Polarized Neutrons

Ross Stewart

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Contents

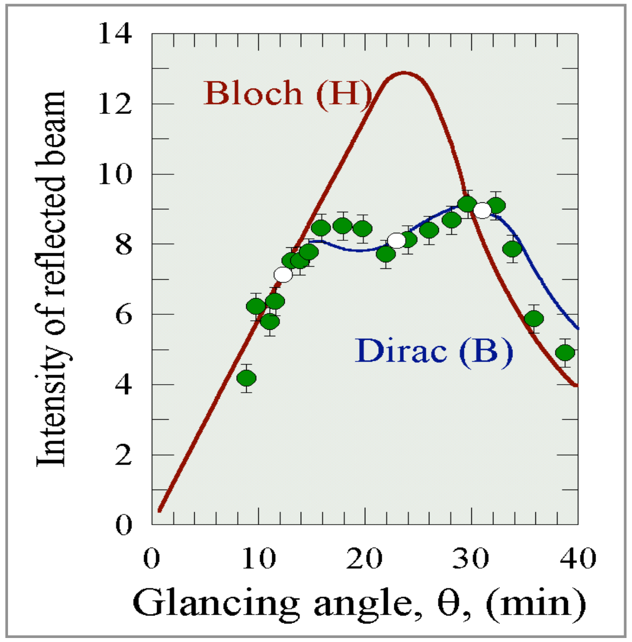
History of Polarized Neutrons

1932 Discovery of the neutron
Chadwick *(Proc Roy Soc **A136** 692)*

1937 Theory of neutron polarization by a ferromagnet
Schwinger *(Phys Rev, **51**, 544)*

1938 Partial polarization of a neutron beam by passage through iron
Frisch et al *(Phys Rev **53**, 719)*, Powers *(Phys Rev 54, 827)*

1940 Magnetic moment of the neutron determined by polarization analysis
Alvarez and Bloch *(Phys Rev **57**,111)*



1937 - 1941 Theory of magnetic neutron scattering (including neutron polarization)
Halpern and Johnson *(Phys Rev 51,* 992; *52,* 52; *55,* 898*)*

1951 Experiments with polarizing mirrors and proof of the neutron's μ.B interaction
Hughes and Burgy *(Phys Rev, **81**, 498)*

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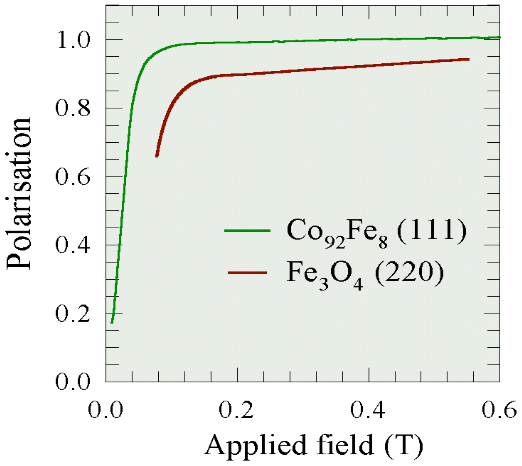
History of Polarized Neutrons

1951 Polarizing crystals (magnetite Fe_3O_4, $Co_{92}Fe_8$)

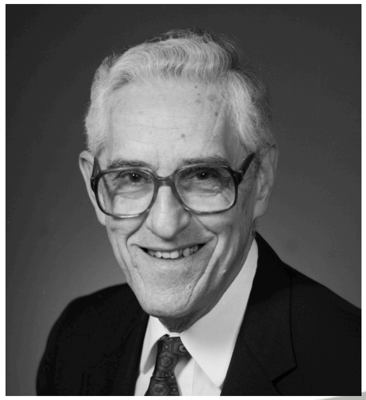
Shull et al (*Phys Rev* ***83****, 333;* ***84****, 912*)

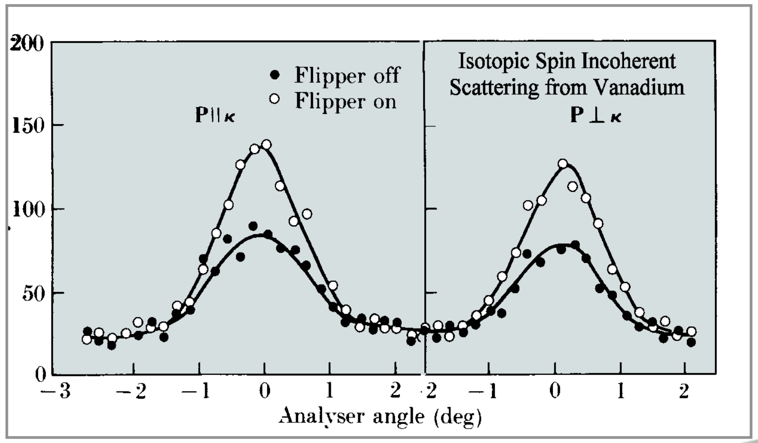


1959 First polarized beam measurements (of magnetic form factors of Ni and Fe)
Nathans et al (*Phys Rev Lett*, ***2****, 254*)

1963 General theory of neutron polarization analysis
Blume (*Phys Rev* ***<u>130</u>, 1670*)**
Maleyev (*Sov. Phys.: Solid State 4, 2533*)

1969 First implementation of neutron polarization analysis, Oak Ridge, USA
Moon, Riste and Koehler (*Phys Rev* ***181****, 920*)





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Polarized neutrons today

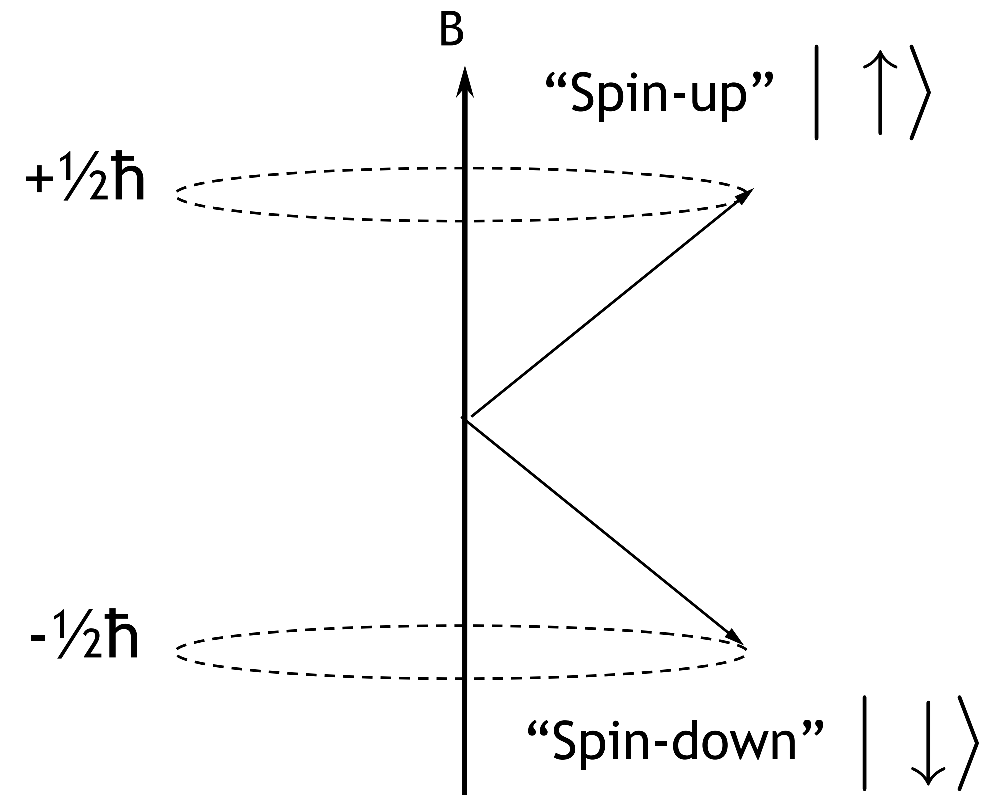
- Single crystal diffraction
- Diffuse scattering
- Inelastic scattering (3-axis and TOF)
- Reflectometry (on and off-specular)
- SANS - magnetic and non-magnetic
- Neutron Spin-Echo
- Neutron Resonance Spin-Echo
- Mieze Spectroscopy
- SESANS, SERGIS
- Larmor Diffraction
- Neutron Depolarization
- Polarized Neutron Tomography
- ……

Polarized neutron beams

Each individual neutron has spin s=½ and an angular momentum of ±½ħ

Each neutron has a spin vector $\vec{s}_n$ and we define the polarization of a neutron beam as the ensemble average over all the neutron spin vectors, normalised to their modulus

$$\vec{P} = \langle \vec{s}_n \rangle / \tfrac{1}{2} = 2 \langle \vec{s}_n \rangle$$



If we apply an external field (quantisation axis) then there are only two possible orientations of the neutrons: parallel and anti-parallel to the field. The polarization can then be expressed as a scalar:

$$P = \frac{N_+ - N_-}{N_+ + N_-}$$

where there are N_+ neutrons with spin-up and N_- neutrons with spin-down

Polarized neutron beams

What we often would like to do in polarized neutron experiments is

$$P = \frac{N_+ - N_-}{N_+ - N_-}$$

$$= \frac{(N_+/N_-) - 1}{(N_+/N_-) + 1}$$

$$= \frac{F - 1}{F + 1}$$

Where $F = \frac{N_+}{N_-}$ is called the Flipping Ratio and is a measurable quantity in a scattering experiment

This description of a polarized beam is OK for experiments in which a single quantisation axis is defined: ***Longitudinal Polarization Analysis***

The technique of 3-dimensional neutron polarimetry, however is termed:
Vector (or Spherical) Polarization Analysis

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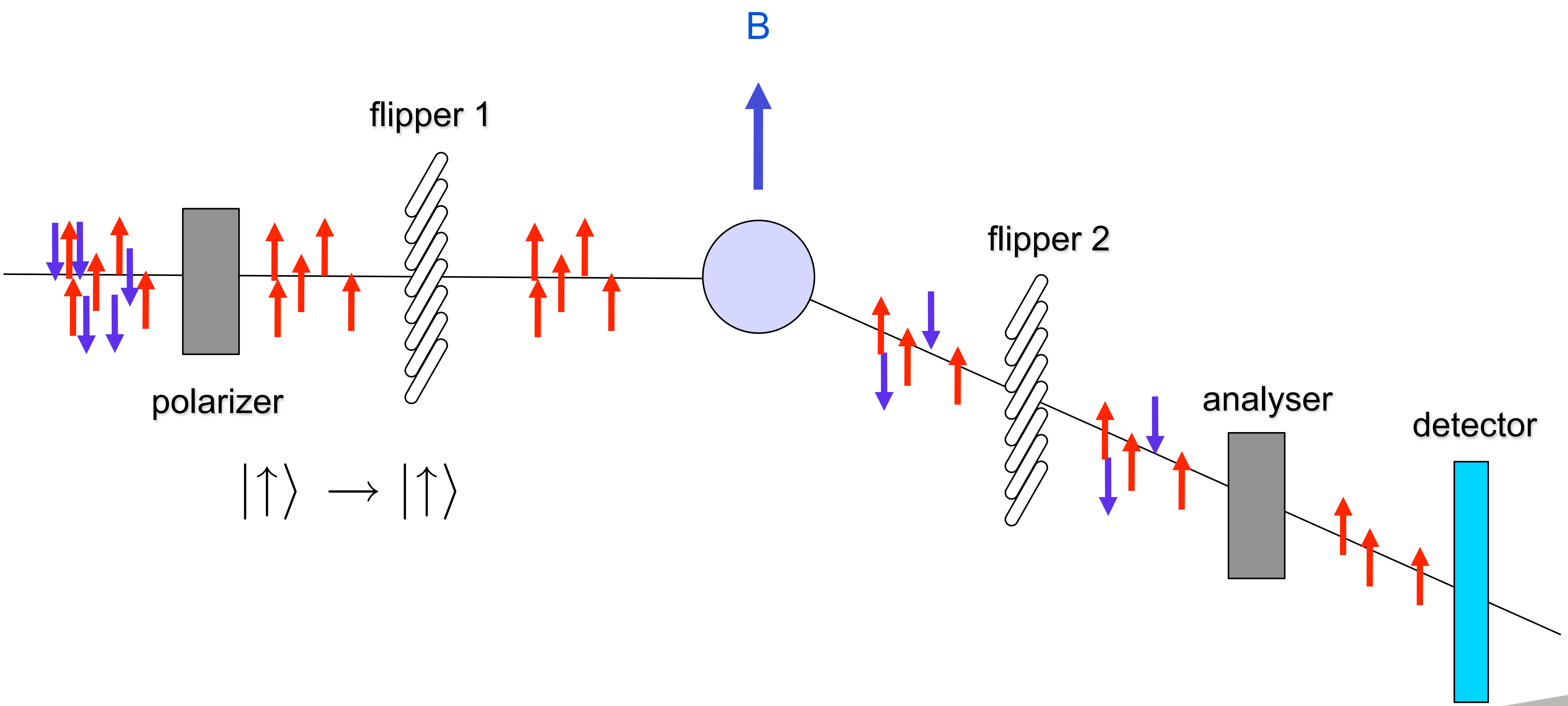
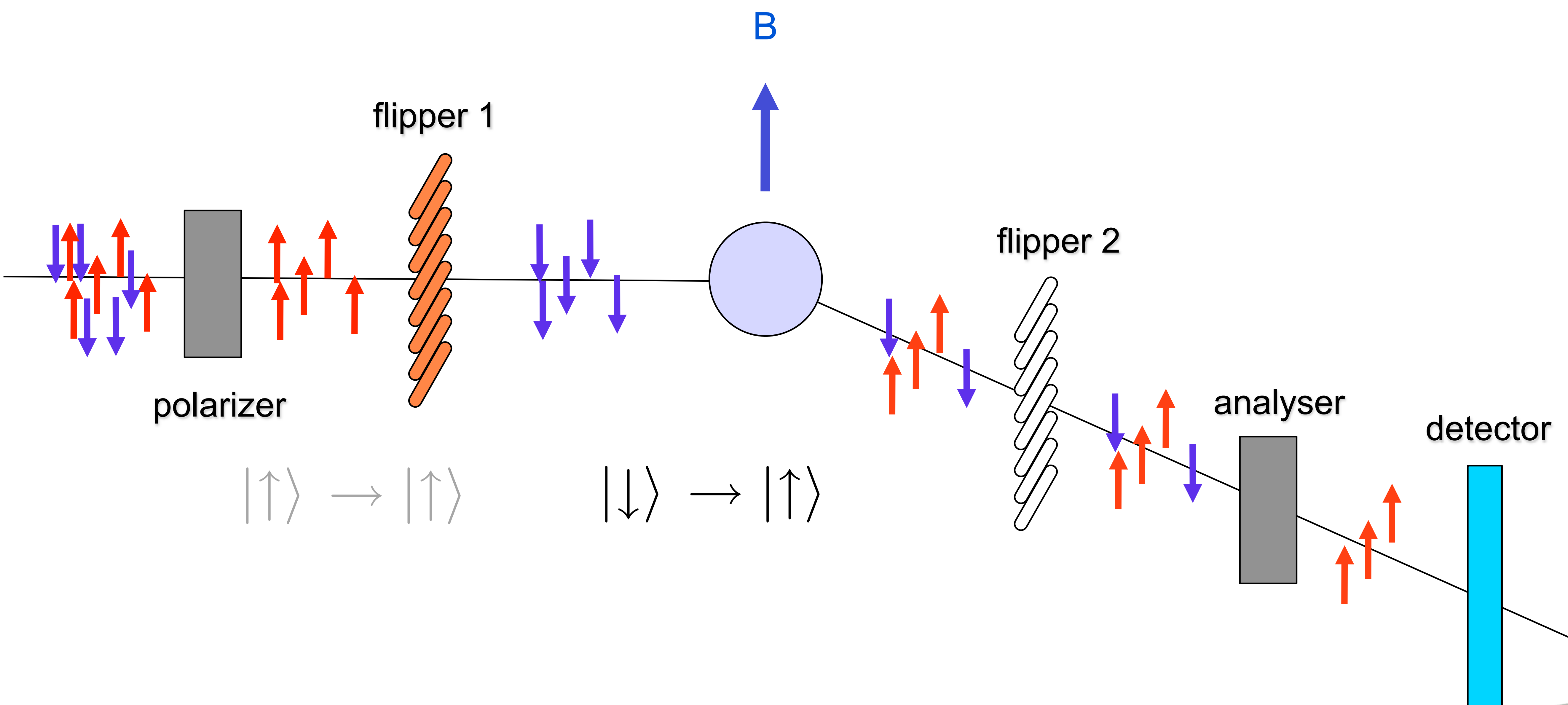
A Uniaxial PA experiment



- First attempted by Moon, Riste and Koehler (Oak Ridge 1969)
  Phys Rev. 181 (1969) 920

A Uniaxial PA experiment



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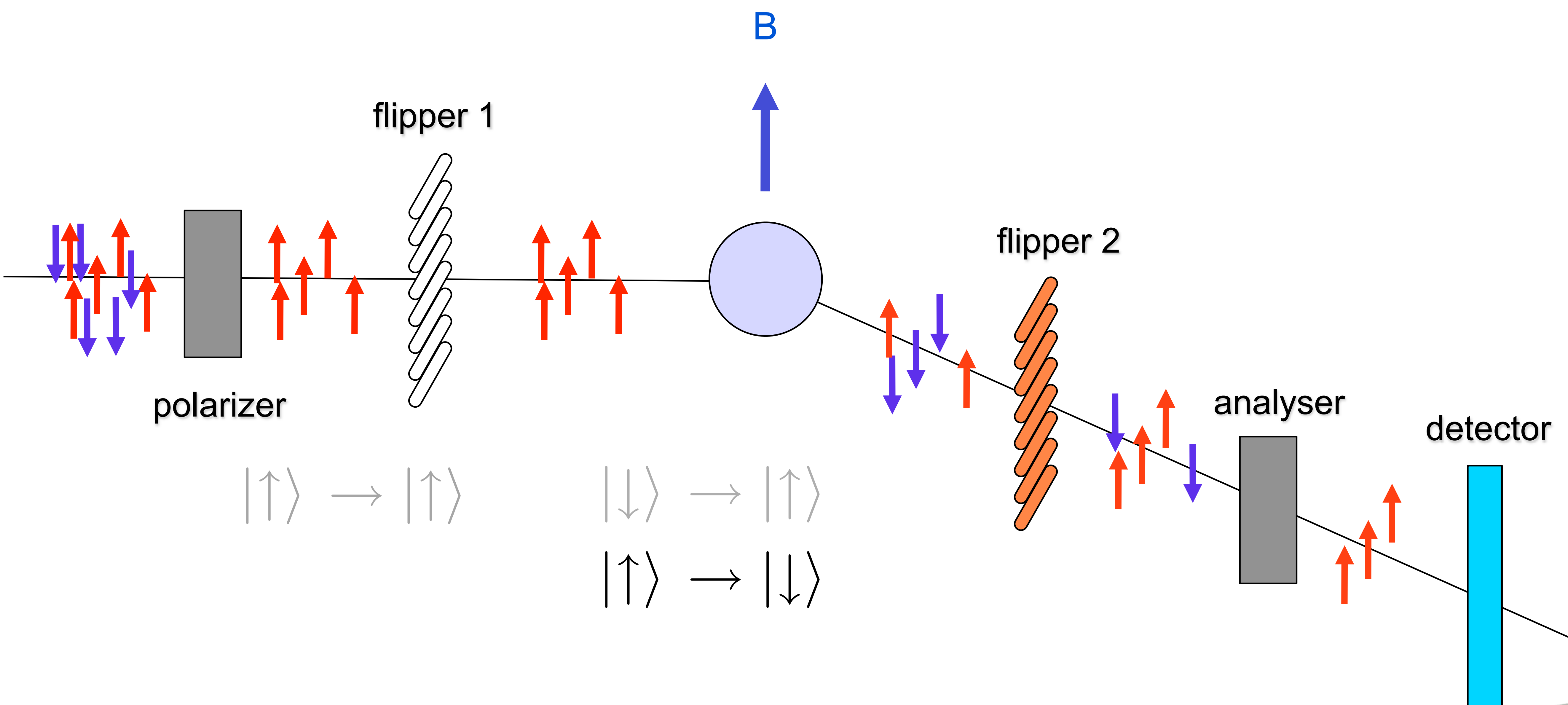

