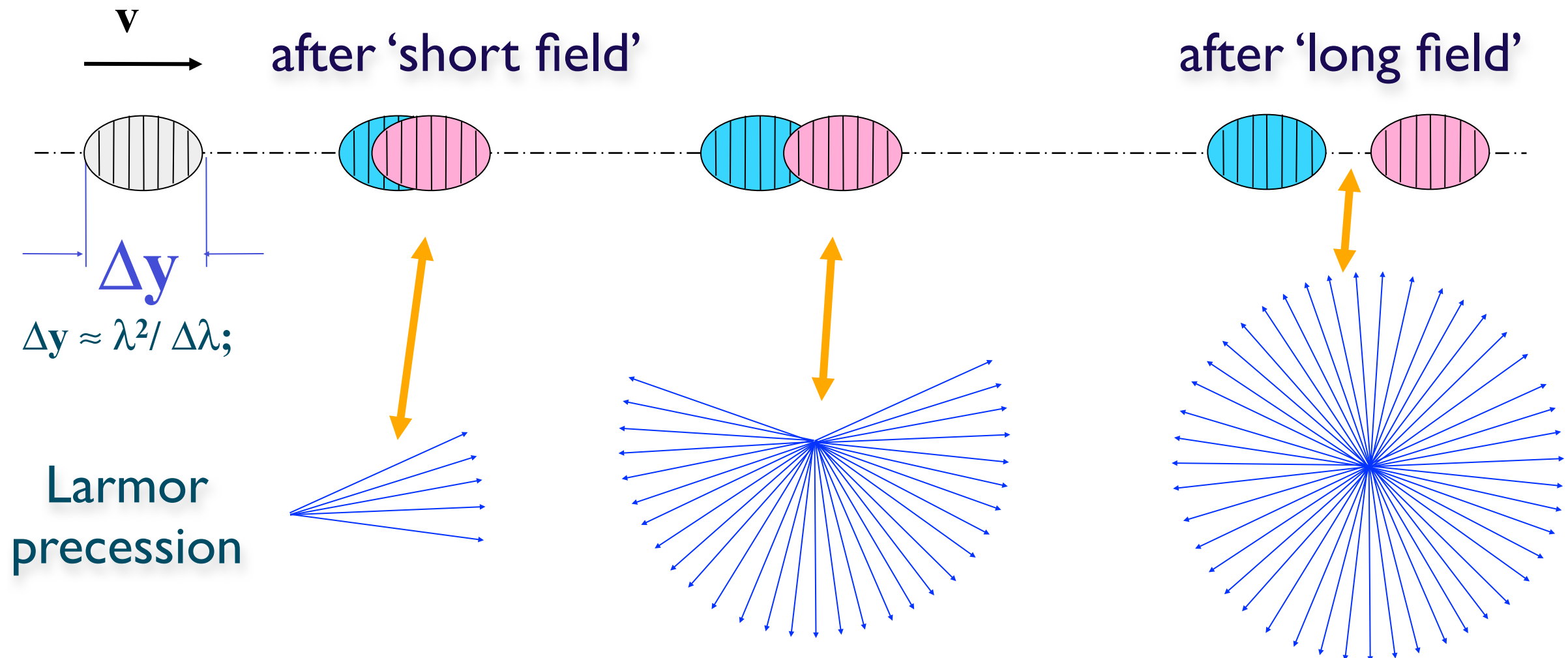
10^{-2}
 10^0
 10^2
 10^4
 10^6
 10^8

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what is the real life - what is fantasy ?

semi-classical picture:



Directions of the individual spins of the polychromatic beam after passage through B fields of different lengths

Complete separation of the two packets implies that no coherent superposition of both states exists any more

what is the real life - what is fantasy ?

what is a spin ?

what is the coherent superposition
of states ?

quantum mechanics

=

plane waves

coherent superposition of states

Basics of QM :