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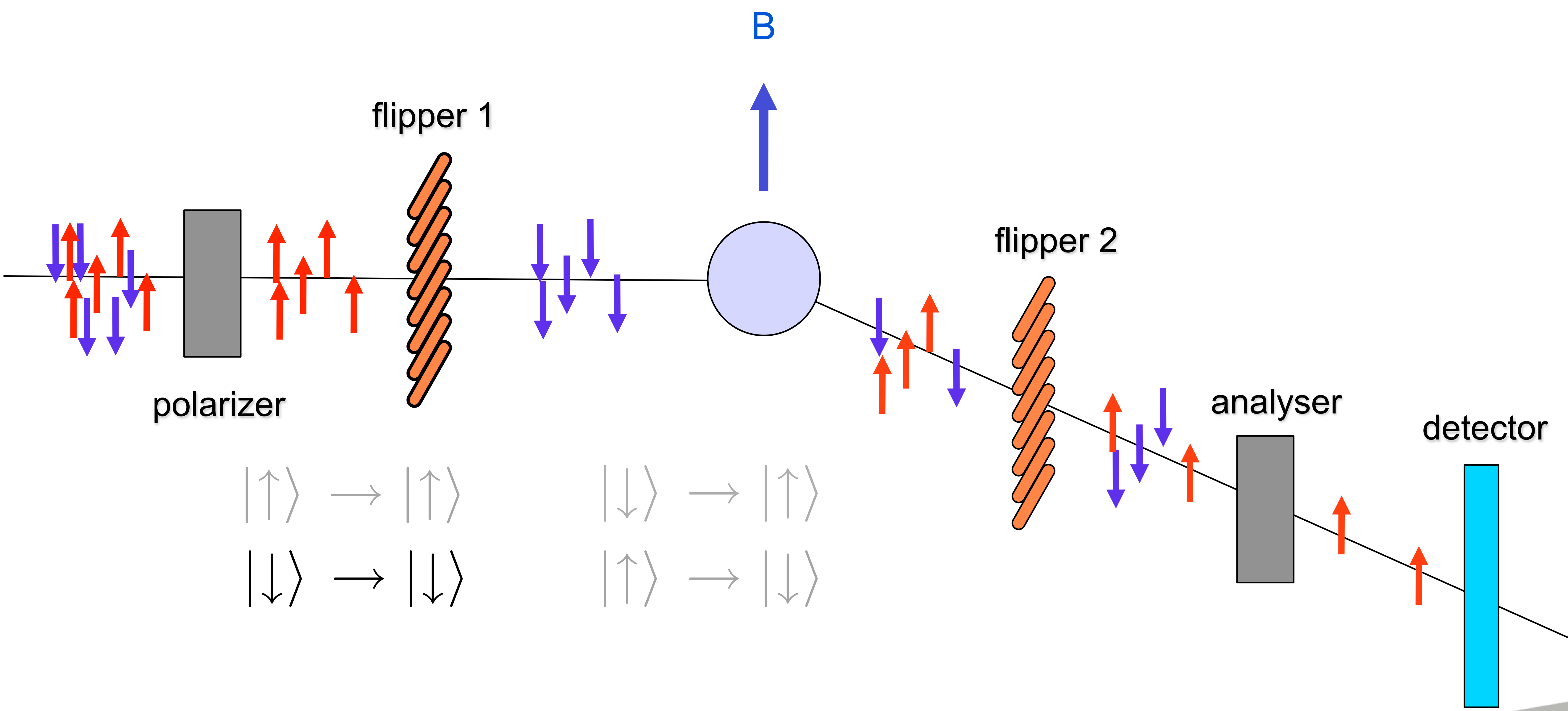

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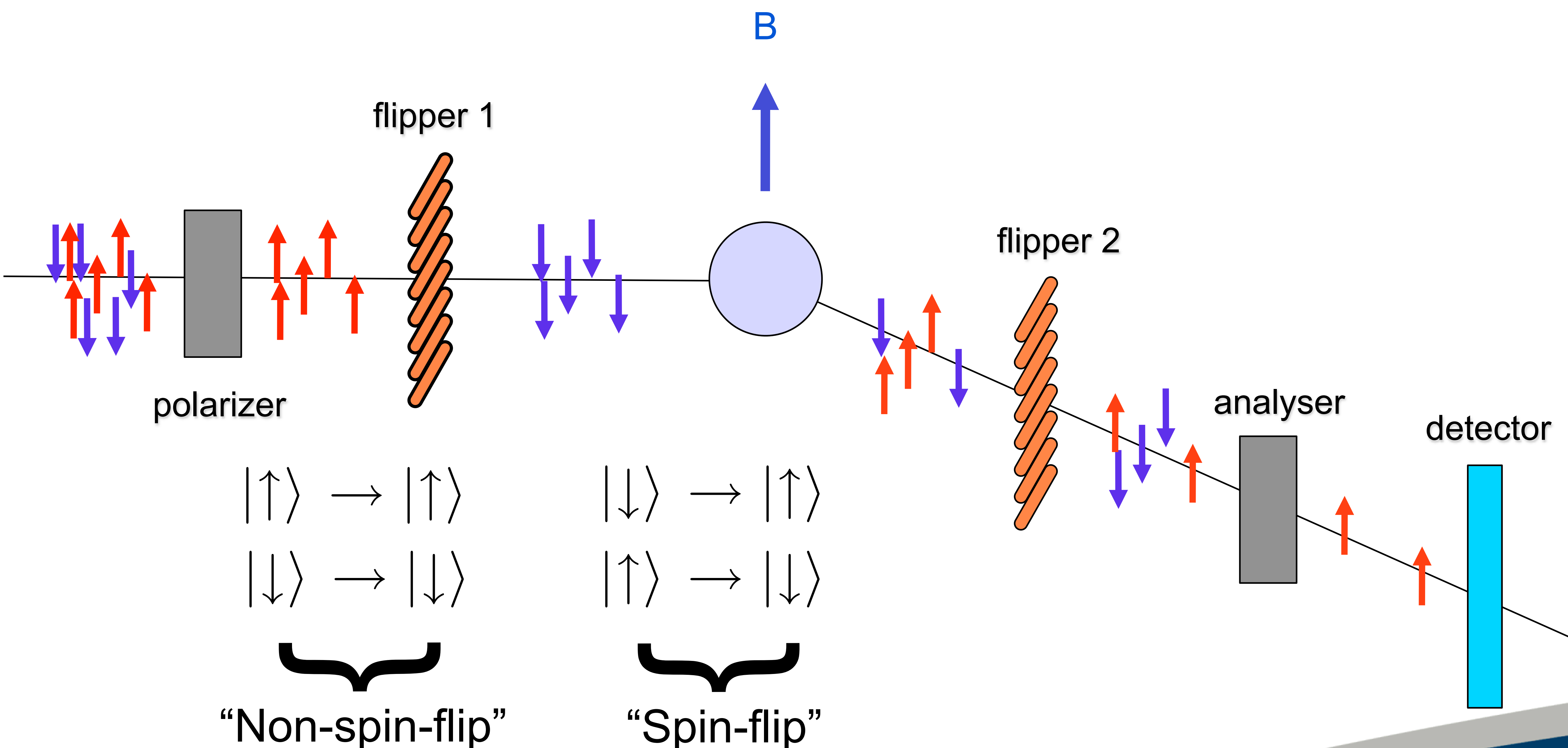
A Uniaxial PA experiment



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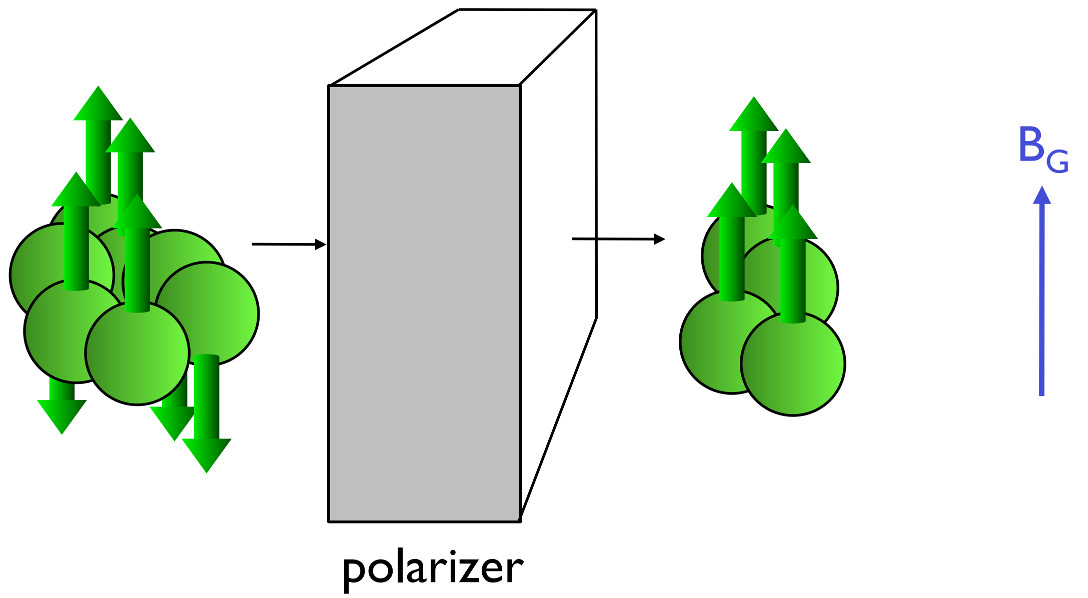
Production of polarized beams



There are three principal (passive) methods of beam polarization, each with specific advantages in particular experimental situations

(a) polarizing filters (e.g. preferential absorption by polarized ^{3}He nuclei)
(b) polarizing mirrors and supermirrors (using preferential reflection)
(c) polarizing crystals (e.g. $Co_{92}Fe_8$, Heusler crystals (Cu_2MnAl)) using preferential Bragg reflection)

Polarizing Filters

The polarizing efficiency of a filter, P, is defined in terms of the transmission of the two spin states, T_+ and T_-

$$P = \frac{T_+ - T_-}{T_+ + T_-}$$

where the total transmission of the filter is $T = (T_+ + T_-) / 2$

The filter performance depends upon both *P* and *T*. *P* can be increased by making the filter thicker, but only at the expense of total transmission, *T*. As a compromise it is the quality factor $P\sqrt{T}$ that is usually optimised (but see *Cussen, J Neutron Res., 7, 15, 1998*)

For a generalised polarizing filter with total absorption cross sections given in terms of the spin-independent absorption cross section, σ_o and a spin-dependent absorption cross section σ_p we have

$$\sigma_\pm = \sigma_0 \pm \sigma_p$$

From which it can be shown (using $T = \exp(-N\sigma t)$ that)

$$P = -\tanh(N\sigma_p t) \qquad \text{and} \qquad T = \exp(-N\sigma_0 t)\cosh(N\sigma_p t)$$

Where N is the number density of atoms/nuclei responsible for spin selection and t is the filter thickness

See, e.g. Williams in Polarized Neutrons, Oxford University Press, 1988

^{3}He spin-filters

Close to a resonance energy the absorption cross sections of nuclei are dependent upon their polarization and the neutron spin direction

$$\sigma_{\pm} = \sigma_a(1 \pm \rho P_N)$$

where σ_a is the mean absorption cross section and P_N is the nuclear polarization,

$$\rho = \frac{I(1-2x)-x}{I+1} \qquad x = \frac{\sigma_{I-\frac{1}{2}}}{\sigma_{I+\frac{1}{2}} + \sigma_{I-\frac{1}{2}}}$$

where $\sigma_{I-1/2}$ and $\sigma_{I+1/2}$ are the absorption cross sections for neutrons with spin down and spin up with respect to the orientation of the nuclear spin ***I***

For ^{3}He nuclei, all the absorption is in the $\sigma_{I-1/2}$ channel, and therefore, x = 1 and ρ = -1 giving

$$\sigma_{\pm} = \sigma_a(1 \mp P_N)$$

Therefore, for fully polarized ^{3}He (P_{He} = 1), one spin state goes through the filter with zero absorption. The other spin state is almost fully absorbed since σ_a = 5000 barns.

We can now set $\sigma_0 = \sigma_a$ and $\sigma_p = -\sigma_a P_N$ which leads to the expressions

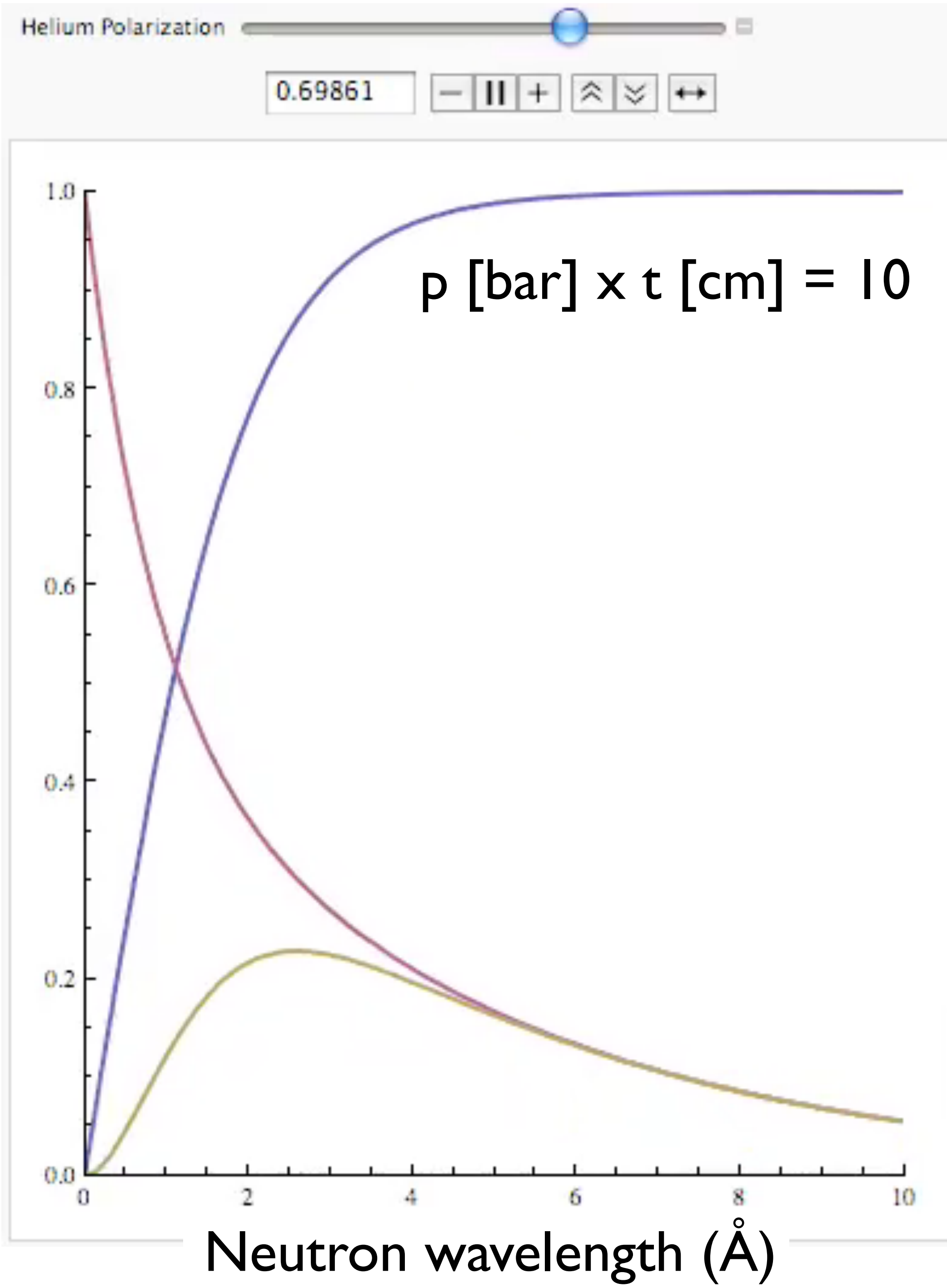
$$P = \tanh(OP_{\mathrm{He}}) \qquad T = \exp(-O)\cosh(OP_{\mathrm{He}})$$

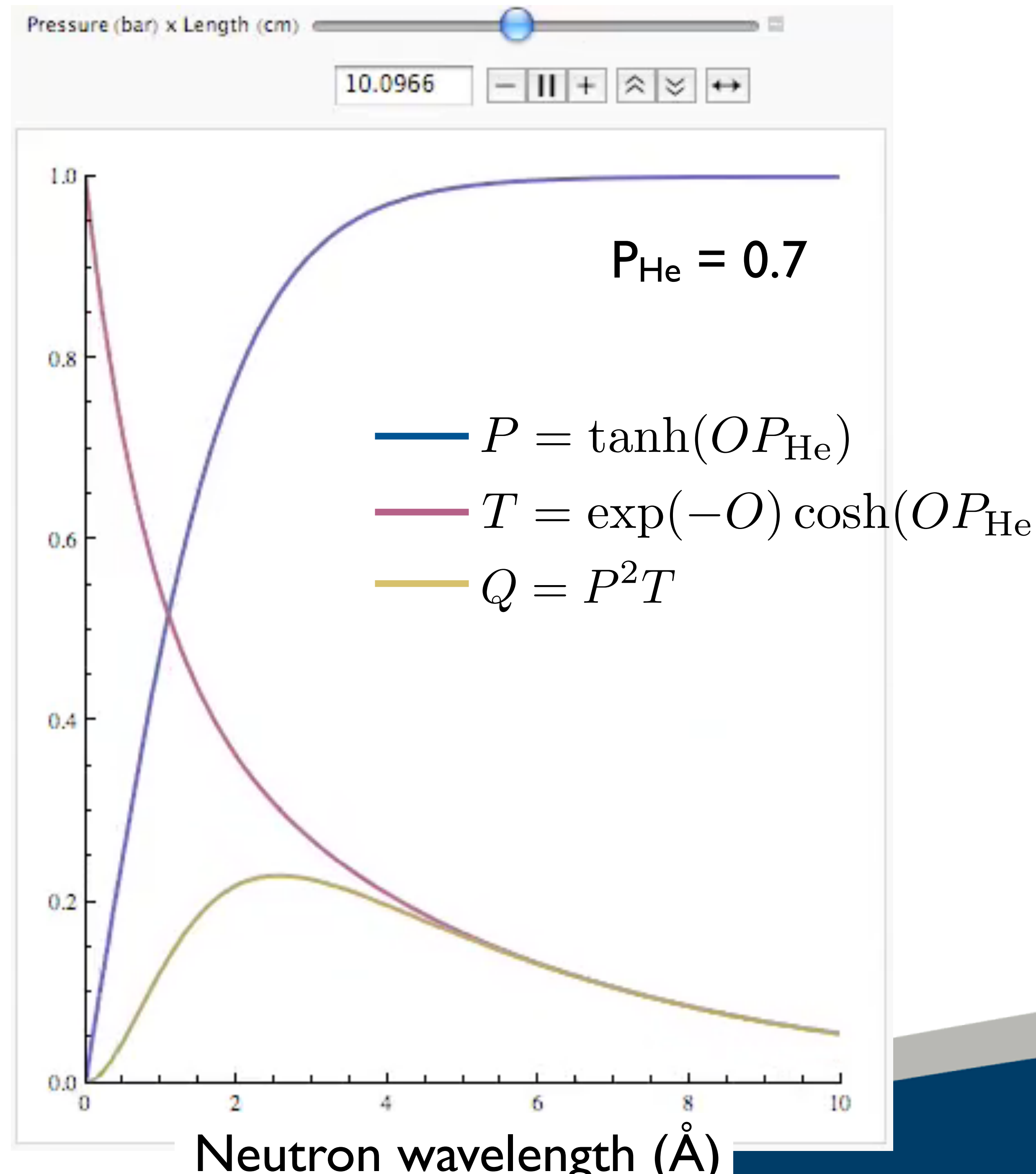
where O is the *opacity* of the spin-filter

$$O = N\sigma_a t$$

$= 0.0732 \times \lambda[\text{Å}] \times p[\text{bar}] \times t[\text{cm}]$

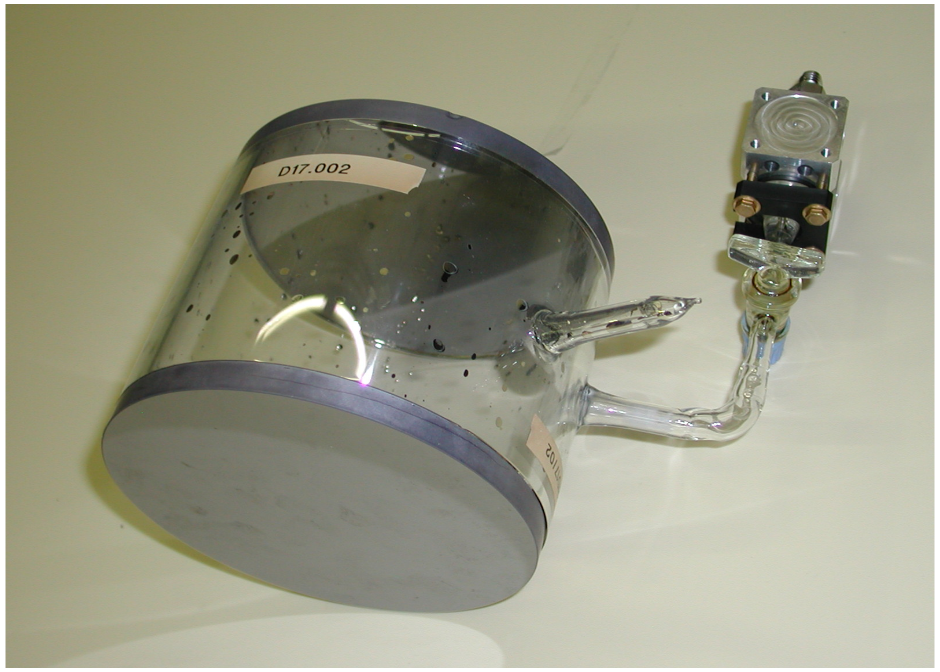
Optimizing Spin-filters



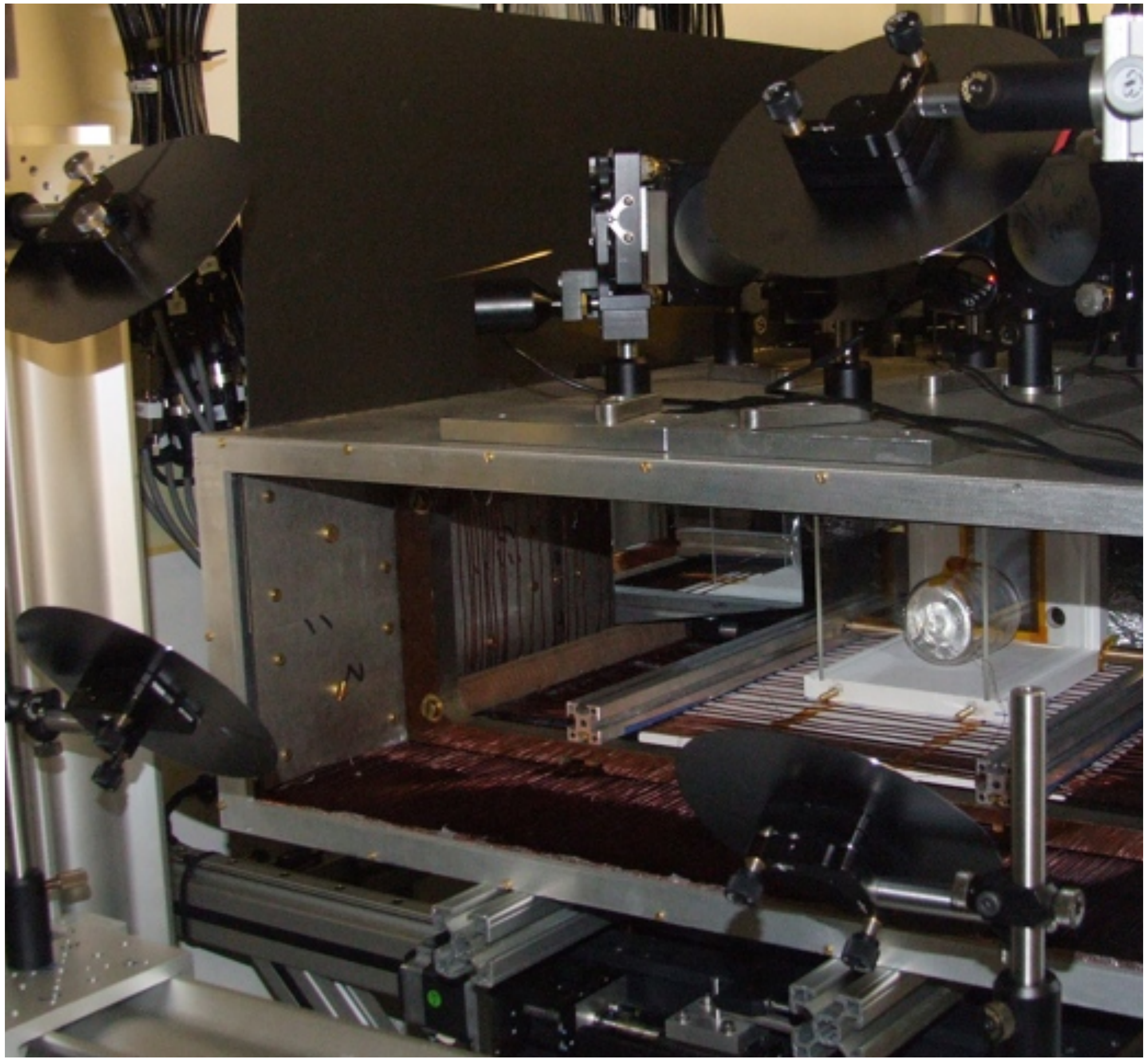


^{3}He polarization produced via optical pumping methods

^{3}He spin-filters in operation



Single crystal Si ^{3}He cell (D17, ILL)