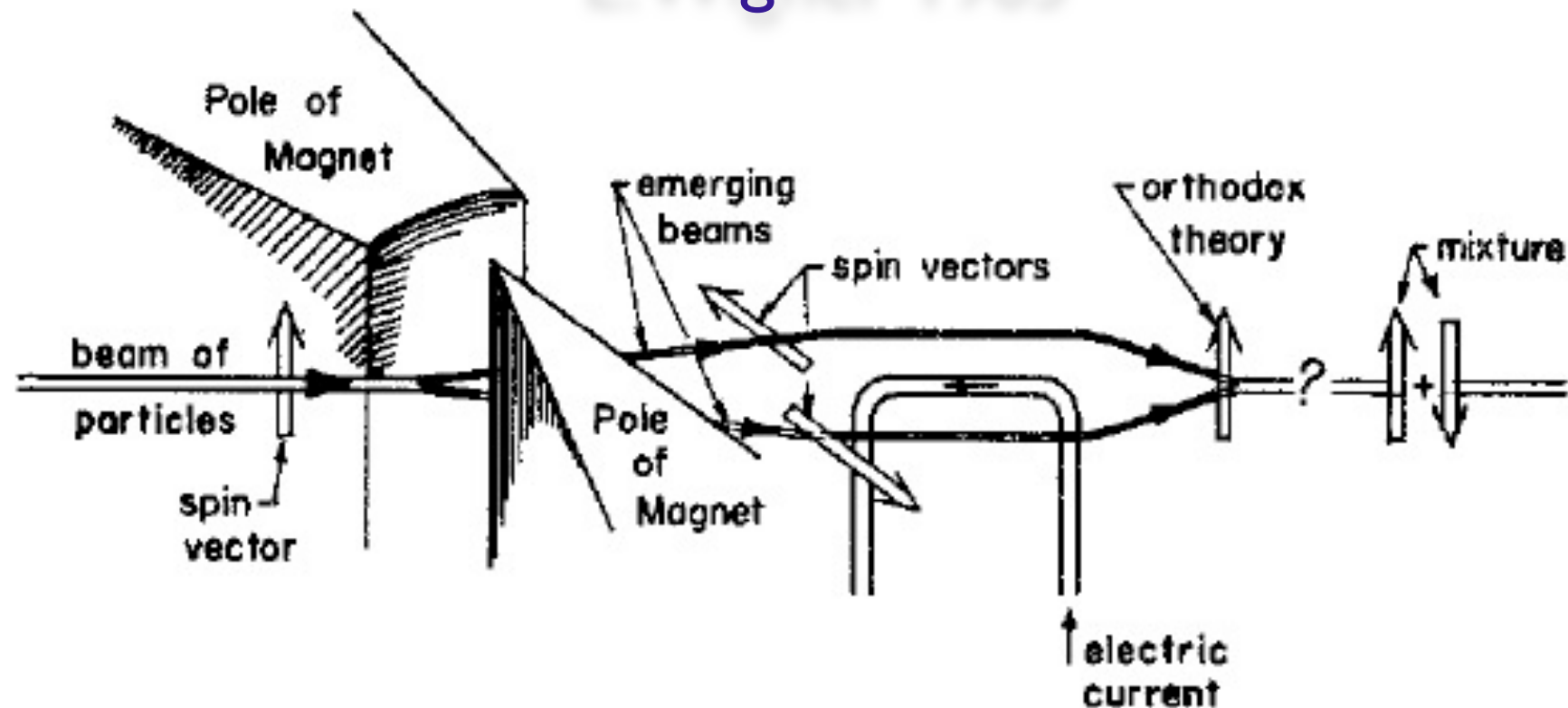
2 slit experiment

wave - particle duality

The classical - “Copenhagen” - description of QM

The Problem of Measurement

E.Wigner 1963



is the coherent superposition of states
the “sum” of the two
or not ?

$$|+\rangle + |-\rangle$$

classical

spin the the plane

quantum
mechanical

the spin is not a vector (classical)

Neutron Spin Echo

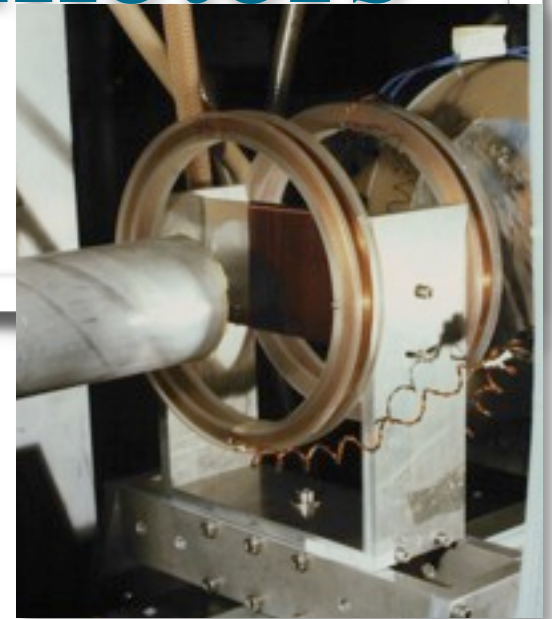
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- VII: NSE and structure