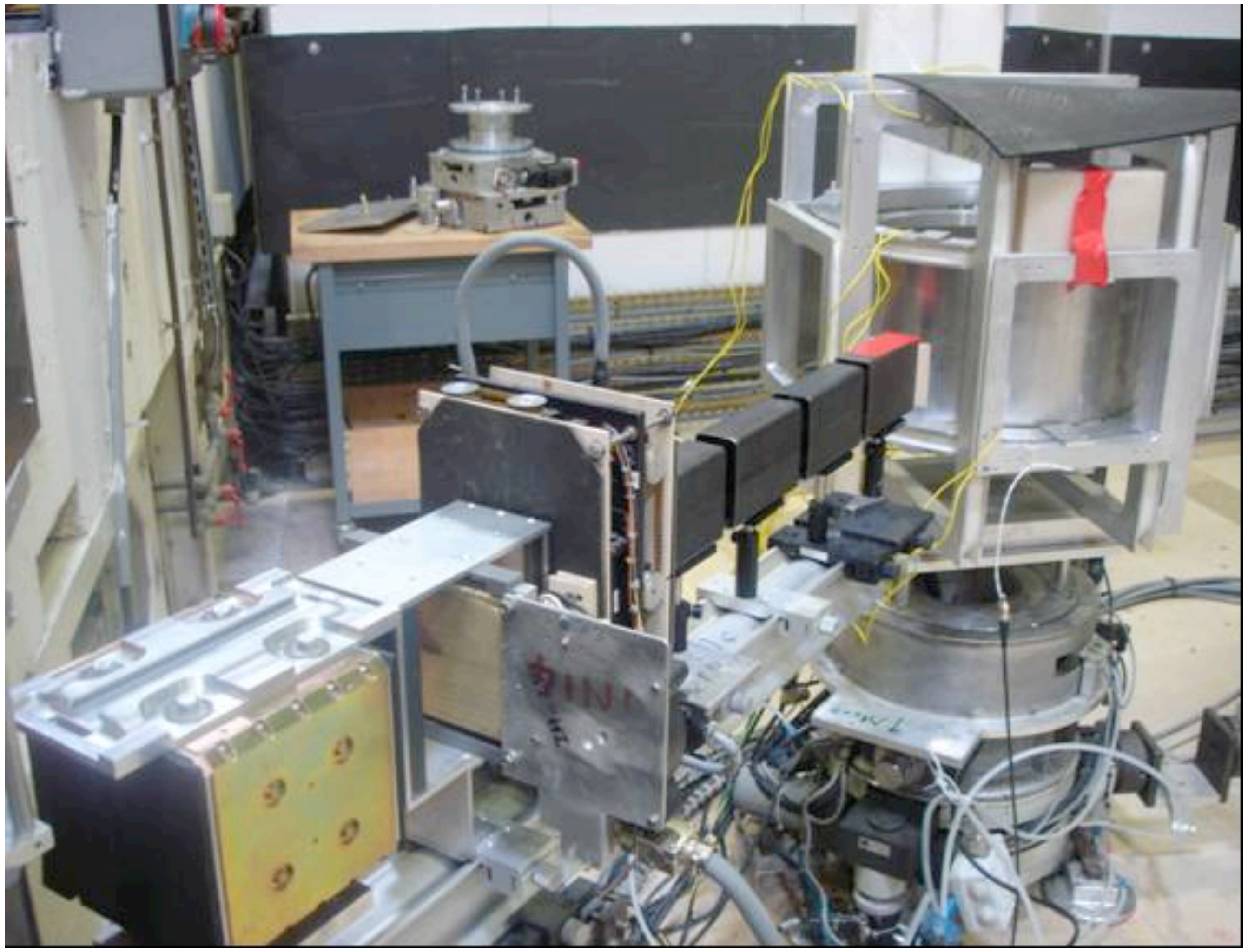
In situ SEOP ^{3}He cell (CRISP, ISIS)

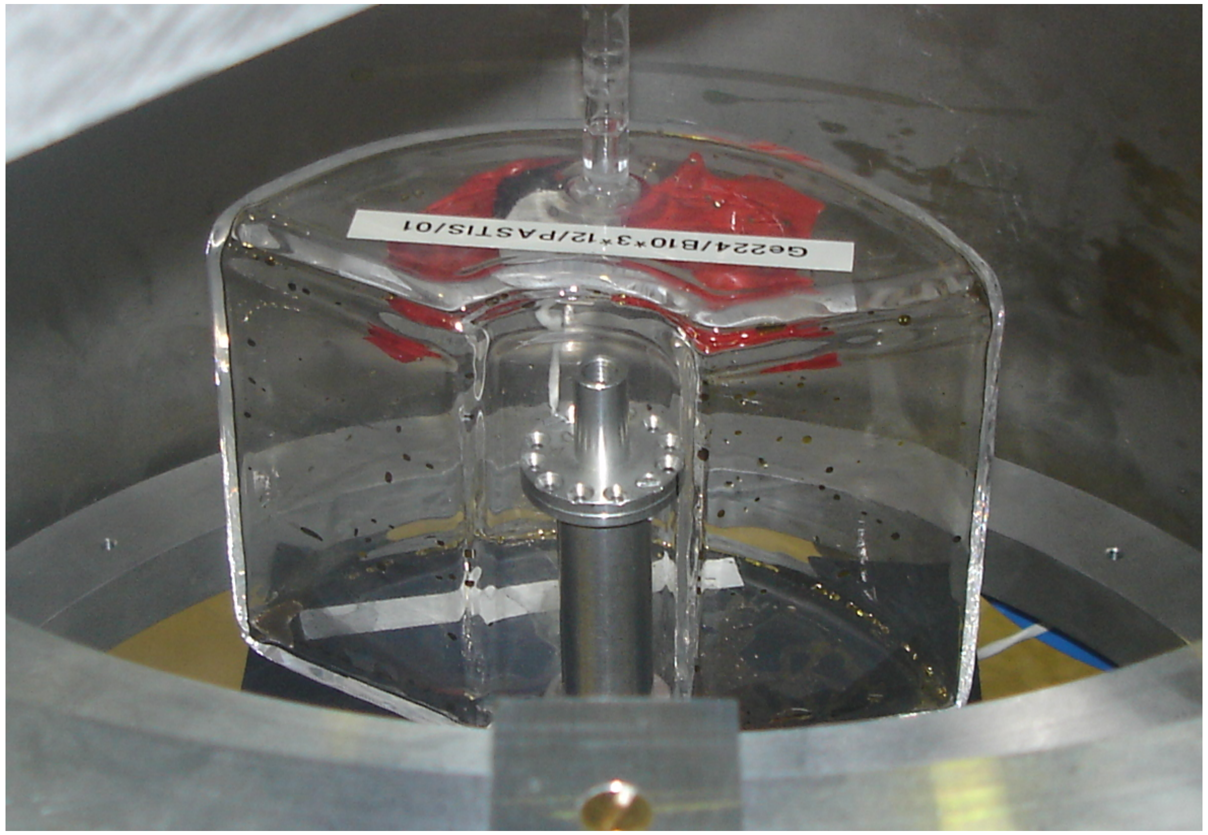
Typical values: 75% initial ^{3}He nuclear polarization with a relaxation time of 100 hours

Relaxation of the ^{3}He polarization arises from collisions with the container walls, from dipole-dipole interactions, and from stray magnetic fields.

^{3}He spin-filters in operation

Time-of-flight spectrometers require a neutron spin-filter that covers a wide solid angle (big detector angles) and is efficient at thermal energies - ^{3}He spin filter is ideal for this





Stewart et.al. Physica B ***385-86*** *(2006) 1142-1145*

Neutron mirror polarizers

All optical phenomena have their neutron counterparts: e.g. refractive index

Optical	Neutron
$n = \frac{c}{v}$	$n = \frac{\lambda_1}{\lambda_2} = \frac{k_2}{k_1}$

If we assume that the neutron experiences a change of energy equivalent to <V> when it enters medium 2, we can write

$$n = \frac{k_2}{k_1} = \left[\frac{E_1 - \langle V \rangle}{E_1}\right]^{\frac{1}{2}} \approx 1 - \frac{\langle V \rangle}{2E_1} \quad \text{assuming that} \quad \langle V \rangle << E_1$$

The potential <V> will in general consist of a nuclear and magnetic part (Fermi pseudopotential)

$$\langle V \rangle_N = \frac{2\pi\hbar^2}{m} N\overline{b} \qquad \langle V \rangle_M = \frac{2\pi\hbar^2}{m} N\overline{p} = \pm\mu_n B$$

Neutron mirror polarizers

Therefore, the spin dependent refractive index of magnetised mirrors for neutrons of wavelength λ is .

$$n_{\pm} = 1 - \left(\frac{N\lambda^2}{2\pi}\right)\left(\overline{b} \pm \overline{p}\right)$$

where b is mean coherent nuclear scattering length, N is the number density of scattering nuclei, and B is the flux density applied in the plane of the surface

Snell's law for refraction states $n_{1,2} = \cos\theta_1 / \cos\theta_2$

And the critical angle of reflection (defined when $\theta_2 = 0$) is $n_{1,2} = \cos\theta_c \approx 1 - \frac{\theta_c^2}{2}$

There are therefore two critical glancing angles $\theta_{c\pm}$ for total (external) reflection:

$$\theta_{c\pm} = \lambda\left[\frac{N}{\pi}\left(\overline{b} \pm \overline{p}\right)\right]^{\frac{1}{2}}$$

Between these two critical angles the reflected beam is effectively fully polarized, and in some circumstances the critical angle for one spin state can be made zero

However, the critical angles are very small (typically 10 arc minutes)

Hughes and Burgy, Phys. Rev. ***81*** *(1951) 498*

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Multi bi-layer mirrors

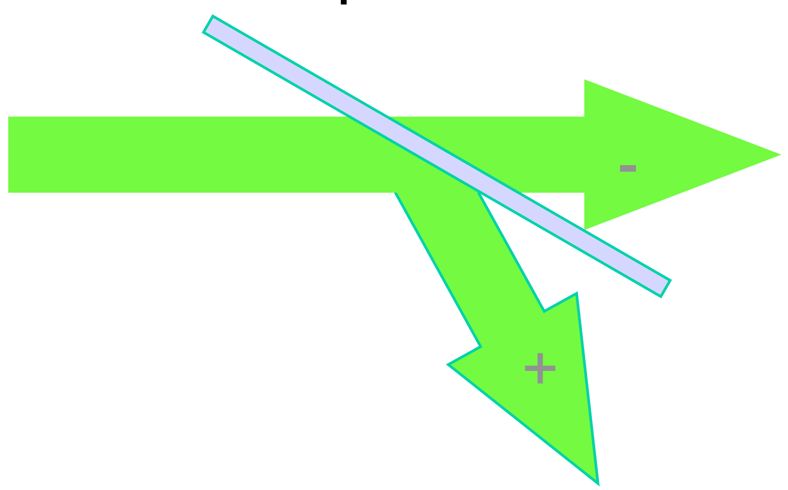
In a polarizing neutron mirror, the basic unit is a non-magnetic/magnetic (A/B) bilayer, which has a reflectivity of the form

$$R_{\pm} \propto \left[N_A b_A - N_B \left(b_B \pm p_B \right) \right]^2$$

Therefore, with a judicious choice of N and b, it can be arranged that $R_{-} = 0$ implying perfect beam polarization (but not reflectivity) at all angles of incidence

In practice this is very difficult to achieve, since the spin-down neutrons have to go somewhere. They are either transmitted or absorbed in an absorbing layer (often Gd, which has a non-zero reflectivity at very low angles)

Transmission polarizer



Very good polarization since clean separation of polarization states is achieved. Can only accept limited divergence and one wavelength

Hughes and Burgy, Phys. Rev. ***81*** *(1951) 498*

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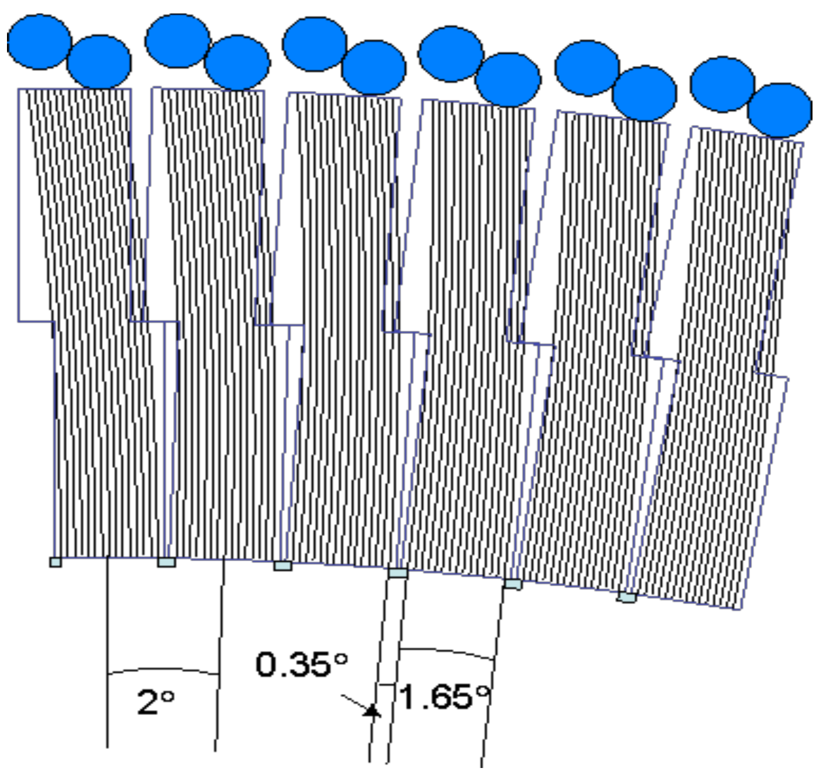
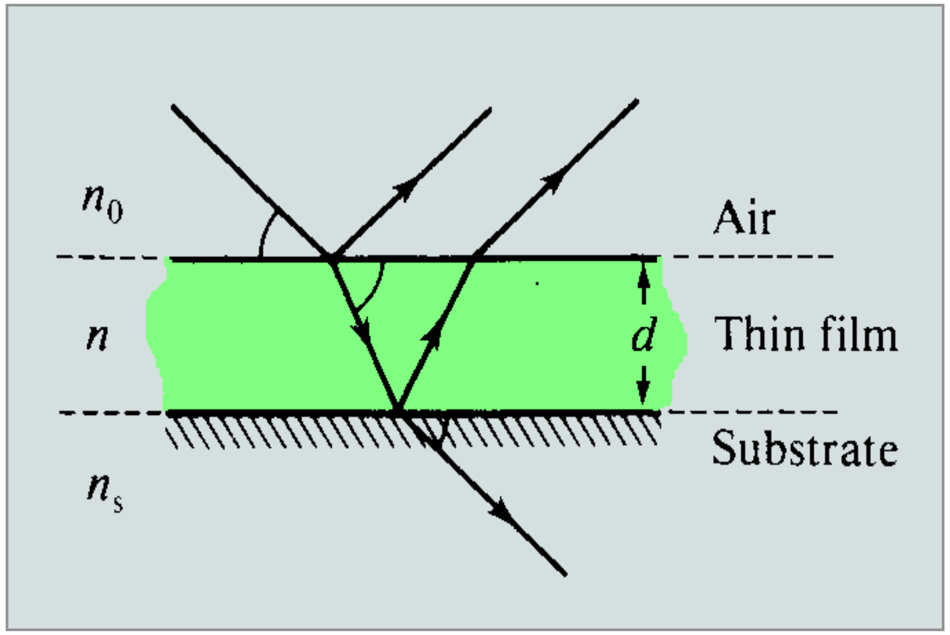


Bender polarizers (D7, ILL)

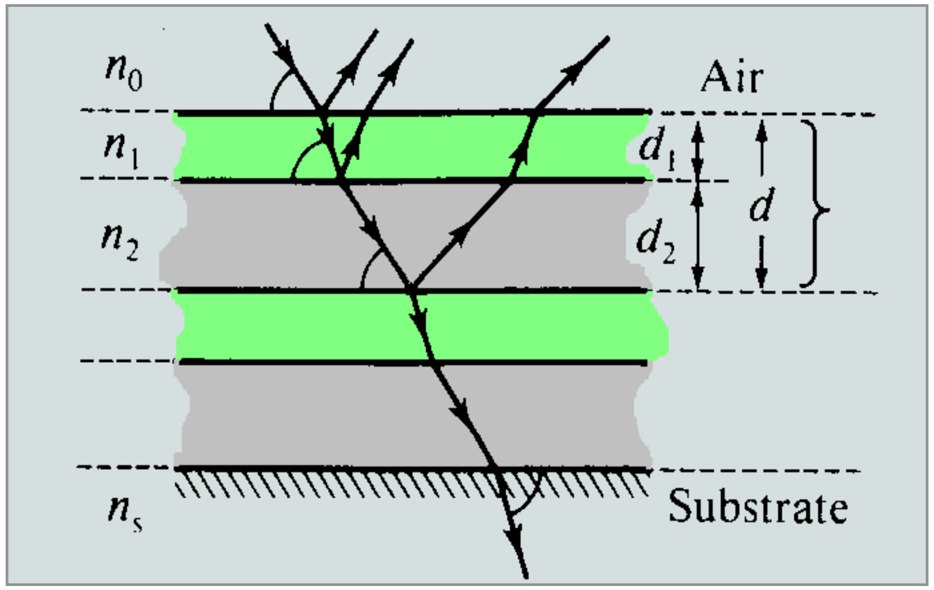
Mirrors are bent to ensure at least one reflection of the neutrons. Polarization less good due to finite reflectivity of absorbing layer. But able to accept large divergence of angles (stacked device)

*O Schärpf, Physica B **174** (1991) 514-527*

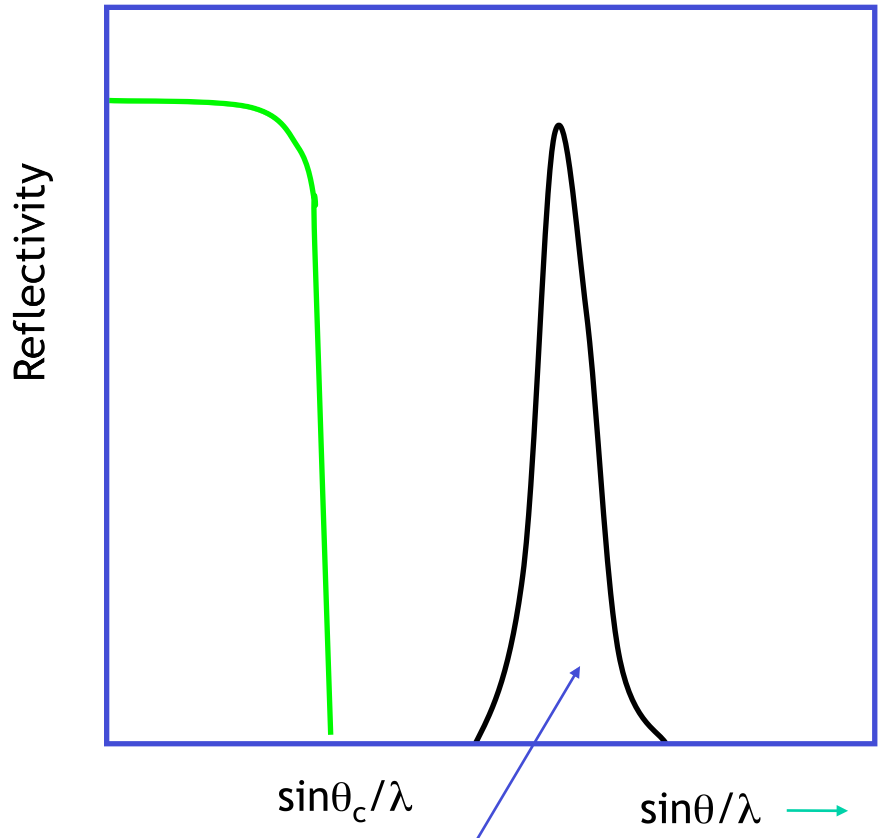
Neutron "supermirrors"