Neutron spin echo spectrometers-



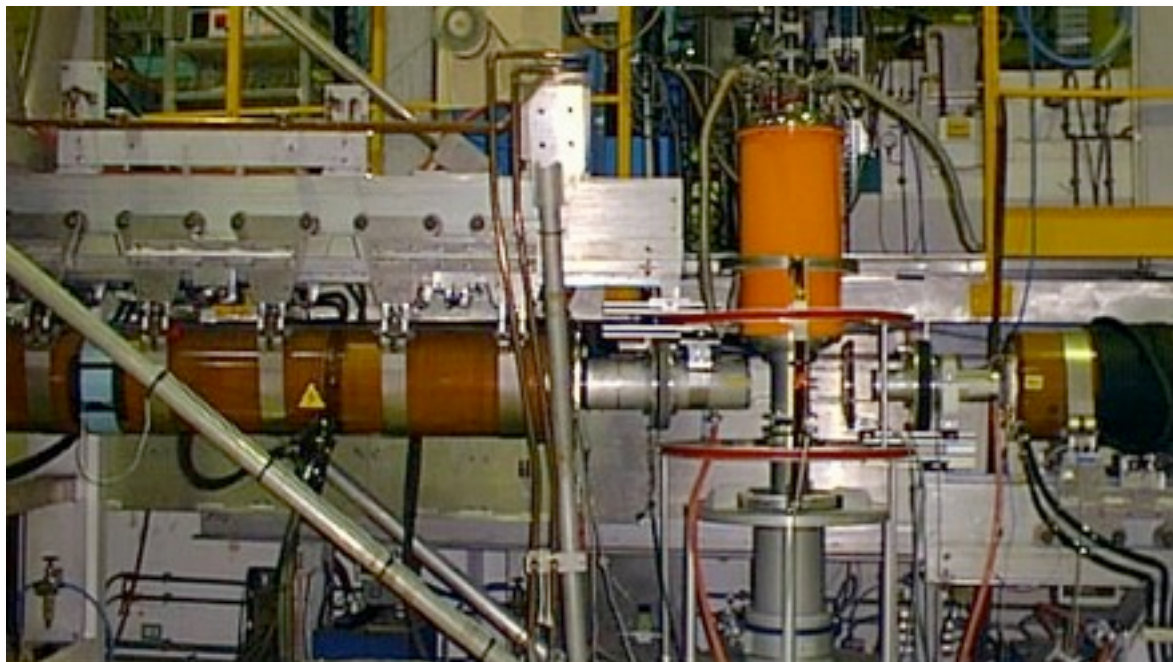
above: Mezei's first
spin echo precession
coils

right: $\pi/2$
flipper coils



HZ Berlin

below: IN11



Examples of neutron studies

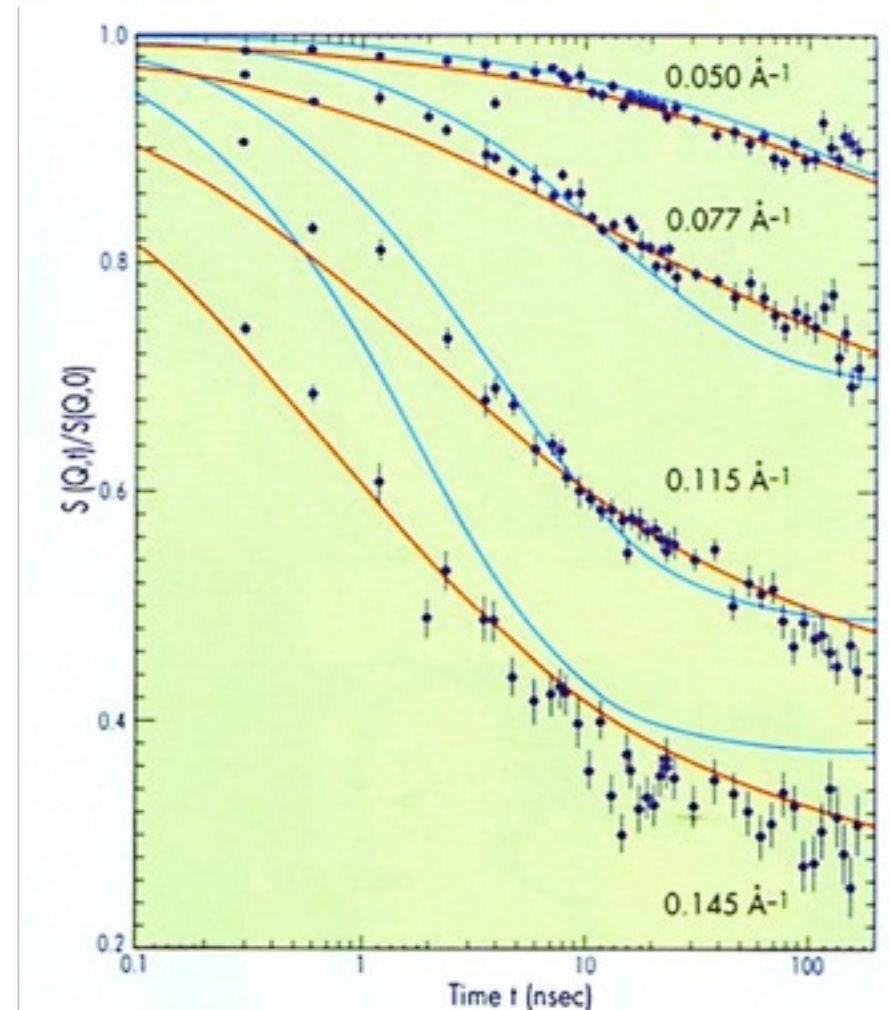
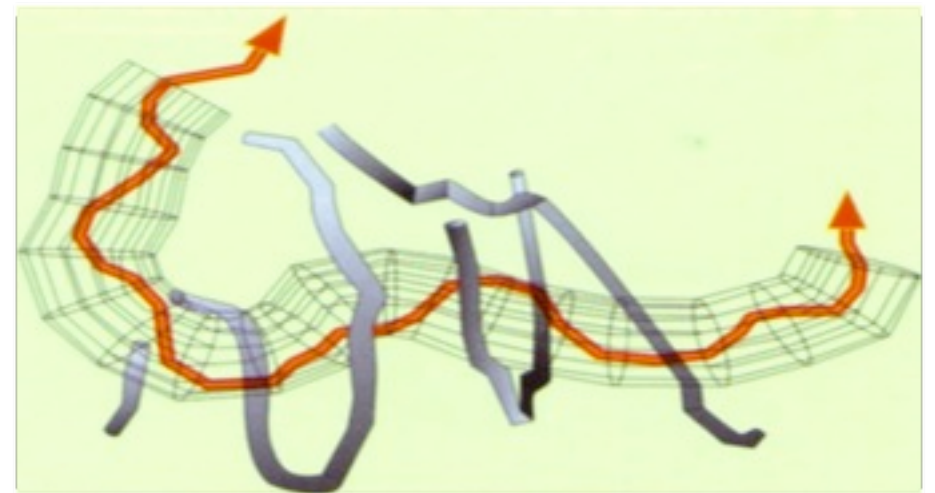
Reptation in polyethylene