Thin film (eg CoFe)

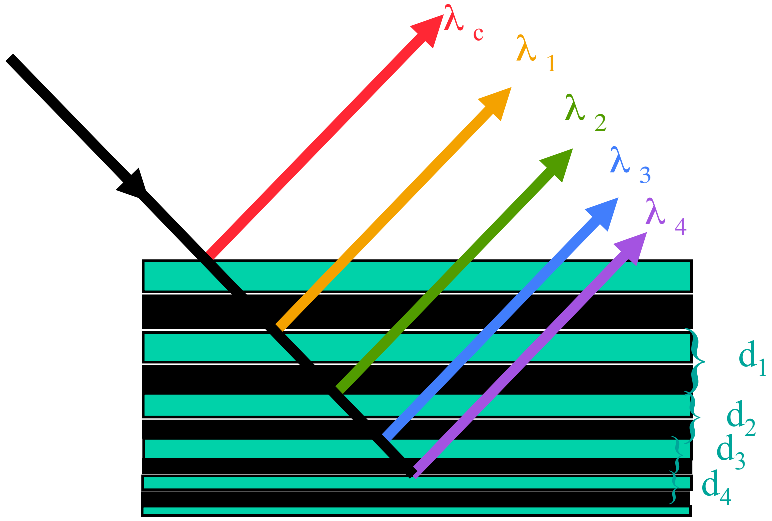


Bi-layers (eg Co/Ti, Fe/Si)

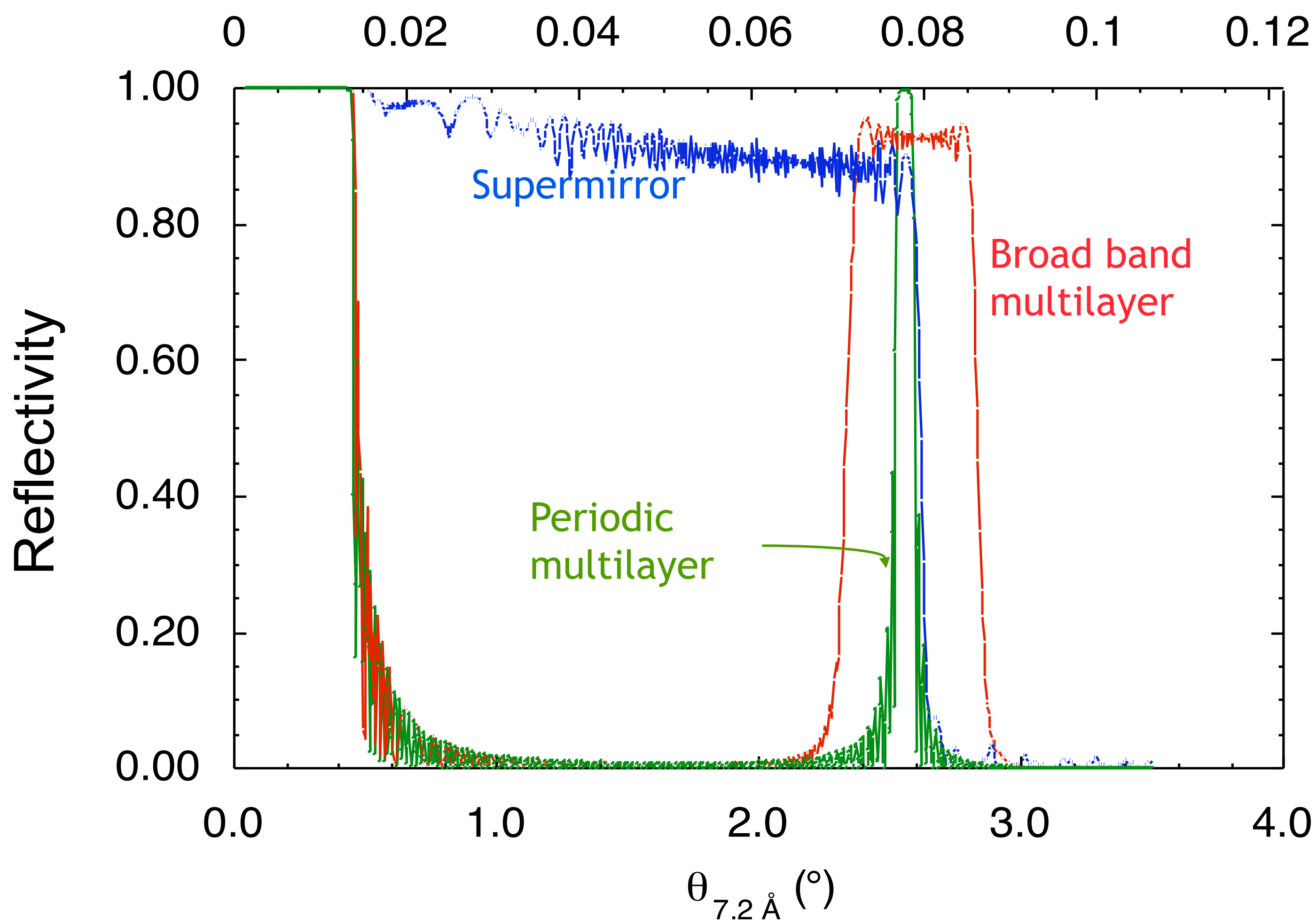


The multi-bilayer system introduces an additional Bragg peak at $\sin\theta/\lambda = 1/2d$

Neutron “supermirrors”



Supermirror systems
(eg Co/Ti, Fe/Si etc)



A gradient in the lattice spacing of the bilayers results in a range of effective Bragg angles, and therefore a reflectivity which extends beyond values expected for normal mirror reflections

Supermirror m-number indicates the range of angles for good reflectivity
For a single (thick) layer of Ni, m = 1 = 0.1°/Å

Mezei, Commun. Phys. ***1*** *(1976) 81*

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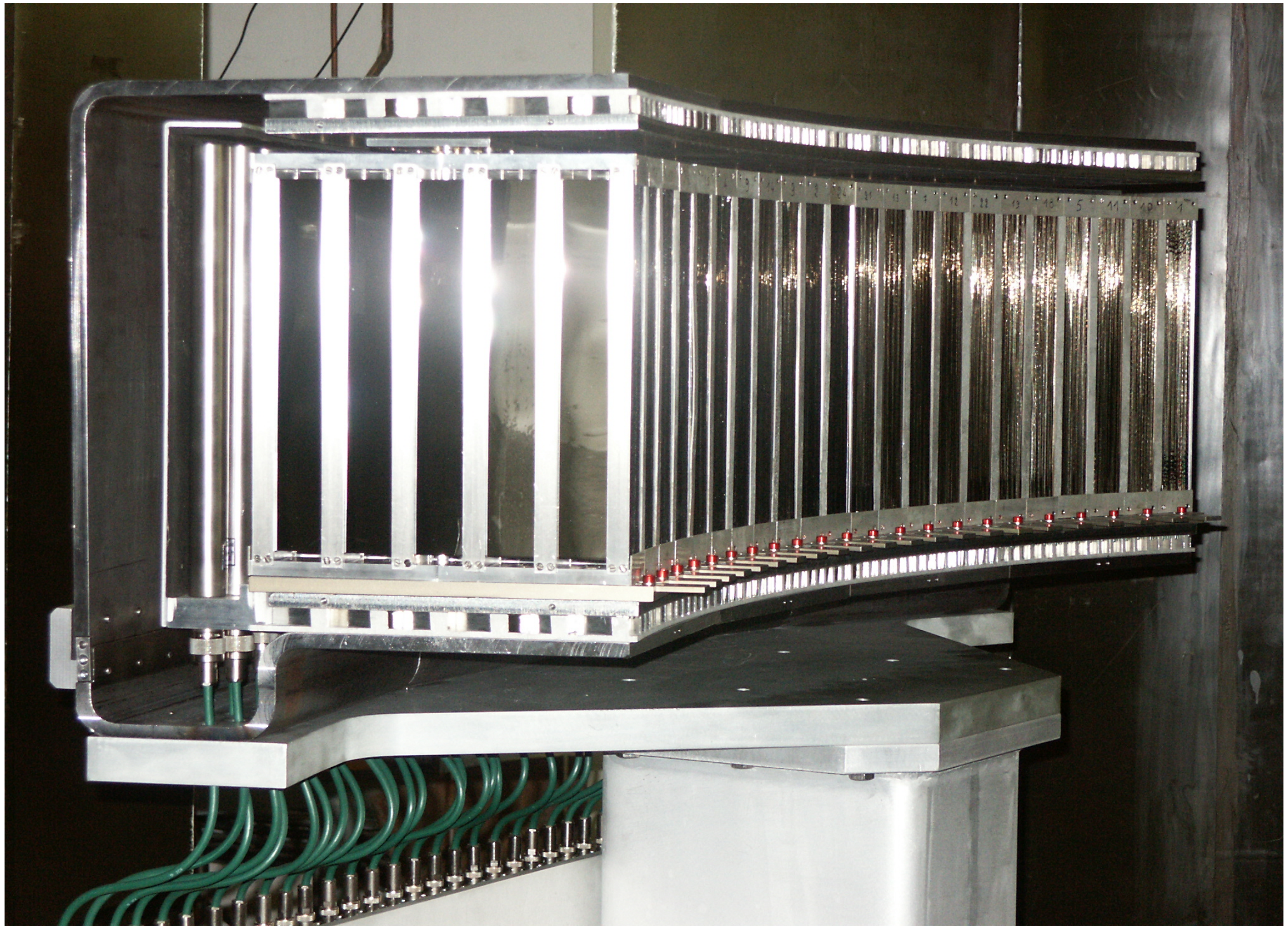
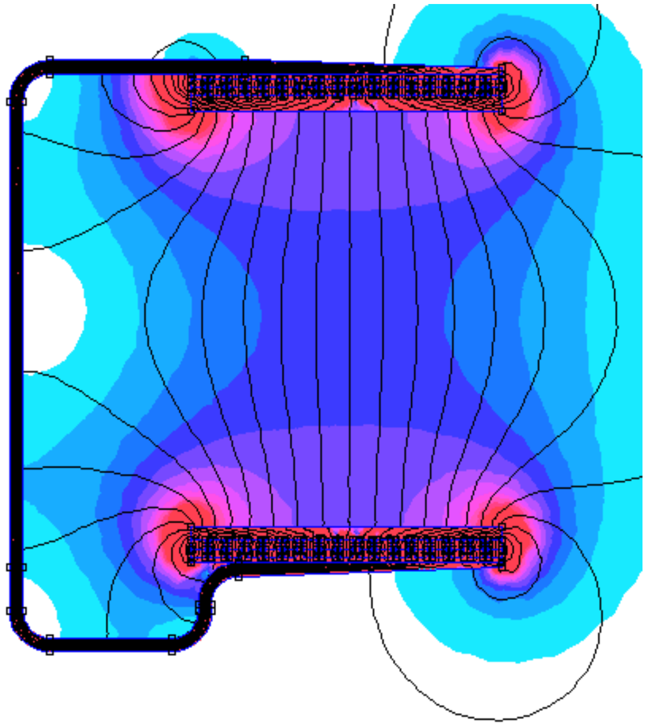
Supermirrors in operation

Supermirror “bender” analyser array on D7, ILL. There is over 250 m^2 of supermirror in the full analyser array. (c.f. doubles tennis court is 260 m^2)

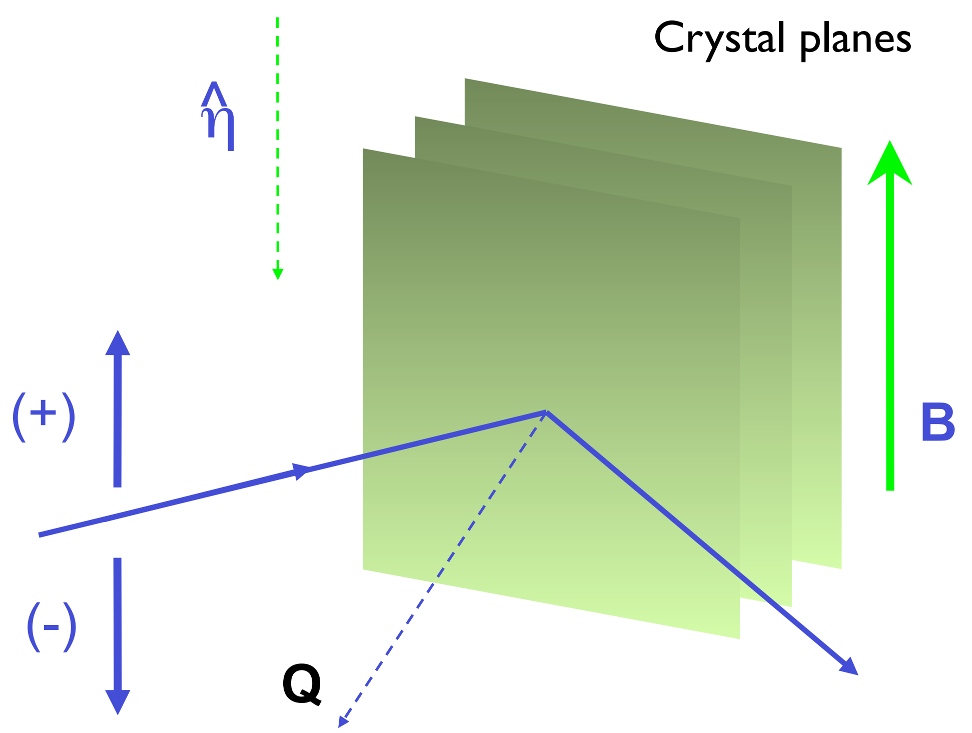


D7, ILL



See also *Boni et al, (Physica B **267-8**(1999) 320)* for the development of remnant supermirrors

Polarizing Crystals



The cross section for Bragg reflection in the geometry shown left, in which the incident beam is considered as a superposition of P=1 and P=-1 states, is

$$d\sigma/d\Omega = F_N^2(\mathbf{Q}) + 2F_N(\mathbf{Q})F_M(\mathbf{Q})(\mathbf{P}\cdot\hat{\boldsymbol{\eta}}) + F_M^2(\mathbf{Q})$$

where $F_{N,M}(\mathbf{Q})$ are the nuclear and magnetic structure factors for the reflection

For $\mathbf{P}\cdot\hat{\boldsymbol{\eta}} = {}^{+}1$: $d\sigma/d\Omega = \left[F_N(\mathbf{Q}) + F_M(\mathbf{Q})\right]^2$ For $\mathbf{P}\cdot\hat{\boldsymbol{\eta}} = -1$: $d\sigma/d\Omega = \left[F_N(\mathbf{Q}) - F_M(\mathbf{Q})\right]^2$

If $|F_N(\mathbf{Q})| = |F_M(\mathbf{Q})|$ the reflected beam will be polarized.

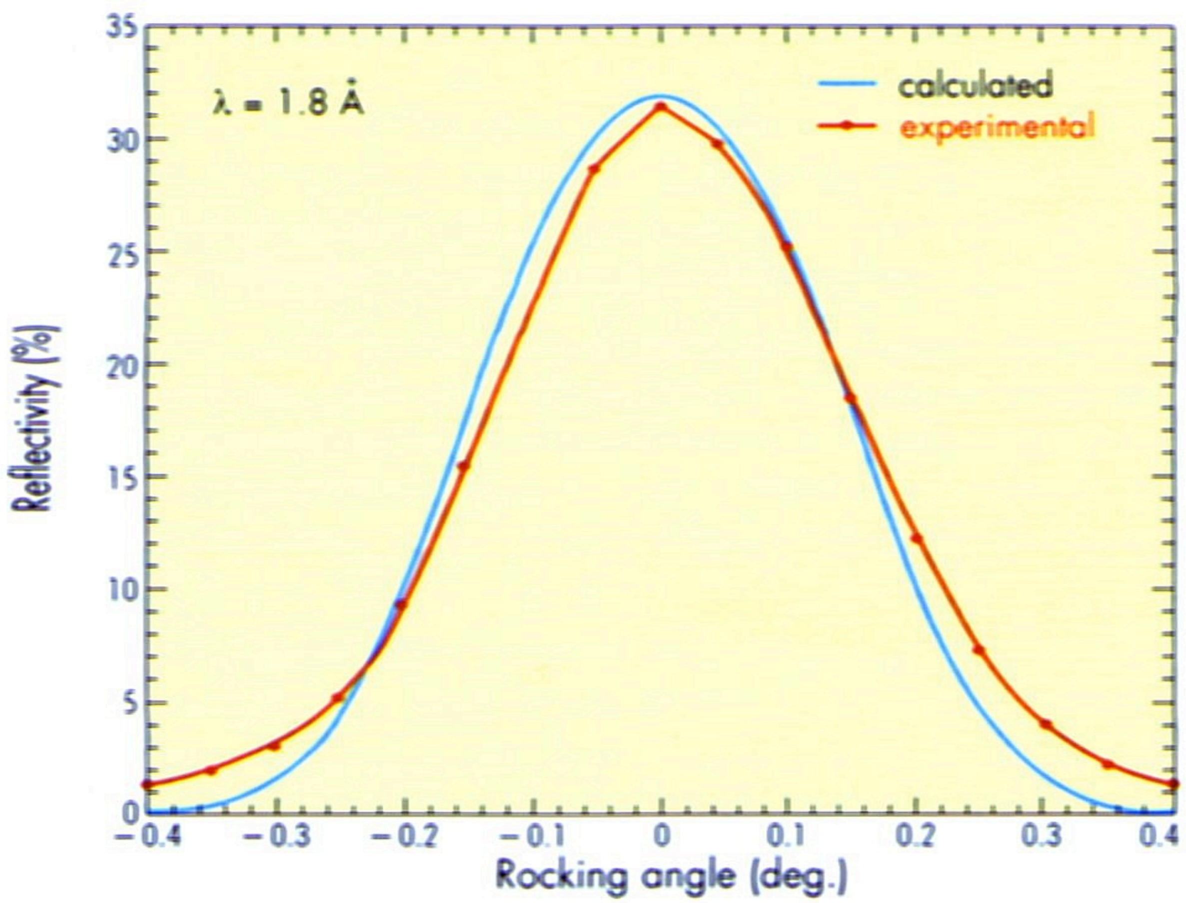
The nuclear and magnetic structure factors can be either positive or negative, so beam polarization, $\mathbf{P}_f$, could be either "up" or "down" with respect to **B** .

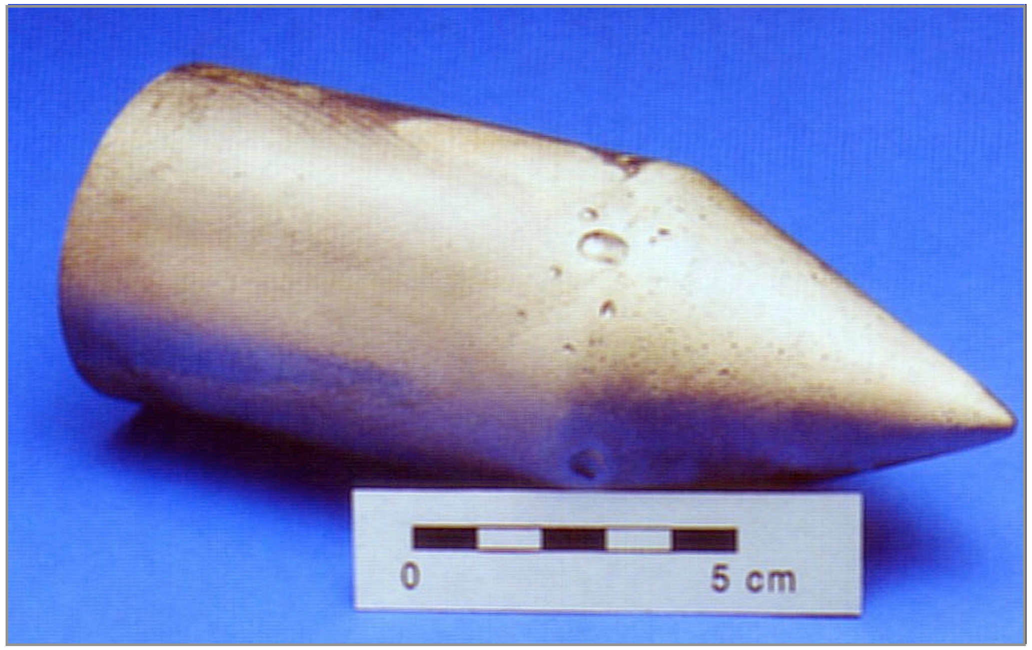
In general the polarizing efficiency of a crystal reflection is given by

$$P_f = \frac{2F_N(\mathbf{Q})F_M(\mathbf{Q})}{\left[F_N^2(\mathbf{Q}) + F_M^2(\mathbf{Q})\right]}$$

Crystal monochromator polarizers

	$Co_{92}Fe_8$	Cu_2MnAl	Fe_3Si
Matched reflection	(200)	(111)	(111)
d-spacing (Å)	1.76	3.43	3.27
2θ at λ=1Å	33.1	16.7	17.6
Maximum λ (Å)	3.5	6.9	6.5





Cu_2MnAl (Heusler) crystal grown at ILL, with associated reflectivity curve.

Cu_2MnAl has a higher reflectivity and lower absorption than $Co_{92}Fe_8$, also F_N=-F_M so the beam is negatively polarized with respect to B

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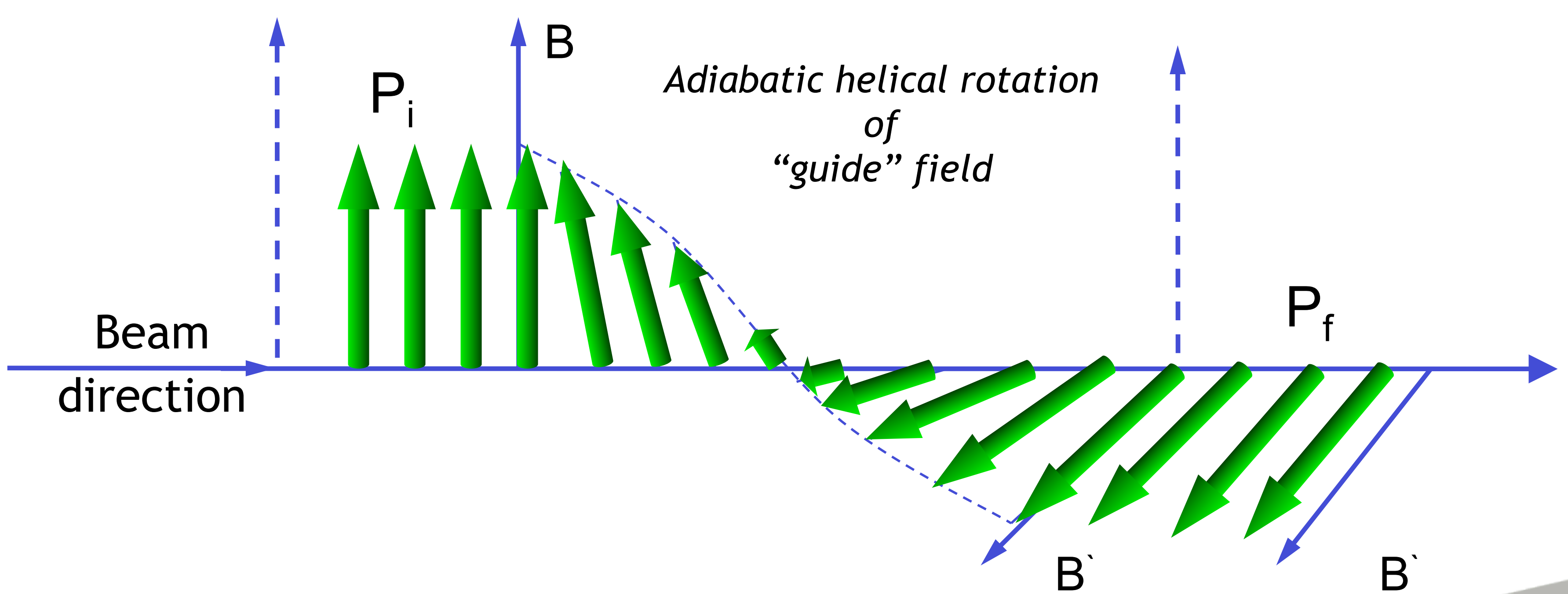
Guiding and flipping neutrons

Guiding the polarization

If the direction of the magnetic field (quantisation axis) remains constant in the rest frame of the neutrons then the direction of the polarization of the neutron beam will be preserved

However, if the direction of the field B changes sufficiently slowly in the rest frame of the neutrons, then the polarization component parallel to B is conserved - ie there is an adiabatic (or reversible) rotation of the polarization

[rem: An adiabatic process is one in which the system is always "infinitesimally close" to equilibrium. Here the field is changed in such a way that the potential energy of the neutrons is close to its initial value – and returns to this value at the end of the process]



Larmor precession

Angular momentum of a spin in a magnetic field (in the z-direction), equation of motion is

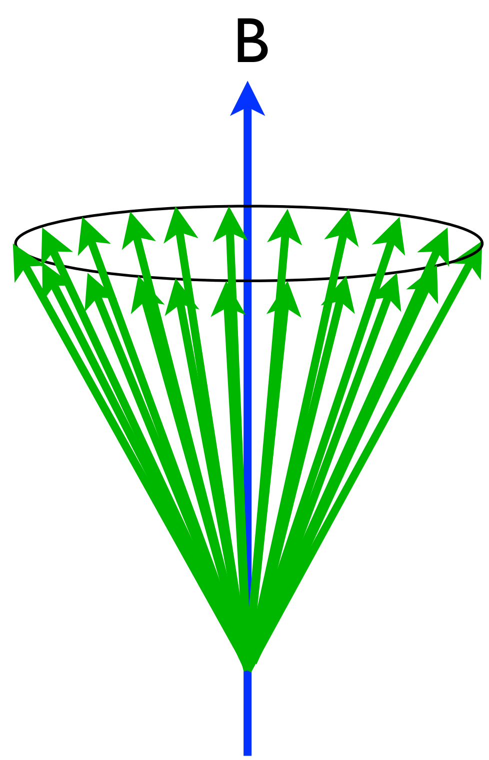
$$\frac{\hbar d\mathbf{I}}{dt} = \mu \times \mathbf{B} \qquad \text{or} \qquad \frac{d\mu}{dt} = \gamma_n \mu \times \mathbf{B}$$

Where the gyromagnetic ratio, γ_n, is the ratio of the magnetic moment to the angular momentum

This gives oscillatory solutions of the form

$$\mu_x = |\mu| \cos \omega_L t \qquad \mu_y = -|\mu| \sin \omega_L t$$

describing a precession of the spin around the field direction, with an angular frequency of ω_L, the **Larmor precession frequency**



$$\omega_L = \gamma_n B$$

Adiabatic rotation

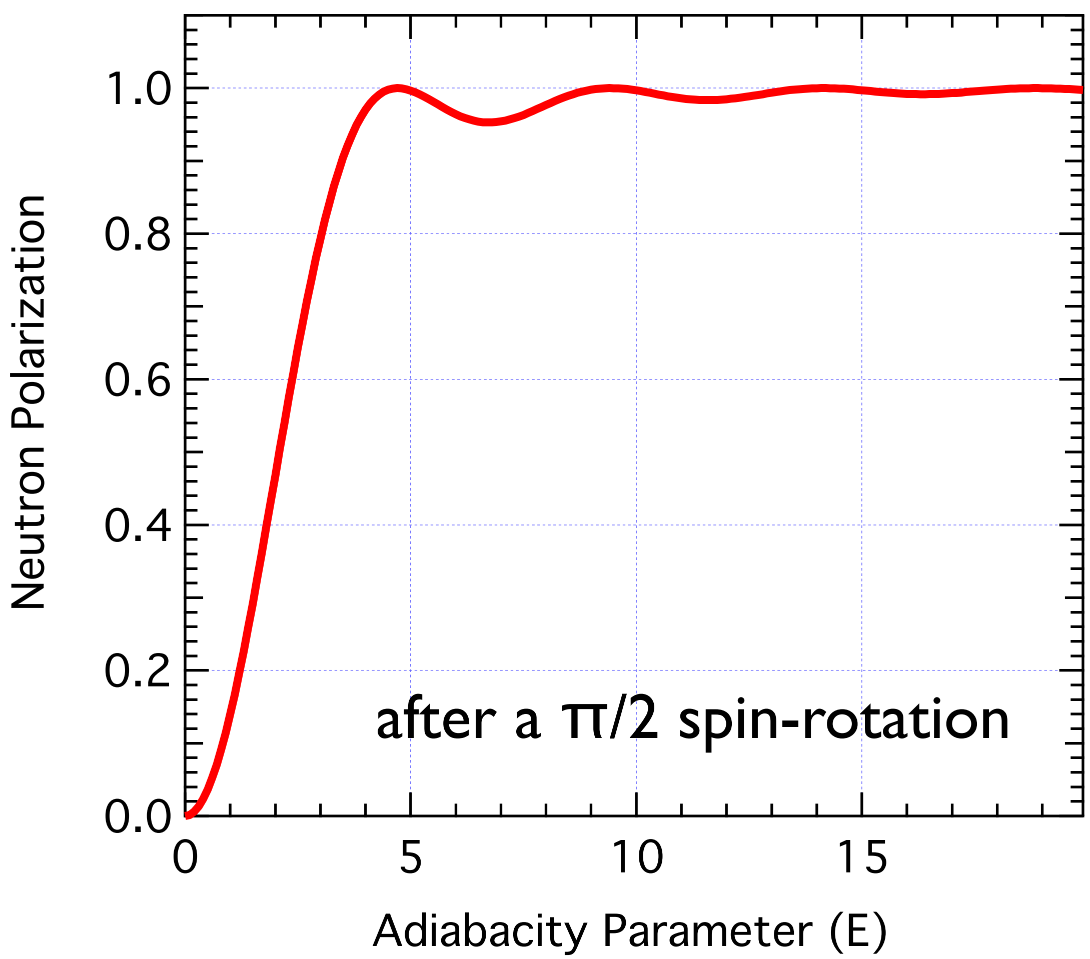
The rate of angular rotation, ω_B, of the field along the y-axis in the rest frame of the neutron is:

$$\omega_B = \frac{d\theta_B}{dt} = \frac{d\theta_B}{dy}.\frac{dy}{dt} = \frac{d\theta_B}{dy}v$$

where v is the neutron velocity

We can therefore define an adiabaticity parameter, E, where

$$E = \frac{\omega_L}{\omega_B} = \frac{|\gamma_n|B}{\frac{d\theta_B}{dy}v}$$



For an adiabatic rotation without loss of polarization we require E>10 (by bitter experience)

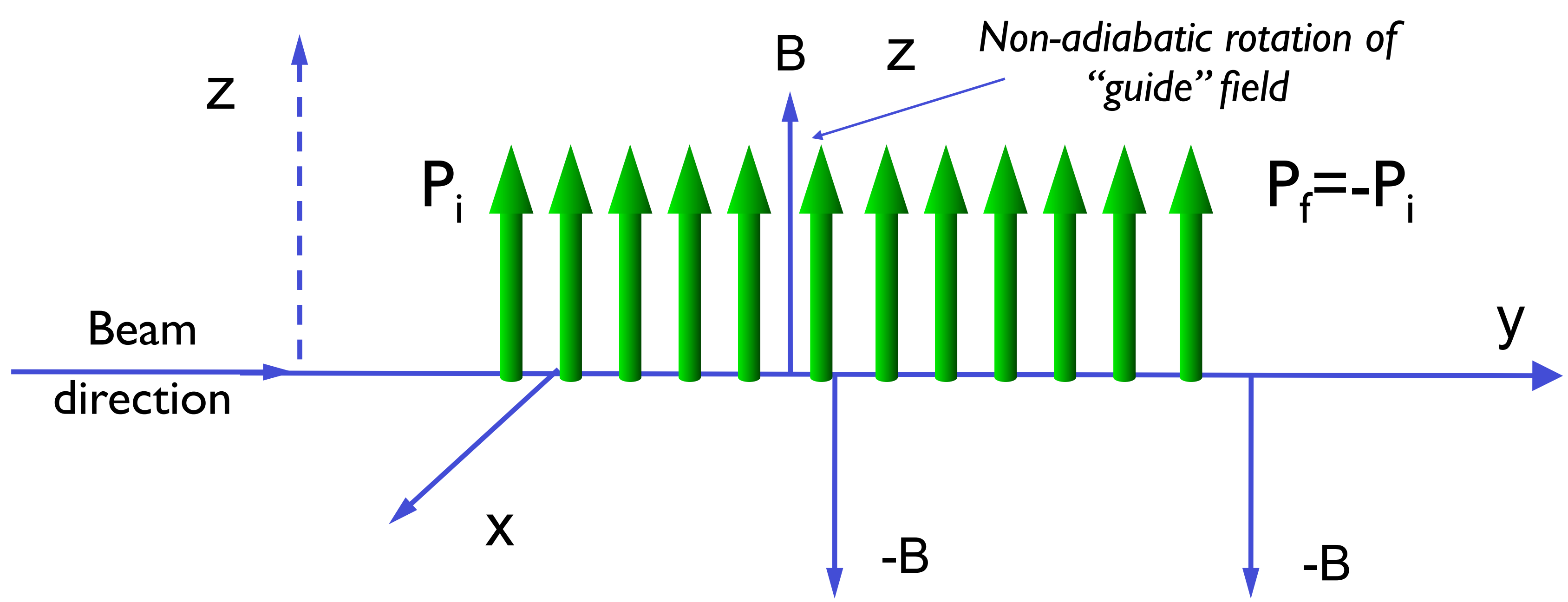
This inequality corresponds to $\frac{d\theta_B}{dy} < 2.65B\lambda$ degrees/cm

with B in mT, θ in degrees, distance y in cm and neutron wavelength, λ in Å

Non-adiabatic transitions

Although we can adiabatically reorient the polarization in the laboratory frame, the polarization of the beam remains constant with respect to the guide field

However, for a non-adiabatic reorientation of the guide field ($\Delta E \neq 0$) the polarization will not re-orient - instead the beam will preserve its initial direction, and begin to precess about the new field direction



Non-adiabatic rotations enable the beam polarization to be effectively "flipped" with respect to the guide field

Non-adiabatic transitions

Although we can adiabatically reorient the polarization in the laboratory frame, the polarization of the beam remains constant with respect to the guide field

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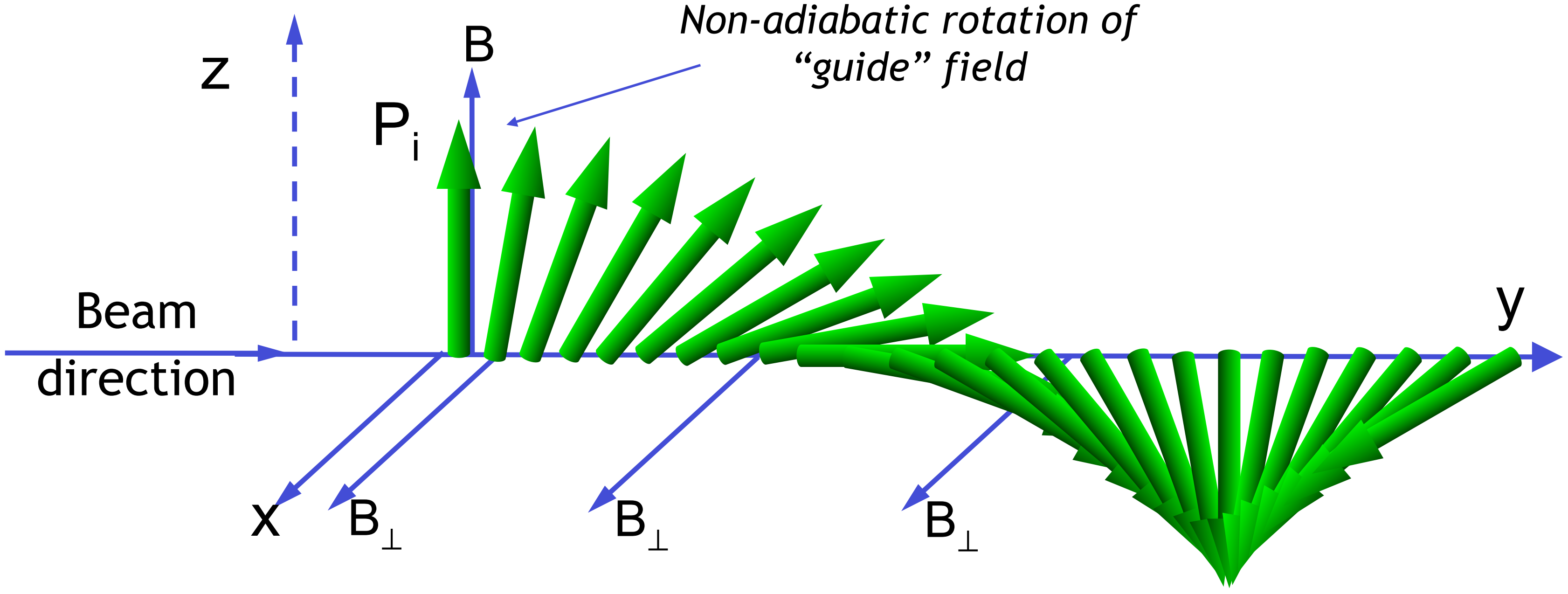
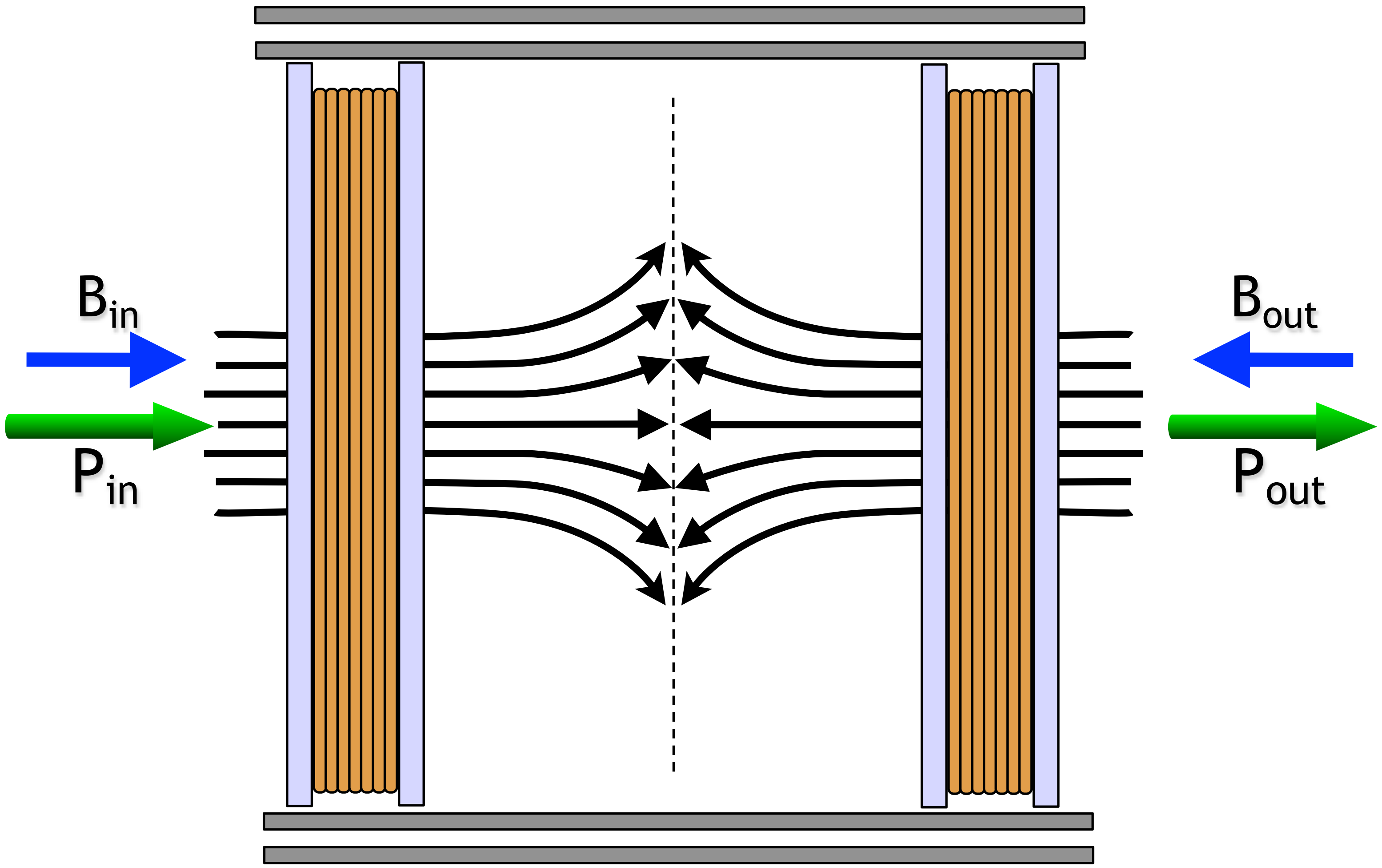


Non-adiabatic rotations enable the beam polarization to be effectively "flipped" with respect to the guide field

Drabkin flipper

A wide variety of devices have been employed as effective non-adiabatic field reversing "spin-flippers":

Drabkin flipper: useful for white beams of limited size
- used on the reflectometers CRISP and SURF at ISIS