The dynamics of dense polymeric systems are dominated by entanglement effects which reduce the degrees of freedom of each chain

de Gennes formulated the reptation hypothesis in which a chain is confined within a “tube” constraining lateral diffusion – although several other models have also been proposed

The measurements on IN15 are in agreement with the reptation model. Fits to the model can be made with one free parameter, the tube diameter, which is estimated to be $45\text{\AA}$

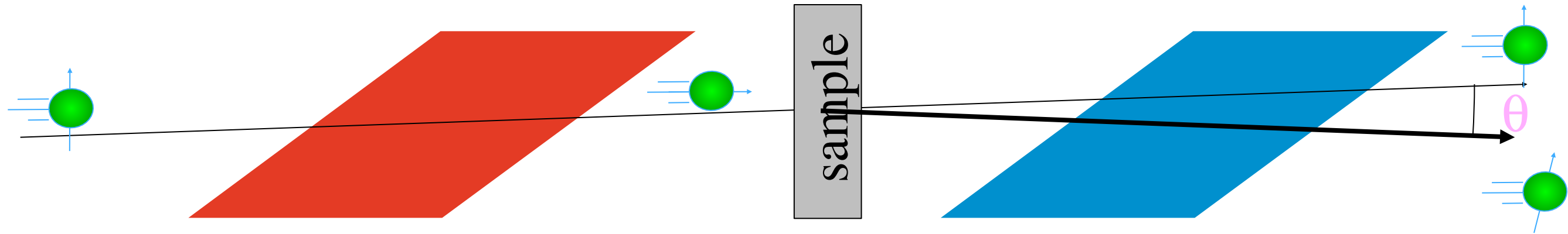
Schleger et al, Phys Rev Lett 81, 124 (1998)



Neutron Spin Echo

- I: polarized neutrons -Larmor precession
- II: NSE : classical
- III: NSE : semi-classical
- IV: movies
- V: quantum mechanical description ??
- VI: examples
- VII: NSE and structure**

Larmor precession codes scattering angle



Unscattered beam gives spin echo at $\varphi = 0$
Independent of height and angle