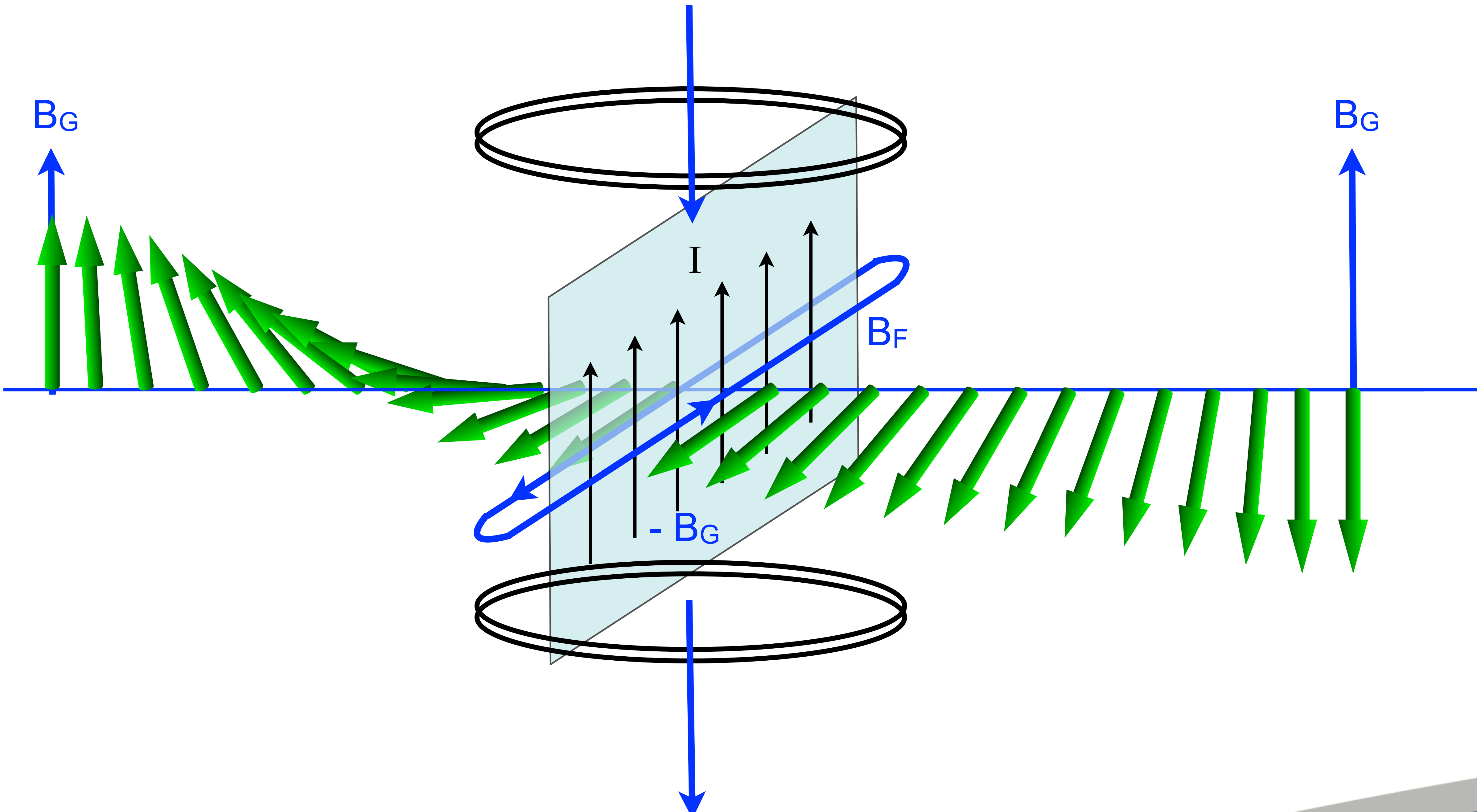
π-flipper

Radial magnetic field off-axis
Results in low flipping ratios

Useful for thin beams only

BUT
No material in beam

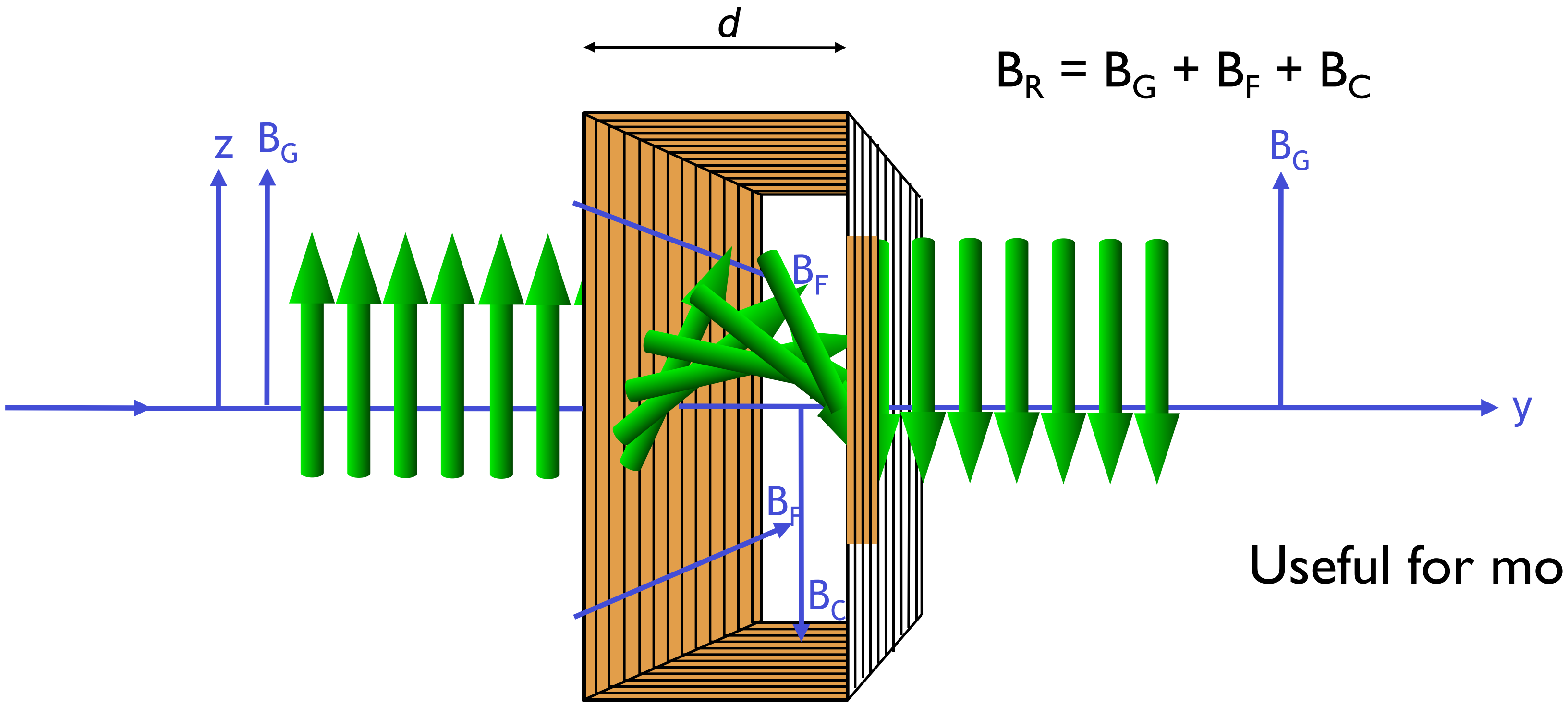
Current Sheet (Dabbs Foil)



Another example of a "sudden field-reversal" π-flipper.
Works for wide wavelength band
(for wavelengths which adiabatically follow the field)

Mezei flipper

The **Mezei flipper** uses a non-adiabatic 90° field change to project the polarization direction of the beam onto any arbitrary field axis:



$B_R = B_G + B_F + B_C$

Useful for monochromatic beams

A flip of φ radians with respect to the guide field can be achieved if the resultant field within the coil, $\mathbf{B}_R$, is perpendicular to $\mathbf{B}_G$ and

$$d = \frac{\phi v}{\gamma_n B_R}$$

e.g. a 1Å neutron will be spin flipped by π radians in a distance of 1cm if B_R=1mT

For further details see, e.g. *Hayter Z Physik B 31, 117 (1978)*

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AFP Flipper

Adiabatic fast passage (see Abragam, Principles of Nuclear Magnetism)

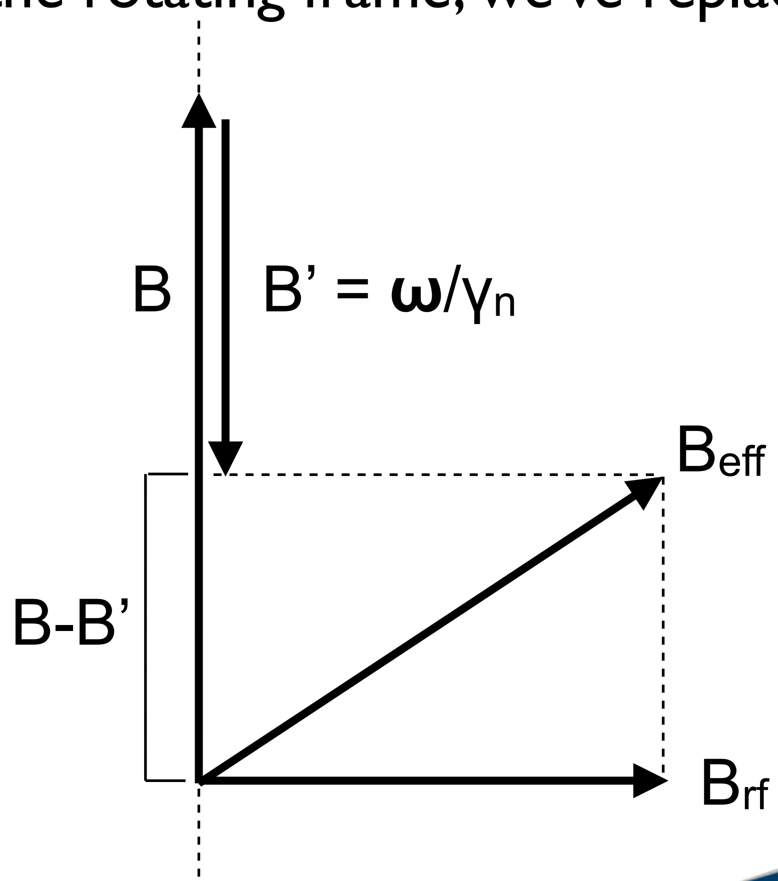
Equation of motion of a magnetic moment in a field **B**

$$\frac{d\mu}{dt} = \gamma_n \mu \times \mathbf{B}$$

Now convert to a rotating frame, with angular velocity $\boldsymbol{\omega}$

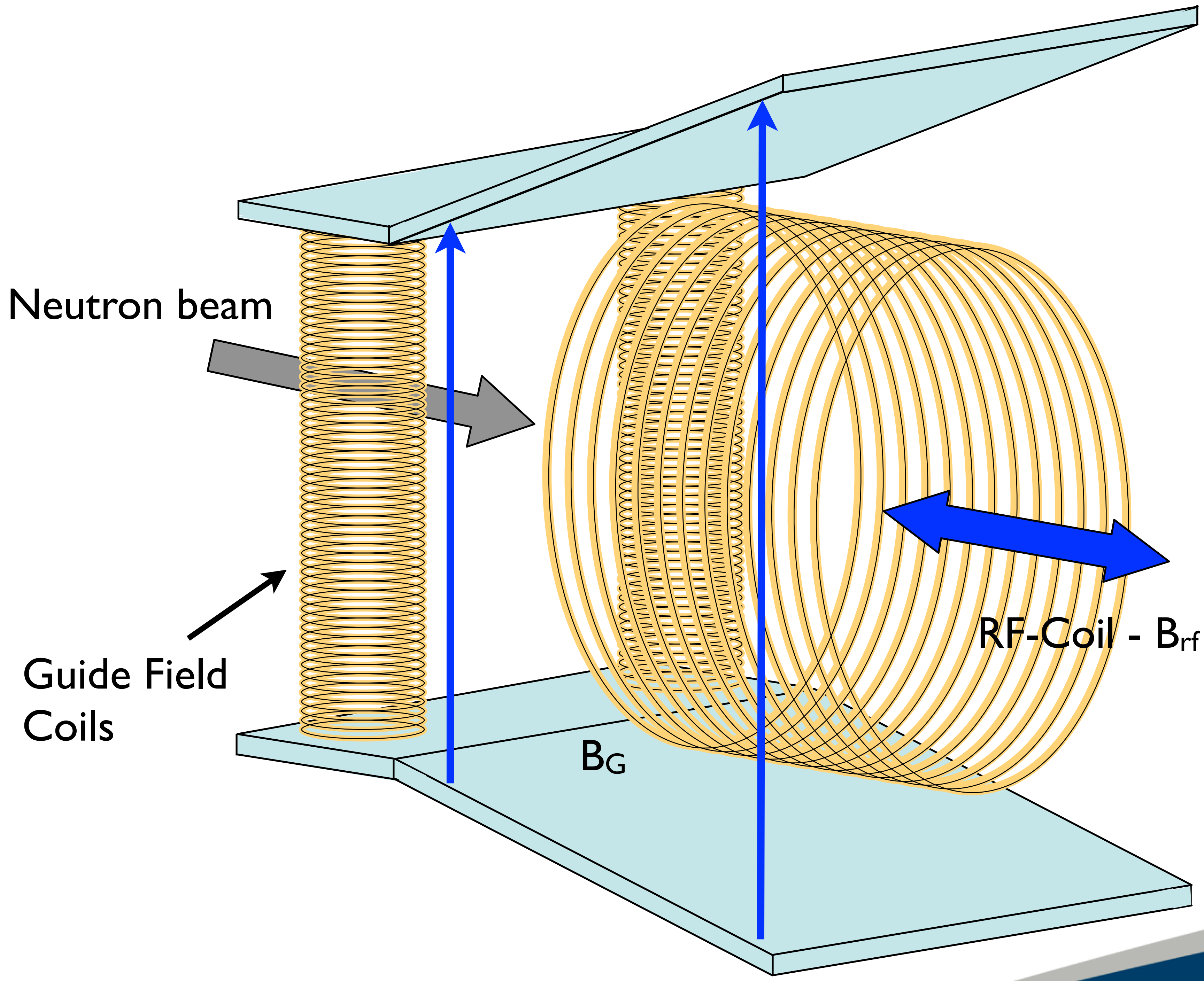
$$\frac{d\mu}{dt} = \gamma_n \mu \times (\mathbf{B} - \frac{\omega}{\gamma_n})$$

In the rotating frame, we've replaced the field, by an effective field $(\mathbf{B} - \frac{\omega}{\gamma_n})$



So, running around a static field B with increasing angular velocity, will reduce the field, then cause it to disappear (when the rotating frame is at Larmor frequency) and then reverse the field.

AFP flipper

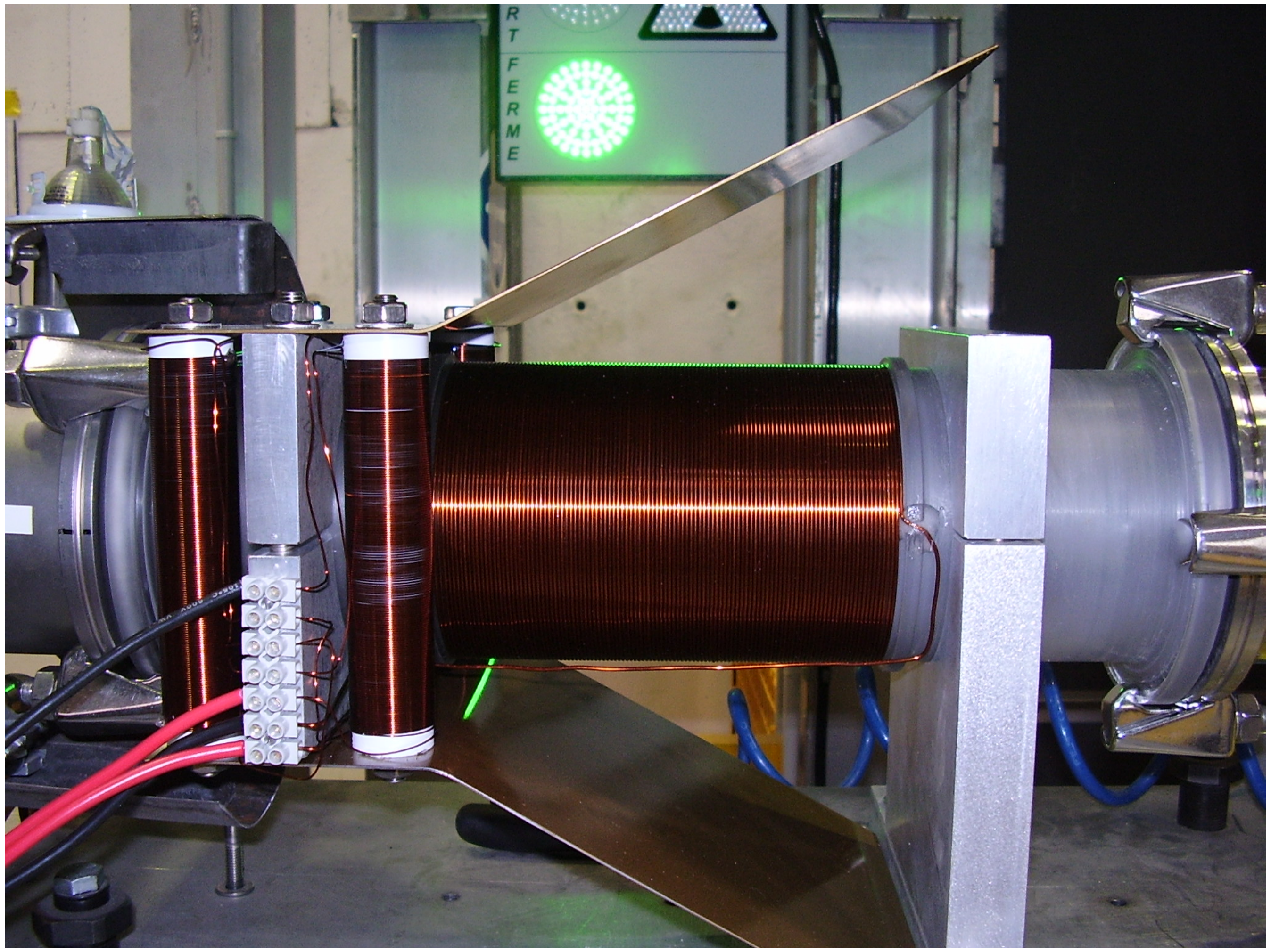


Rf-field at constant frequency

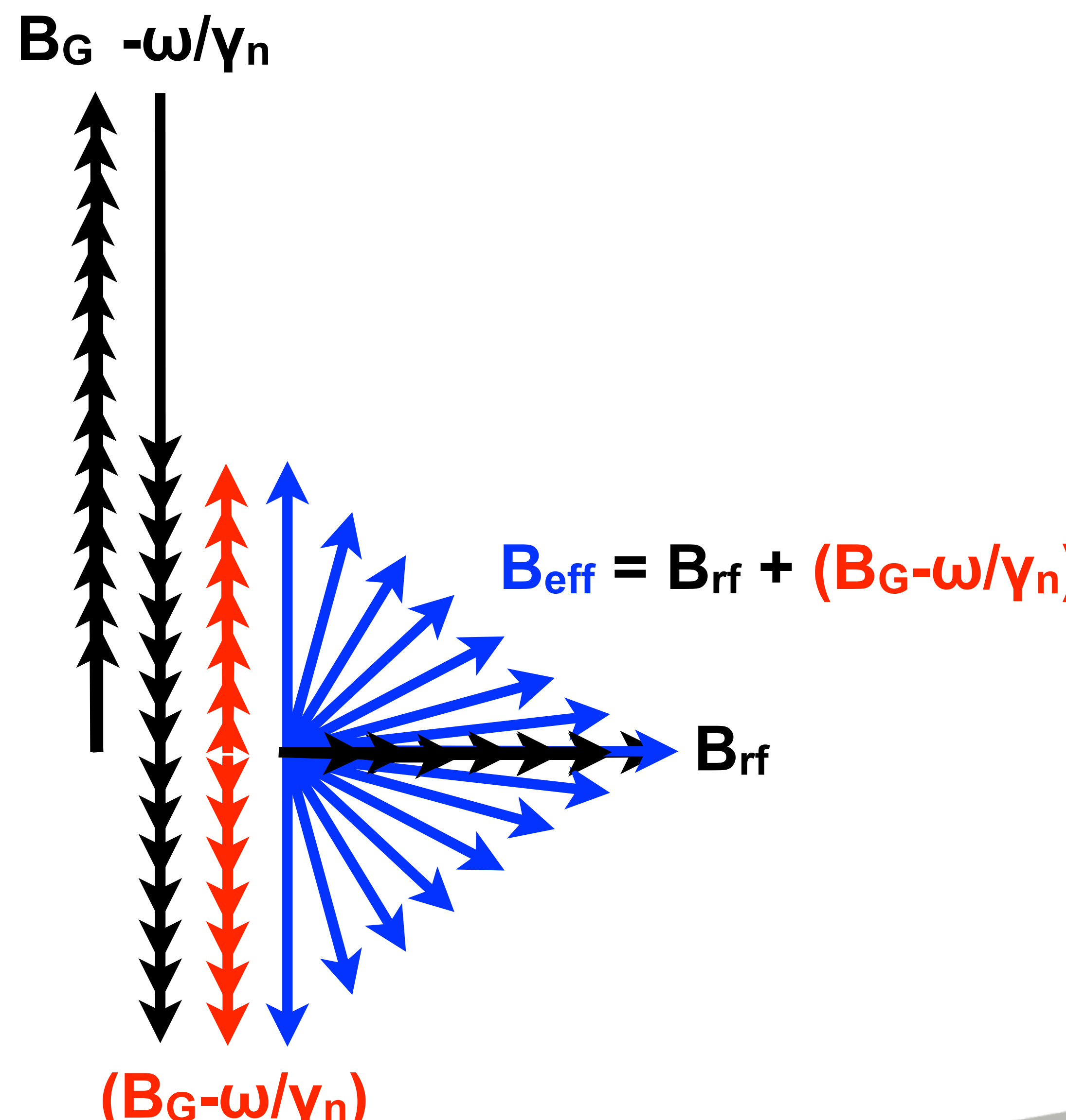
B_G swept from high to low fields, through resonance with B_{rf}

So in rotating frame the effective field B_{eff} reverses - flipping the spins

AFP flipper



AFP flipper installed on D22 at ILL



AFP can also be used to flip polarized ^3He nuclei is neutron spin-filters, using a *frequency sweep*

Uniaxial Polarization Analysis

Neutron polarization and scattering

We start with the (elastic - $|k_i| = |k_f|$) scattering cross-section

$$\frac{d\sigma}{d\Omega} = \left(\frac{m_n}{2\pi\hbar^2}\right)\left|\langle \mathbf{k}'\mathbf{S}'|V|\mathbf{kS}\rangle\right|^2$$

Where the spin-state of the neutron **S** is either spin-up $|\uparrow\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ or spin down $|\downarrow\rangle = \begin{pmatrix}0\\1\end{pmatrix}$

1) <u>Nuclear (I = 0) scattering</u>

V is the Fermi pseudopotential, and the matrix element is

$$\langle \mathbf{S}'|b|\mathbf{S}\rangle = b\langle \mathbf{S}'|\mathbf{S}\rangle = \begin{cases} b & \underbrace{\left\{\begin{matrix}|\uparrow\rangle \rightarrow |\uparrow\rangle \\ |\downarrow\rangle \rightarrow |\downarrow\rangle\end{matrix}\right\}}_{\text{Non-spin-flip}} \\ 0 & \underbrace{\left\{\begin{matrix}|\uparrow\rangle \rightarrow |\downarrow\rangle \\ |\downarrow\rangle \rightarrow |\uparrow\rangle\end{matrix}\right\}}_{\text{Spin-flip}} \end{cases}$$

where we have used the fact that the spin states are orthogonal and normalised

$$\langle\uparrow|\downarrow\rangle = \langle\downarrow|\uparrow\rangle = 0, \quad \langle\uparrow|\uparrow\rangle = \langle\downarrow|\downarrow\rangle = 1$$

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Neutron polarization and scattering

2) Magnetic scattering

V is the magnetic scattering potential given by

$$V_m(\mathbf{Q}) = -\frac{\gamma_n r_0}{2\mu_B}\boldsymbol{\sigma}\cdot\mathbf{M}_\perp(\mathbf{Q}) = -\frac{\gamma_n r_0}{2\mu_B}\sum_\zeta \sigma_\zeta \cdot M_{\perp\zeta}(\mathbf{Q})$$ (see e.g. Squires)

where ζ = x, y, z. Here $M_\perp(\mathbf{Q})$ represents the component of the Fourier transform of the magnetisation of the sample, which is perpendicular to the scattering vector **Q** - i.e. the neutron sensitive part. σ_ζ are the Pauli spin matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Substitution of these into the magnetic potential gives us the matrix elements

$$\langle \mathbf{S}' | V_m(\mathbf{Q}) | \mathbf{S} \rangle = -\frac{\gamma_n r_0}{2\mu_B}\begin{cases} M_{\perp z}(\mathbf{Q}) & |\uparrow\rangle \to |\uparrow\rangle \\ -M_{\perp z}(\mathbf{Q}) & |\downarrow\rangle \to |\downarrow\rangle \\ M_{\perp x}(\mathbf{Q}) - iM_{\perp y}(\mathbf{Q}) & |\uparrow\rangle \to |\downarrow\rangle \\ M_{\perp x}(\mathbf{Q}) + iM_{\perp y}(\mathbf{Q}) & |\downarrow\rangle \to |\uparrow\rangle \end{cases}$$

Non-spin-flip

Spin-flip

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Magnetic scattering rule

The non-spin-flip scattering is sensitive only to those components of the magnetisation parallel to the neutron spin

The spin-flip scattering is sensitive only to those components of the magnetisation perpendicular to the neutron spin

NB This is one of those points that you should take away with you. It is the basis of all magnetic polarization analysis techniques

Neutron polarization and scattering

3) Nuclear spin-dependent scattering

In general a bound state is formed between the nucleus and the neutron during scattering with either spins antiparallel (spin-singlet) or spins parallel (spin-triplet). The scattering lengths for these situations are different and are termed b_- and b_+.