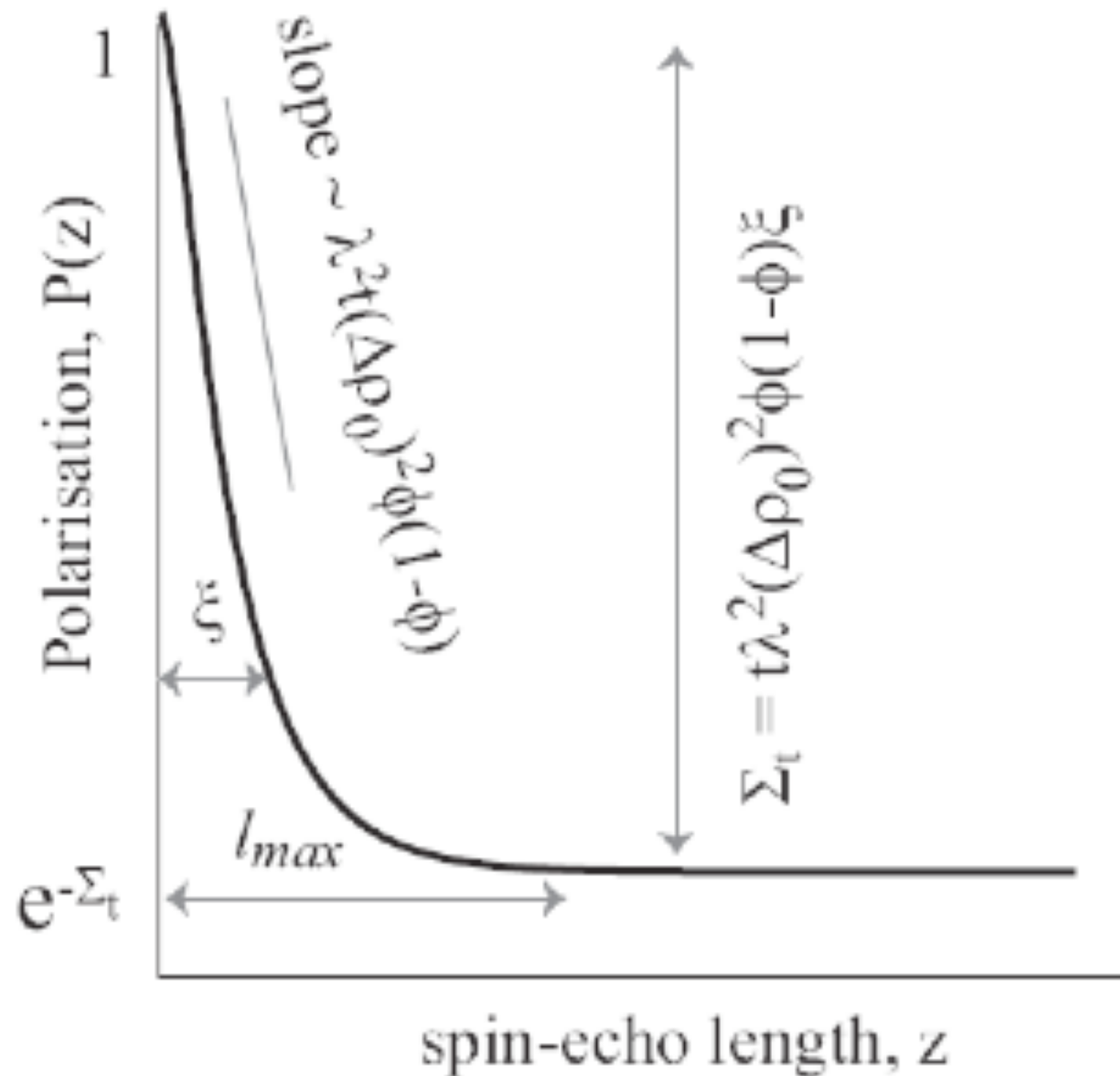
Scattered wave vector transfer Q_z
results in precession ϕ $\phi \cong zQ_z$

Proportional to the **spin echo length** z

Measure polarisation: $P(z) = \cos(zQ_z)$

Fourier transform scattering cross-section:
real space density correlation function $G(z)$