We define the scattering length operator

$$\hat{\mathbf{b}} = A + B\boldsymbol{\sigma}\cdot\mathbf{I}$$

$$A = \frac{(I+1)b_+ + Ib_-}{2I+1}, \quad B = \frac{b_+ - b_-}{2I+1}$$

(see e.g. Squires, p173)

The calculation of the matrix elements now proceeds analogously to the case of magnetic scattering

$$\langle \mathbf{S}'|\hat{\mathbf{b}}|\mathbf{S}\rangle = \begin{cases} A + BI_z & |\uparrow\rangle \to |\uparrow\rangle \\ A - BI_z & |\downarrow\rangle \to |\downarrow\rangle \\ B(I_x - iI_y) & |\uparrow\rangle \to |\downarrow\rangle \\ B(I_x + iI_y) & |\downarrow\rangle \to |\uparrow\rangle \end{cases}$$

Non-spin-flip

Spin-flip

Since the nuclear spins are (normally) random $\langle I_x\rangle = \langle I_y\rangle = \langle I_z\rangle = 0$

Therefore with the coherent scattering amplitude proportional to $\bar{b}$, we can write

$\bar{b} = A$ i.e. the coherent scattering is entirely non-spin-flip

Moon-Riste-Koehler Equations

Bringing all this together, we get

$$|\uparrow\rangle \rightarrow |\uparrow\rangle = \bar{b} - \frac{\gamma_n r_0}{2\mu_B} M_{\perp z} + BI_z$$

$$|\downarrow\rangle \rightarrow |\downarrow\rangle = \bar{b} + \frac{\gamma_n r_0}{2\mu_B} M_{\perp z} - BI_z$$

$$|\uparrow\rangle \rightarrow |\downarrow\rangle = -\frac{\gamma_n r_0}{2\mu_B}\left(M_{\perp x} - iM_{\perp y}\right) + B\left(I_x - iI_y\right)$$

$$|\downarrow\rangle \rightarrow |\uparrow\rangle = -\frac{\gamma_n r_0}{2\mu_B}\left(M_{\perp x} + iM_{\perp y}\right) + B\left(I_x + iI_y\right)$$

Moon, Riste and Koehler (*Phys Rev* ***181*** *(1969) 920*)

Remember that: $\mathbf{M}_{\perp} = -2\mu_B\left[\hat{\boldsymbol{\eta}} - (\hat{\boldsymbol{\eta}}\cdot\hat{\mathbf{Q}})\hat{\mathbf{Q}}\right]$ (see e.g. Squires)

If the polarization is parallel to the scattering vector, then the magnetisation in the direction of the polarization will not be observed since the magnetic interaction vector is zero. i.e. all magnetic scattering will be spin-flip

Spin-incoherent scattering

Now, let's take another look at the nuclear incoherent scattering. We know that this is given by $\overline{b^2} - \left(\overline{b}\right)^2$

Applying this to the $|\uparrow\rangle \rightarrow |\uparrow\rangle$ transition, and neglecting magnetic scattering, we get

$$\overline{b^2} = \left\langle \left(\overline{b} + BI_z\right)^2 \right\rangle$$

$$= \left\langle \left(\overline{b}\right)^2 \right\rangle + \left\langle B^2 I_z^2 \right\rangle + 2\left\langle \overline{b} B I_z \right\rangle$$

Now, for a randomly oriented distribution of nuclei of spin I, we have

$$\langle \mathbf{I} \rangle = \sqrt{I(I+1)} = \sqrt{I_x^2 + I_y^2 + I_z^2}$$

$$\Rightarrow I_x^2 = I_y^2 = I_z^2 = \frac{1}{3} I(I+1)$$ since the distribution is isotropic

Therefore we can write

$$\overline{b^2} - \left(\overline{b}\right)^2 = \left\langle \left(\overline{b}\right)^2 \right\rangle - \left(\overline{b}\right)^2 + \frac{1}{3} B^2 I(I+1)$$

Isotope incoherent scattering

spin incoherent scattering

The other transitions are dealt with in a similar way

Moon-Riste-Koehler II

Finally, we get

$$\left|\uparrow\right\rangle \rightarrow \left|\uparrow\right\rangle = \overline{b} - \frac{\gamma_n r_0}{2\mu_B} M_{\perp z} + b_{II} + \frac{1}{3} b_{SI}$$

$$\left|\downarrow\right\rangle \rightarrow \left|\downarrow\right\rangle = \overline{b} + \frac{\gamma_n r_0}{2\mu_B} M_{\perp z} + b_{II} + \frac{1}{3} b_{SI}$$

$$\left|\uparrow\right\rangle \rightarrow \left|\downarrow\right\rangle = -\frac{\gamma_n r_0}{2\mu_B} \left(M_{\perp x} - iM_{\perp y}\right) + \frac{2}{3} b_{SI}$$

$$\left|\downarrow\right\rangle \rightarrow \left|\uparrow\right\rangle = -\frac{\gamma_n r_0}{2\mu_B} \left(M_{\perp x} + iM_{\perp y}\right) + \frac{2}{3} b_{SI}$$

where

$$b_{II} = \sqrt{\left\langle \left(\overline{b}\right)^2 \right\rangle - \left(\overline{b}\right)^2}$$

$$b_{SI} = \sqrt{B^2 I(I+1)}$$

The details of the magnetic scattering will in general depend on the direction of the neutron polarization with respect to the scattering vector, and also on the nature of the orientation of the magnetic moments

Scientific Examples

Magnetic form-factors in ferromagnets

For a ferromagnet the cross section for unpolarized neutrons is

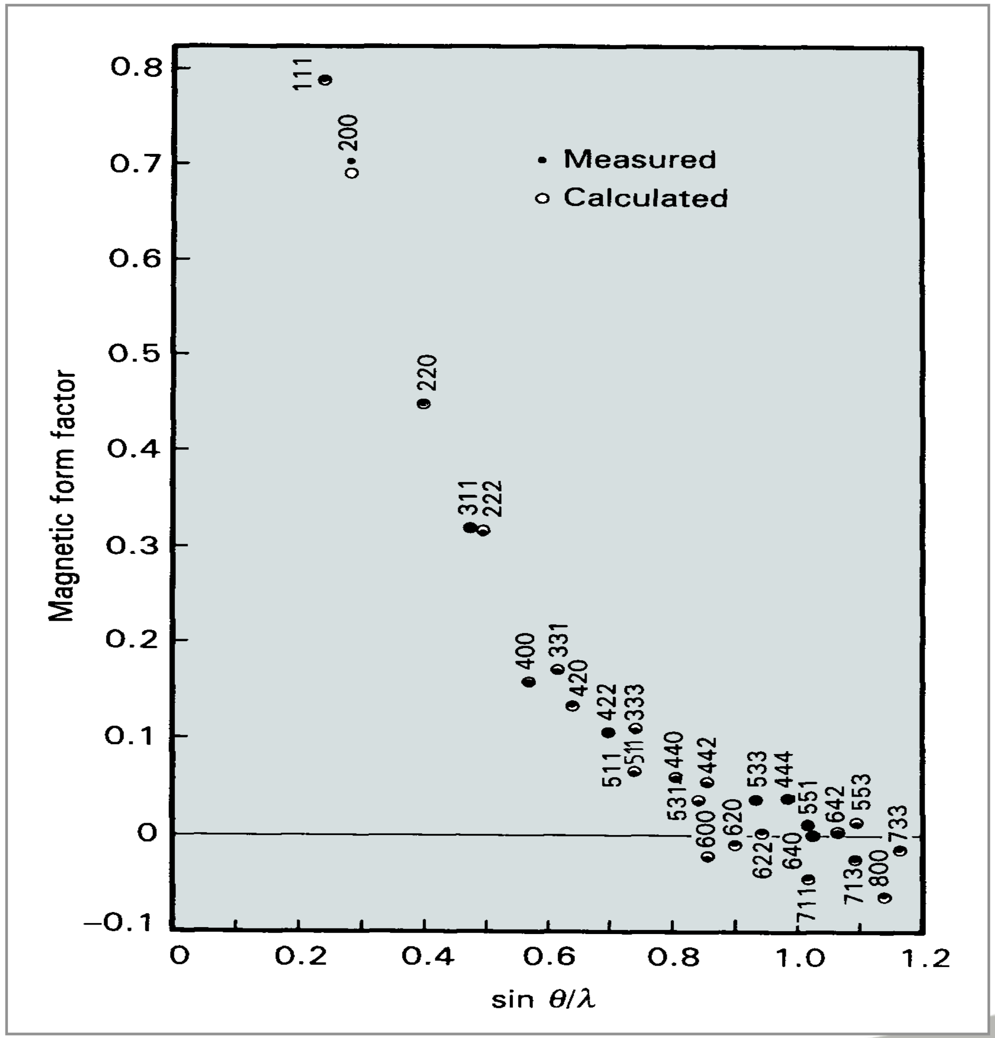
$$\frac{d\sigma}{d\Omega} = F_N^2(\mathbf{Q}) + F_M^2(\mathbf{Q})$$

If we take a simple ferromagnet such as Ni (b_{Ni}=1.03×10^{-12}cm, μ_{Ni}= 0.6μ_B) aligned such that the magnetisation is perpendicular to **Q** (i.e. magnetic interaction vector = 1) the ratio of the magnetic to the nuclear contribution to the intensity of even the lowest angle (111) reflection is only

$$F_M^2(\mathbf{Q}) / F_N^2(\mathbf{Q}) = 0.017$$

By the (400) reflection this ratio has fallen as a consequence of the magnetic form factor to

$$F_M^2(\mathbf{Q}) / F_N^2(\mathbf{Q}) = 6 \times 10^{-4}$$



Polarized magnetic diffraction

For a ferromagnetic sample aligned in a field perpendicular to the scattering vector we have

$$\mathbf{M}_\perp = -2\mu_B\left[\hat{\boldsymbol{\eta}} - (\hat{\boldsymbol{\eta}}\cdot\hat{\mathbf{Q}})\hat{\mathbf{Q}}\right] = -2\mu_B\hat{\boldsymbol{\eta}}$$

Squires: pp 129 - 135

and $M_\perp$ has no component in the xy-plane, so that the spin-flip scattering is zero. This implies that we don't need to analyse the neutron spin, it will always end up in the same direction it started in. Therefore

$d\sigma/d\Omega = \left[F_N(\mathbf{Q}) - F_M(\mathbf{Q})\right]^2$ for neutrons polarized parallel to the field

$d\sigma/d\Omega = \left[F_N(\mathbf{Q}) + F_M(\mathbf{Q})\right]^2$ for neutrons polarized antiparallel to the field

where

$$F_N(\mathbf{Q}) = \sum_i b_i \exp(i\mathbf{Q}\cdot\mathbf{r}_i)$$

$$F_M(\mathbf{Q}) = \gamma_n r_0 \sum_i g_{J_i} J_i f_i(\mathbf{Q}) \exp(i\mathbf{Q}\cdot\mathbf{r}_i)$$

Notice that to simulate an unpolarized measurement, we simply average the two polarized cross sections

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\left[\left(F_N(\mathbf{Q}) - F_M(\mathbf{Q})\right)^2 + \left(F_N(\mathbf{Q}) + F_M(\mathbf{Q})\right)^2\right]$$
$$= F_N^2(\mathbf{Q}) + F_M^2(\mathbf{Q})$$

NB we have neglected incoherent scattering here

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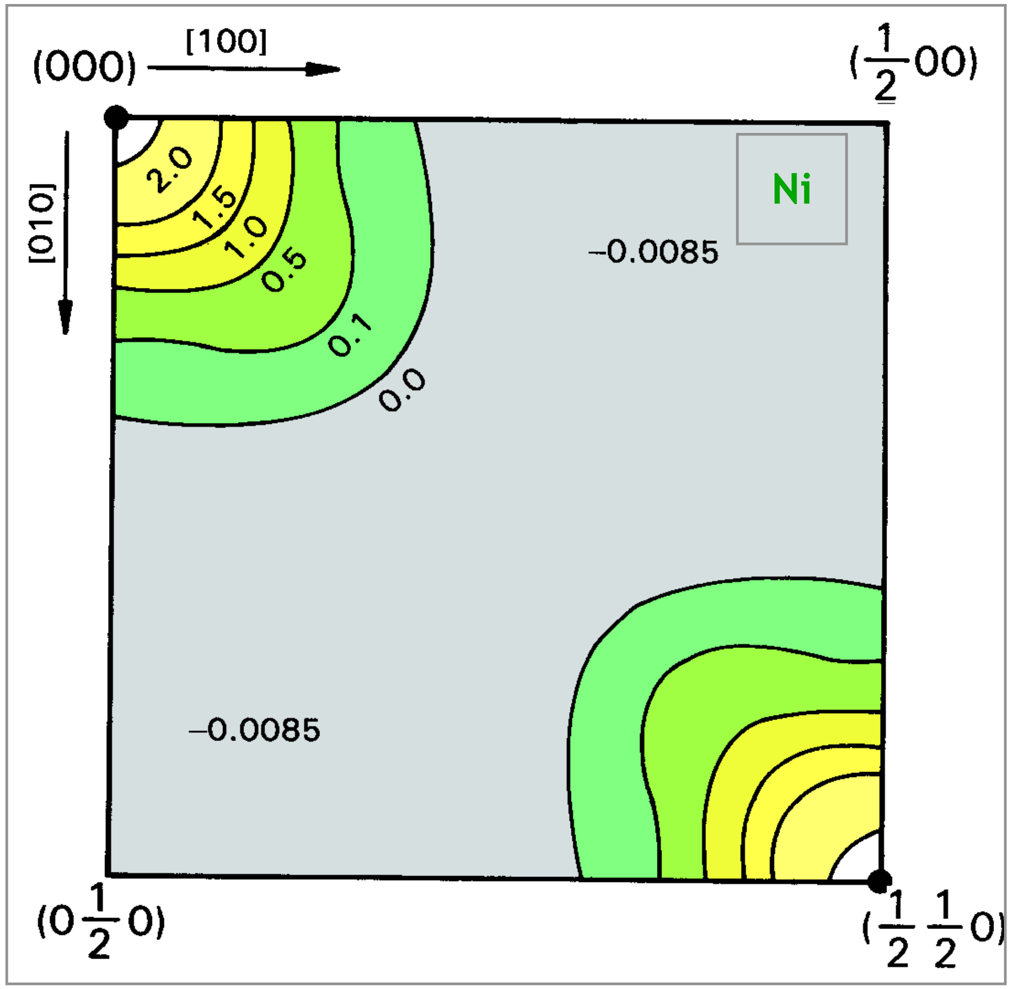
Polarized magnetic diffraction

Using a spin flipper to access these two polarized cross sections we can determine the "flipping ratio", R, of a particular Bragg reflection:

$$R = \frac{(d\sigma/d\Omega)_{\Downarrow}}{(d\sigma/d\Omega)_{\Uparrow}} = \frac{[F_N(\mathbf{Q}) + F_M(\mathbf{Q})]^2}{[F_N(\mathbf{Q}) - F_M(\mathbf{Q})]^2} = \left(\frac{1+\gamma}{1-\gamma}\right)^2$$

with $\gamma = \frac{F_M(\mathbf{Q})}{F_N(\mathbf{Q})}$



So, for example, in the case of Ni we measure a flipping ratio of 1.7 at the (111) reflection and 1.1 at the (400) reflection

The goal is, of course, to determine $F_M(\mathbf{Q})$ which can then be Fourier transformed to give a full 3-dimensional spin density distribution

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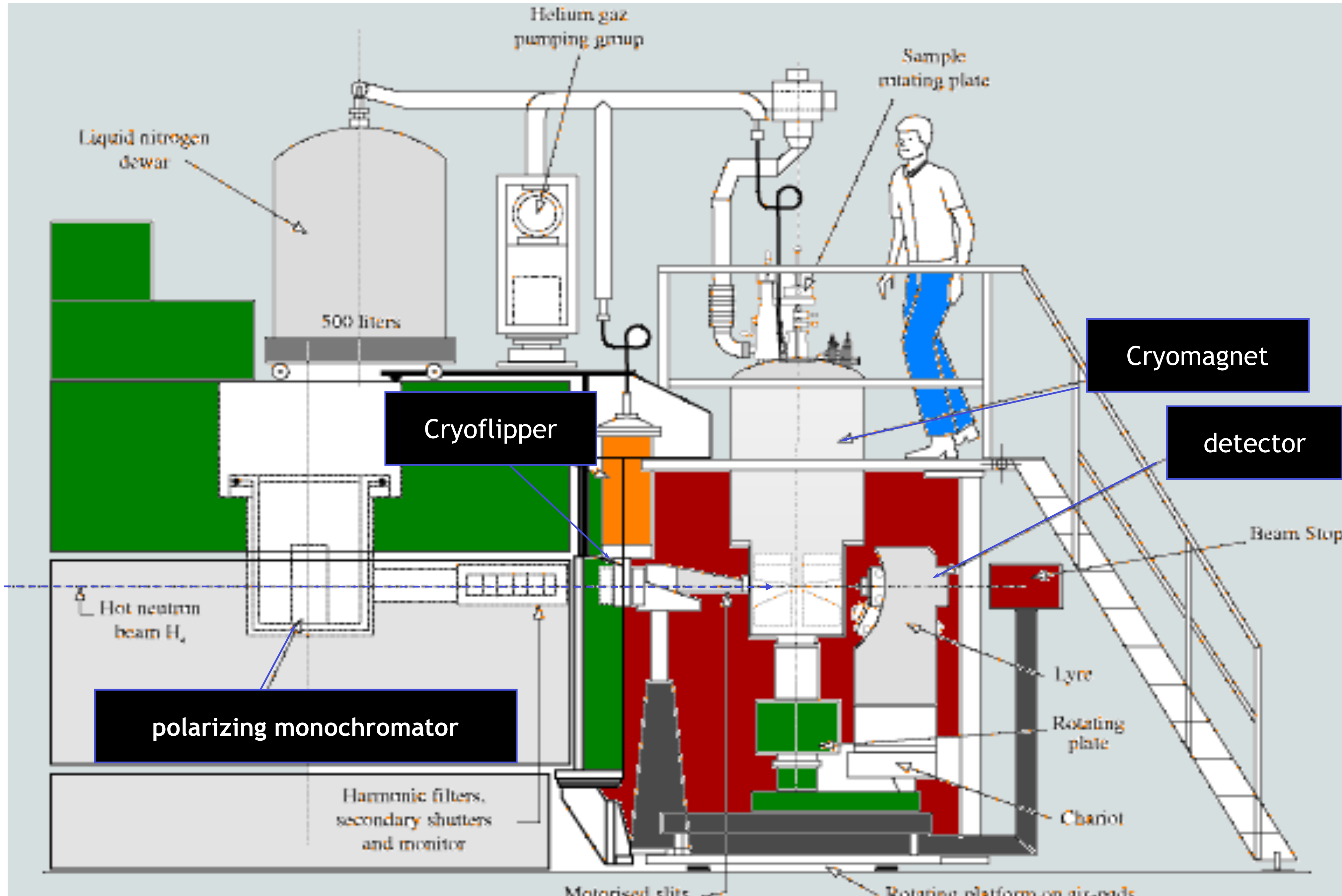
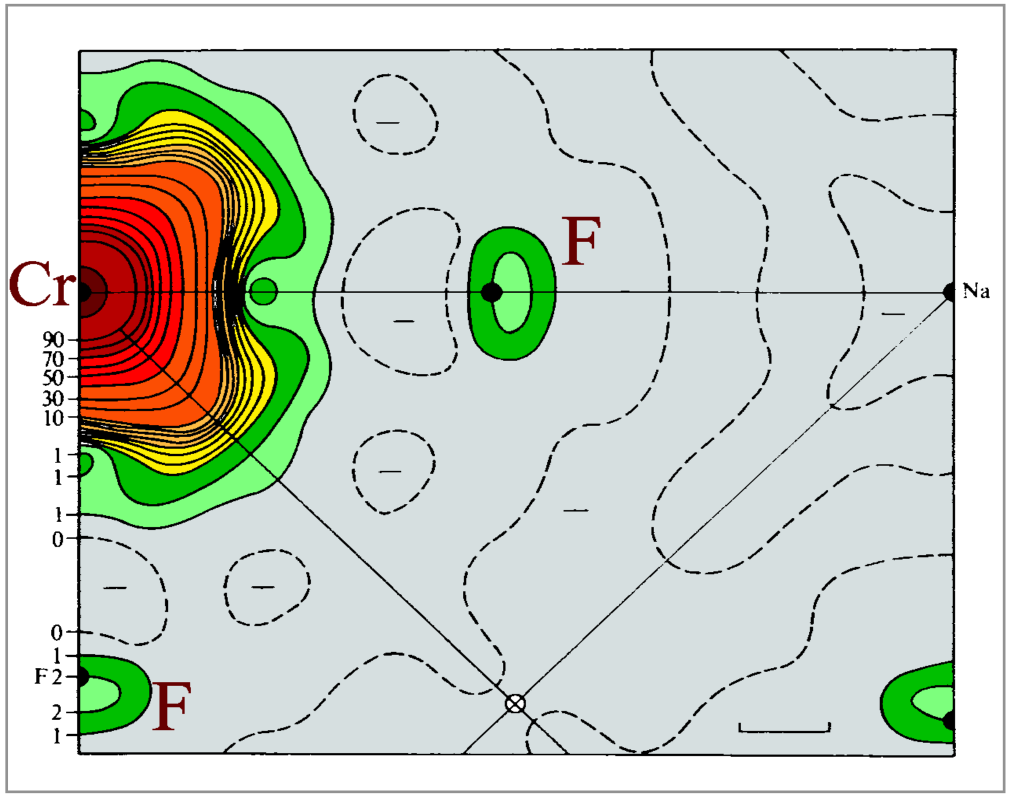