Direct information in measurement



Ordering

Compactness

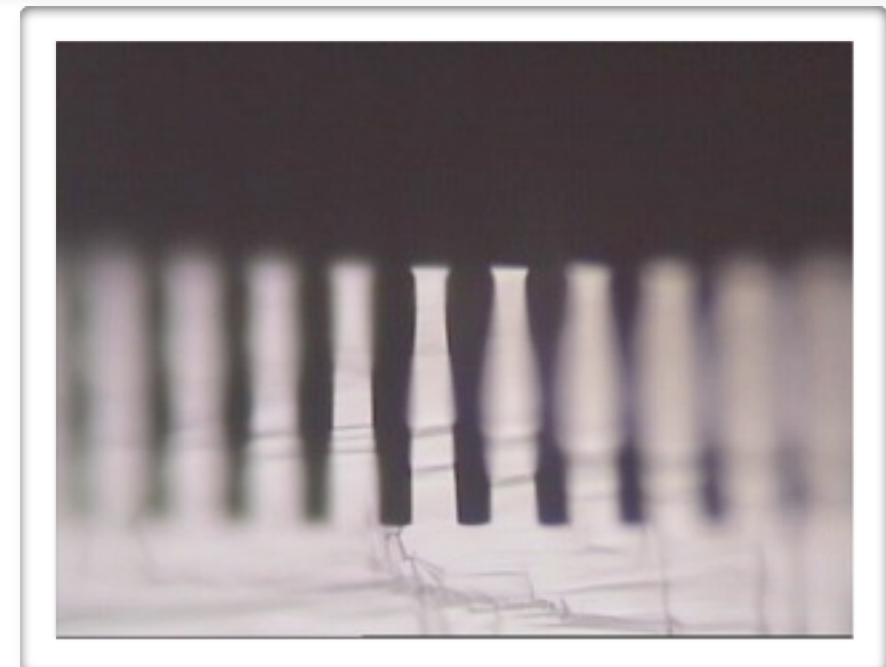
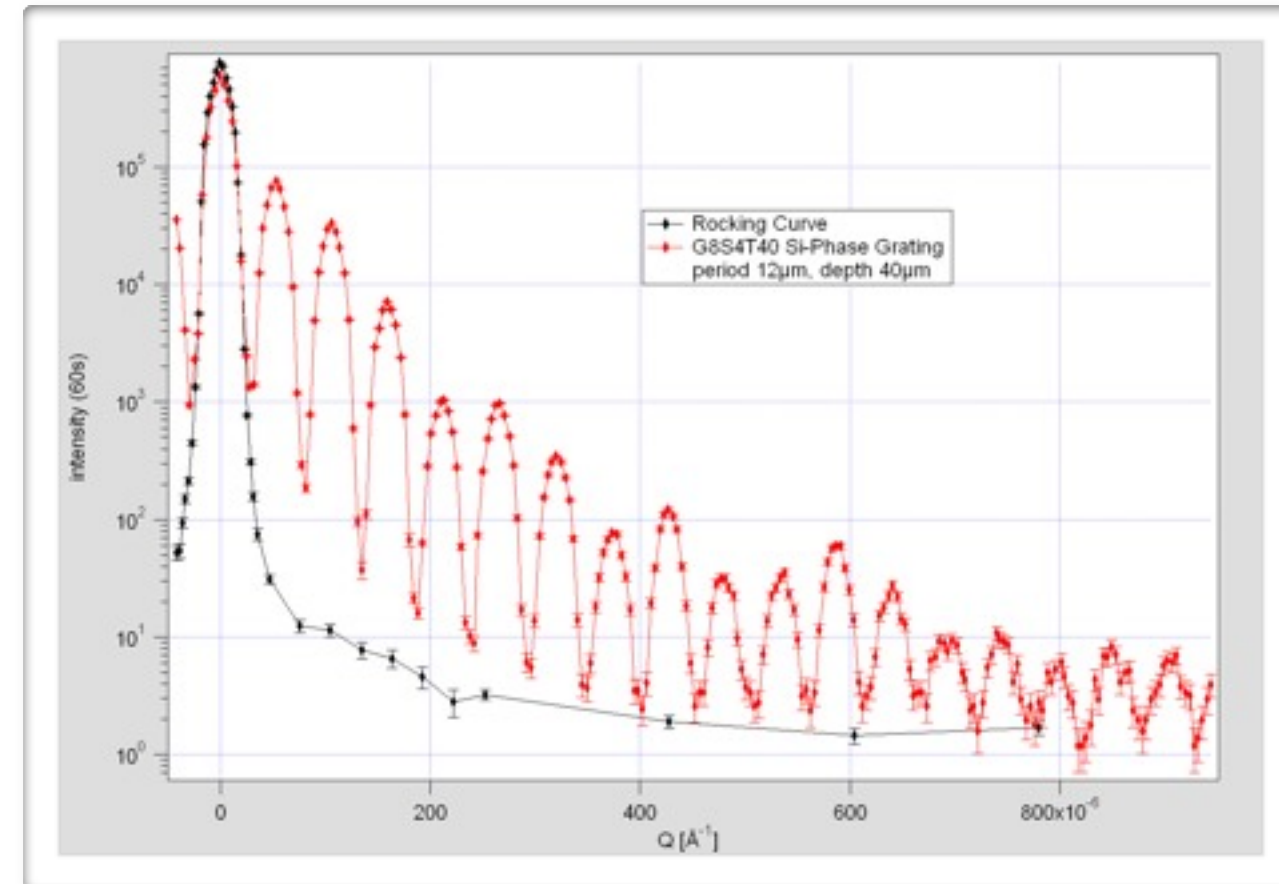
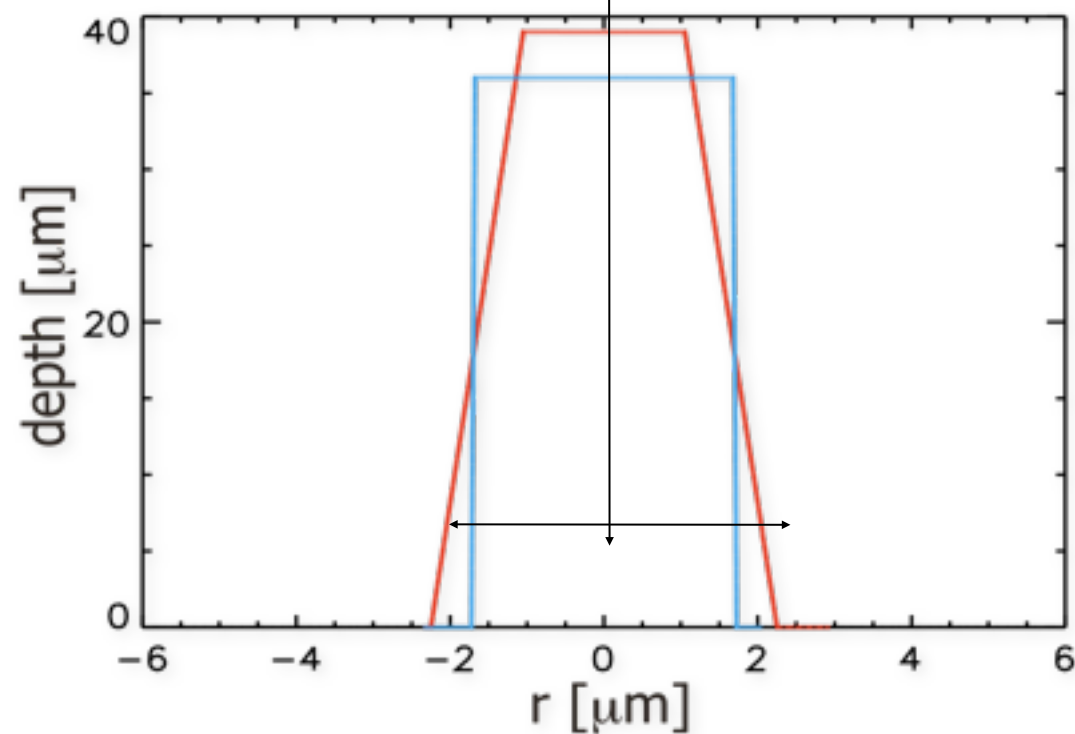
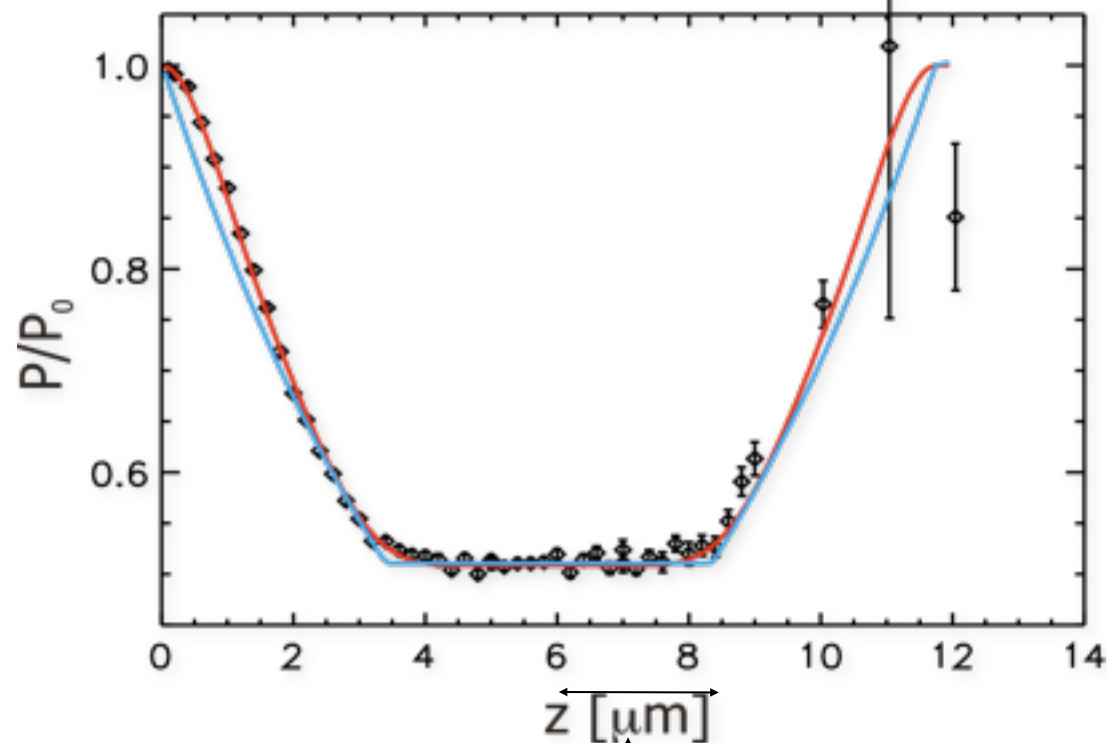
Correlation lengths

Fractal dimensions

Specific surfaces

Etc...

SESANS measures directly shape ridges



Realisation SESANS

monochromator

polariser

magnet 1

field stepper

guide field

analyser

detector

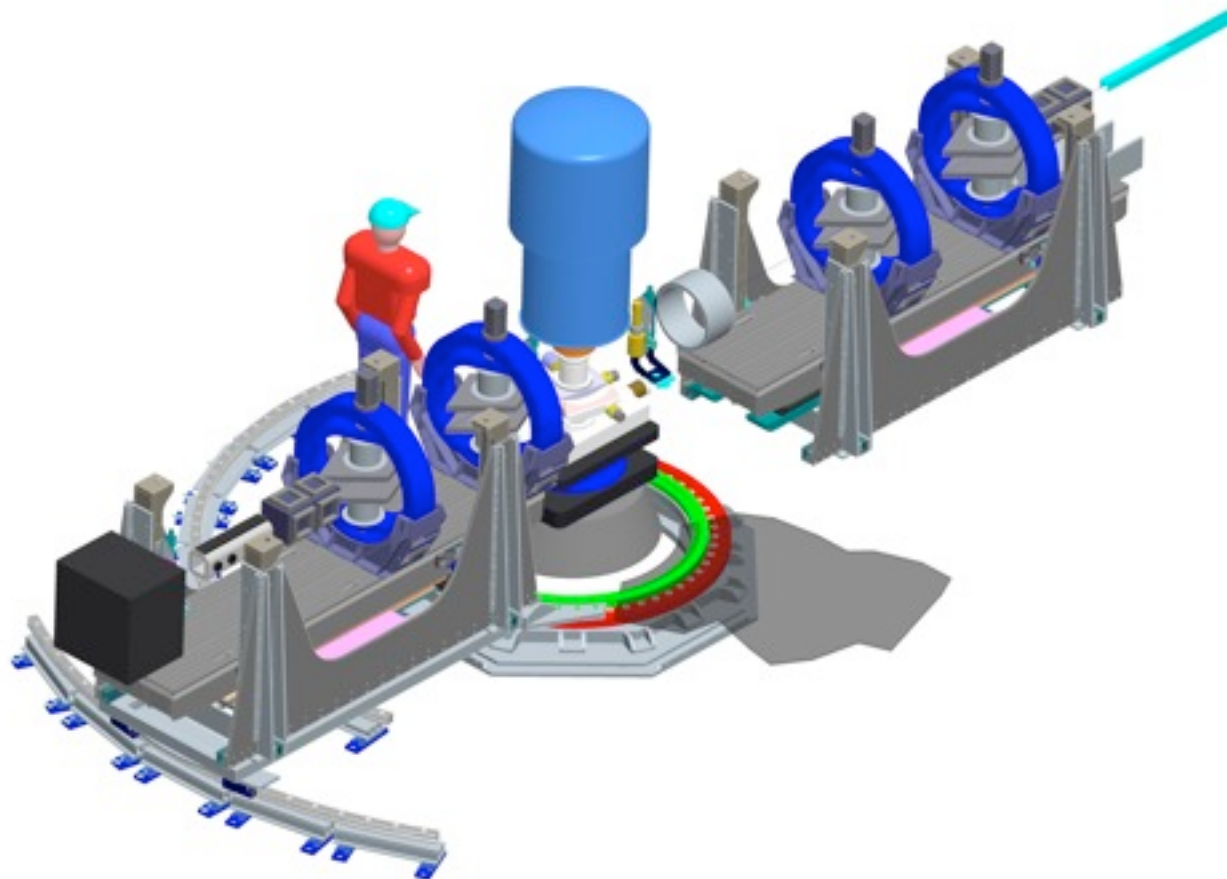
polariser

sample

polariser

Spin-echo reflectometer OffSPEC

- Off-specular to measure in-plane scattering
- Specular reflectivity of bent surfaces high-resolution
- Separation specular and off-specular



ISIS 2nd target station
Off specular reflectometer
Spin-echo components for
High resolution without
collimation

Thank you !