Helium gaz
pumping group
Sample
rotating plate
Liquid nitrogen
dewar
500 liters
Cryomagnet
Cryoflipper
detector
Beam Stop
Hot neutron
beam H
Lyre
polarizing monochromator
Rotating
plate
Harmonic filters,
secondary shutters
and monitor
Chariot
Motorised slits
Rotating platform on air-pads

Spin-density maps

K_2NaCrF_6

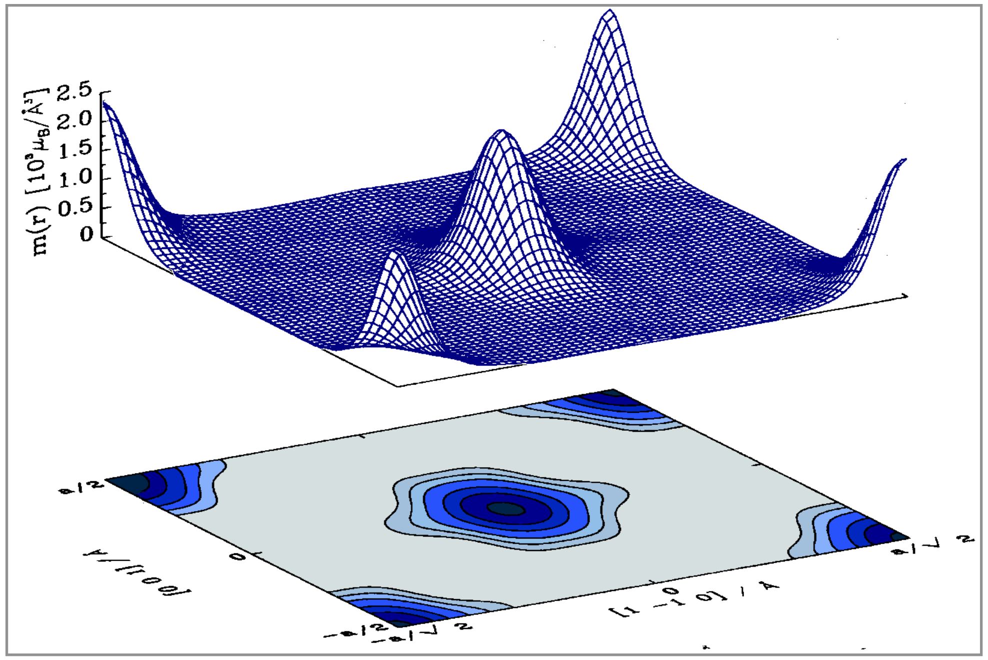


K_2NaCrF_6 is a cubic insulator in which Cr^{3+} ions are at the centre of an octahedron of F^-

T_{2g} symmetry around Cr and islands of spin density around F indicative of spin transfer through covalent bonding

Wedgwood, Proc Roy Soc A349, 447 (1976)

Antiferromagnetic Cr metal



Pure Cr is an itinerant electron spin density wave antiferromagnet.
However a field induced magnetic form factor can be measured
Results (analysed using Max. Ent. methods) are consistent with 60% orbital and 40% spin contributions

Strempfer et al Physica B 267-8 56 (1999)

Polarization analysis with multi-detectors

In order to separate nuclear, magnetic and spin-incoherent scattering on a multi-detector neutron instrument, we need to employ ***XYZ polarization analysis***

Since it is difficult to get a wide angle spin-flipper to cover the multi-detector, we make do with one flipper before the sample. Therefore we see the following transitions

$$|\uparrow\rangle \rightarrow |\uparrow\rangle = b - \frac{\gamma_n r_0}{2\mu_B} M_{\perp z} + b_{II} + \frac{1}{3} b_{SI}$$

$$|\downarrow\rangle \rightarrow |\uparrow\rangle = -\frac{\gamma_n r_0}{2\mu_B}\left(M_{\perp x} + iM_{\perp y}\right) + \frac{2}{3} b_{SI}$$

The cross-sections (considering magnetic part only) are therefore

$$\left(\frac{d\sigma}{d\Omega}\right)_{NSF} = \left(\frac{\gamma_n r_0}{2\mu_B}\right)^2 \left\langle M^*_{\perp z} M_{\perp z} \right\rangle$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{SF} = \left(\frac{\gamma_n r_0}{2\mu_B}\right)^2 \left\langle \left(M_{\perp x} + iM_{\perp y}\right)^* \left(M_{\perp x} + iM_{\perp y}\right) \right\rangle$$

$$= \left(\frac{\gamma_n r_0}{2\mu_B}\right)^2 \left\langle M^*_{\perp x} M_{\perp x} + M^*_{\perp y} M_{\perp y} \right\rangle$$

XYZ-Polarization analysis

It can be shown (see Squires p 179) that in the case of a fully disordered paramagnet these expressions reduce to

$$\left(\frac{d\sigma}{d\Omega}\right)^{\zeta}_{NSF} = \frac{1}{3}\left(\frac{\gamma r_0}{2}\right)^2 g^2 f^2(\mathbf{Q}) J(J+1)\left[1-\left(\hat{\mathbf{P}}\cdot\hat{\mathbf{Q}}\right)^2\right]$$

$$\left(\frac{d\sigma}{d\Omega}\right)^{\zeta}_{SF} = \frac{1}{3}\left(\frac{\gamma r_0}{2}\right)^2 g^2 f^2(\mathbf{Q}) J(J+1)\left[1+\left(\hat{\mathbf{P}}\cdot\hat{\mathbf{Q}}\right)^2\right]$$

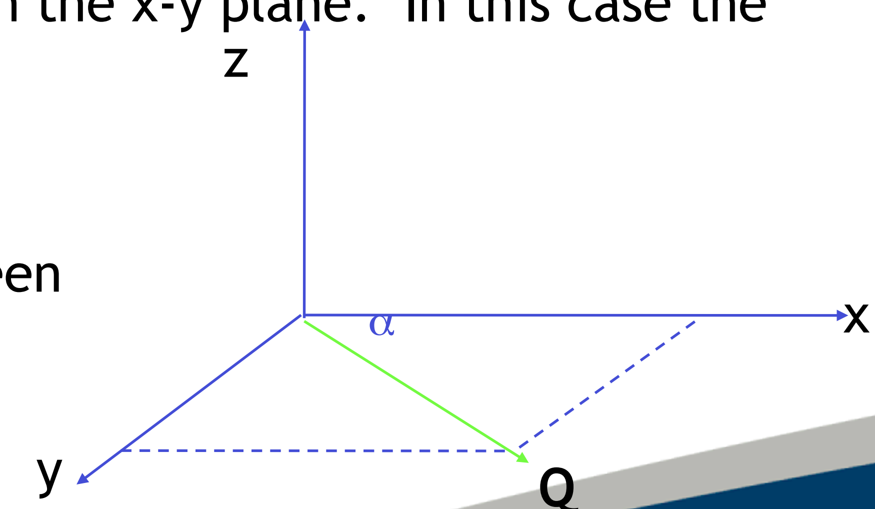
where we have replaced the z-direction with the general direction ζ = x, y, or z

We can immediately see that setting the polarization (ζ) direction along the scattering vector has the desired effect of rendering all the magnetic scattering in the spin-flip cross-section.

Now we suppose that we have a multi-detector in the x-y plane. In this case the unit scattering vector is

$$\hat{\mathbf{Q}} = \begin{pmatrix} \cos\alpha \\ \sin\alpha \\ 0 \end{pmatrix}$$

where α is the angle between **Q** and an arbitrary x-axis - the "Schärpf angle"



The Schärpf Equations

Substituting this unit scattering vector into the NSF and SF cross sections leads then to 6 equations (including now the nuclear coherent, isotope incoherent and spin-incoherent terms)

$$\left(\frac{d\sigma}{d\Omega}\right)_X^{NSF} = \frac{1}{2}\sin^2\alpha\left(\frac{d\sigma}{d\Omega}\right)_{mag} + \frac{1}{3}\left(\frac{d\sigma}{d\Omega}\right)_{SI} + \left(\frac{d\sigma}{d\Omega}\right)_{nuc+II}$$

x-direction

$$\left(\frac{d\sigma}{d\Omega}\right)_X^{SF} = \frac{1}{2}\left(\cos^2\alpha + 1\right)\left(\frac{d\sigma}{d\Omega}\right)_{mag} + \frac{2}{3}\left(\frac{d\sigma}{d\Omega}\right)_{SI}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_Y^{NSF} = \frac{1}{2}\cos^2\alpha\left(\frac{d\sigma}{d\Omega}\right)_{mag} + \frac{1}{3}\left(\frac{d\sigma}{d\Omega}\right)_{SI} + \left(\frac{d\sigma}{d\Omega}\right)_{nuc+II}$$

y-direction

$$\left(\frac{d\sigma}{d\Omega}\right)_Y^{SF} = \frac{1}{2}\left(\sin^2\alpha + 1\right)\left(\frac{d\sigma}{d\Omega}\right)_{mag} + \frac{2}{3}\left(\frac{d\sigma}{d\Omega}\right)_{SI}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_Z^{NSF} = \frac{1}{2}\left(\frac{d\sigma}{d\Omega}\right)_{mag} + \frac{1}{3}\left(\frac{d\sigma}{d\Omega}\right)_{SI} + \left(\frac{d\sigma}{d\Omega}\right)_{nuc+II}$$

z-direction

$$\left(\frac{d\sigma}{d\Omega}\right)_Z^{SF} = \frac{1}{2}\left(\frac{d\sigma}{d\Omega}\right)_{mag} + \frac{2}{3}\left(\frac{d\sigma}{d\Omega}\right)_{SI}$$

Schärpf and Capellmann Phys Stat Sol a135 (1993) 359

Separation of scattering

The principal cross sections can be extracted by combining the six partials:

The magnetic cross-section is independently calculated in 2 ways

$$\left.\begin{aligned}\left(\frac{d\sigma}{d\Omega}\right)_{mag} &= 2\left[\left(\frac{d\sigma}{d\Omega}\right)_{SF}^{X} + \left(\frac{d\sigma}{d\Omega}\right)_{SF}^{Y} - 2\left(\frac{d\sigma}{d\Omega}\right)_{SF}^{Z}\right] \\ \left(\frac{d\sigma}{d\Omega}\right)_{mag} &= 2\left[2\left(\frac{d\sigma}{d\Omega}\right)_{NSF}^{Z} - \left(\frac{d\sigma}{d\Omega}\right)_{NSF}^{X} - \left(\frac{d\sigma}{d\Omega}\right)_{NSF}^{Y}\right]\end{aligned}\right\} = \frac{2}{3}\left(\frac{\gamma_n r_0}{2}\right)^2 g_J^2 f^2(Q) J(J+1)$$

For the other cross-sections, we have

$$\left(\frac{d\sigma}{d\Omega}\right)_{SI} = \frac{1}{2}\left(\frac{d\sigma}{d\Omega}\right)_{TSF} - \left(\frac{d\sigma}{d\Omega}\right)_{mag} = B^2 I(I+1)$$

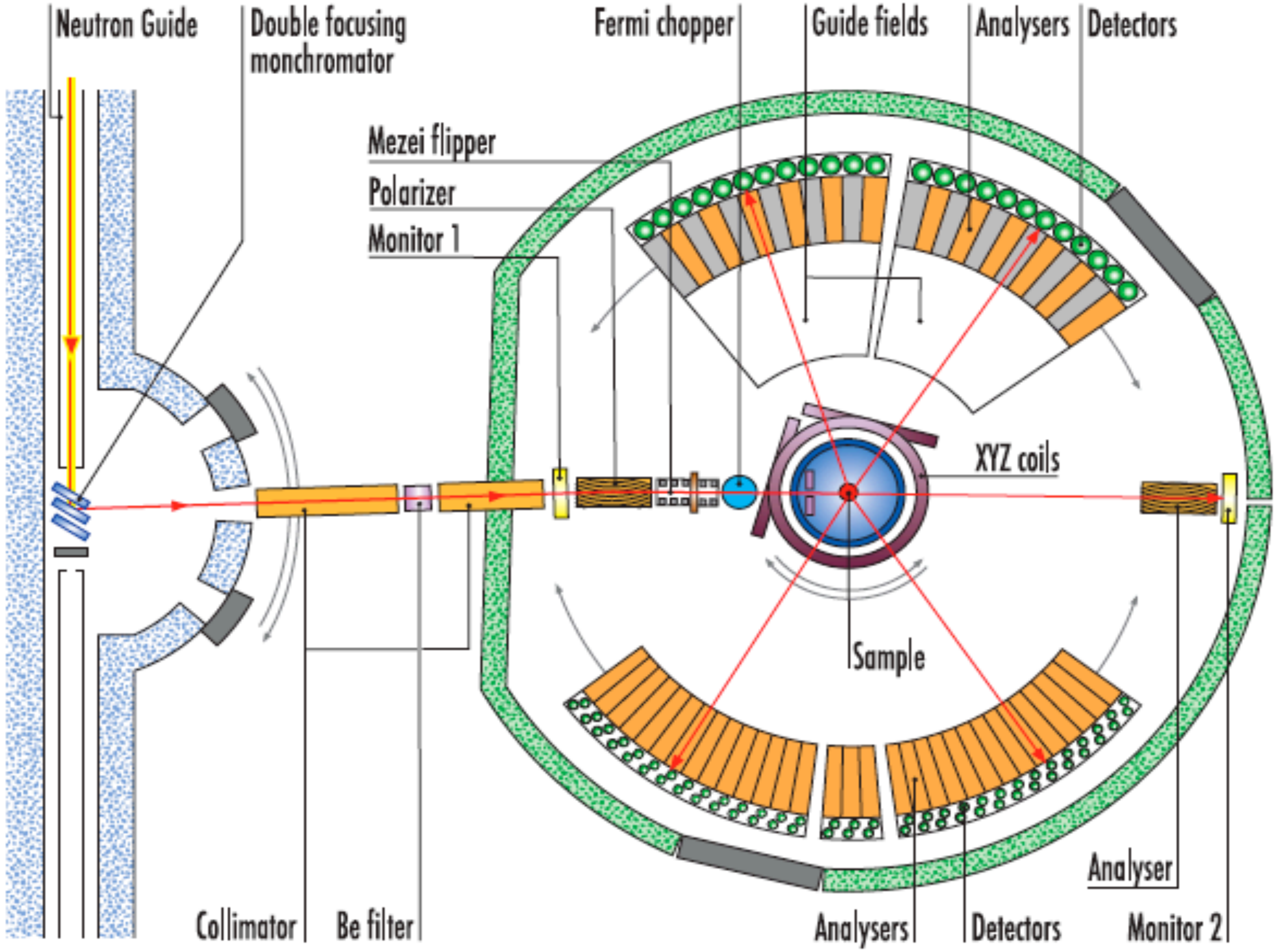
$$\left(\frac{d\sigma}{d\Omega}\right)_{nuc+II} = \frac{1}{6}\left[2\left(\frac{d\sigma}{d\Omega}\right)_{TNSF} - \left(\frac{d\sigma}{d\Omega}\right)_{TSF}\right] = b^2 S(Q) + \overline{b^2} - \left(\overline{b}\right)^2$$

where the subscript, TNSF or TSF refers to the total (x + y + z) NSF or SF scattering cross sections

The 3-directional, or xyz- difference method is most widely used for diffuse scattering studies of magnetic correlations in spin glasses, antiferromagnets and frustrated systems – the general requirement being that these equations only work if there are no collinear (or chiral) components to the magnetisation

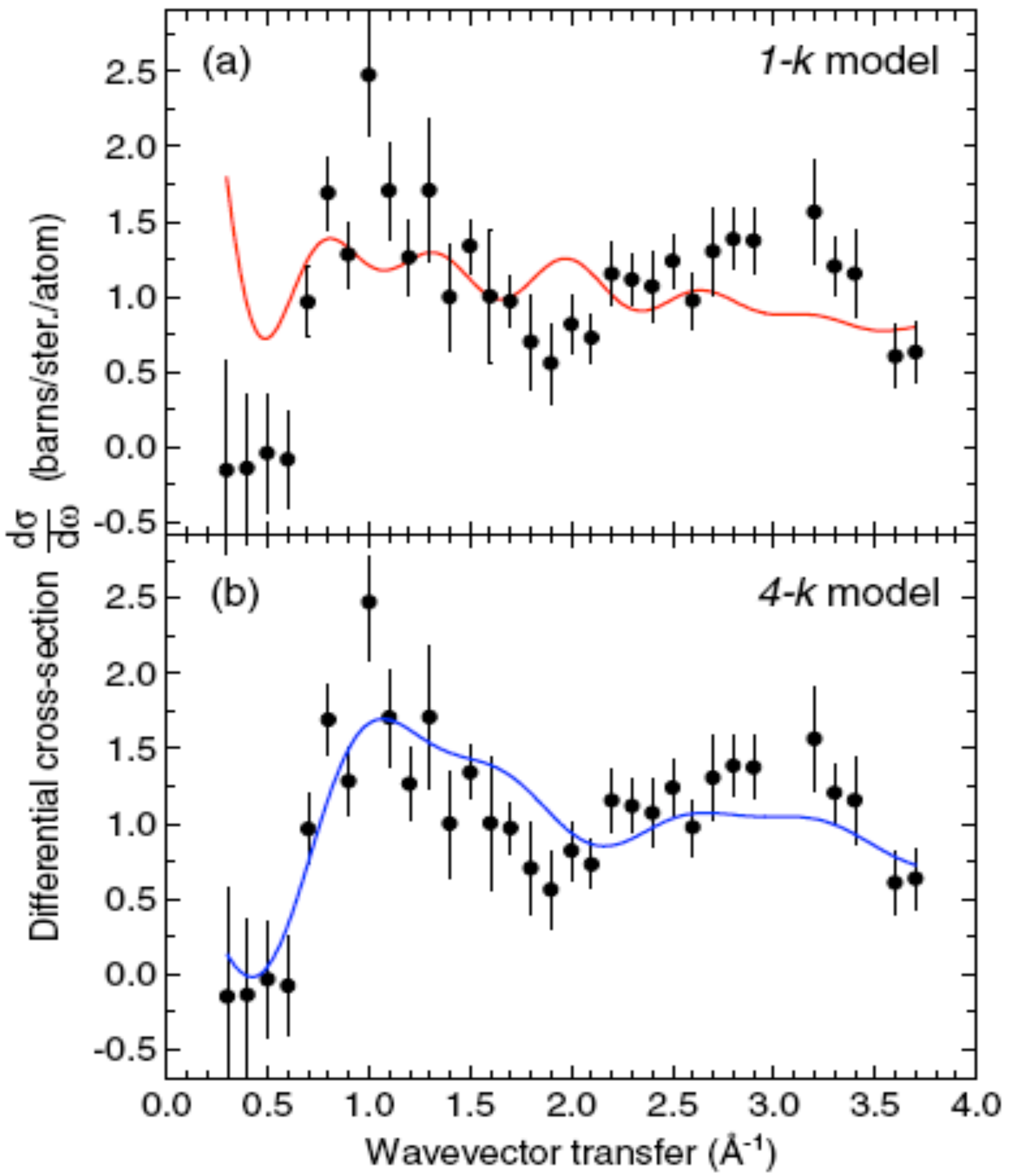
Stewart, et. al. J. Applied Phys. ***87*** *(2000)*

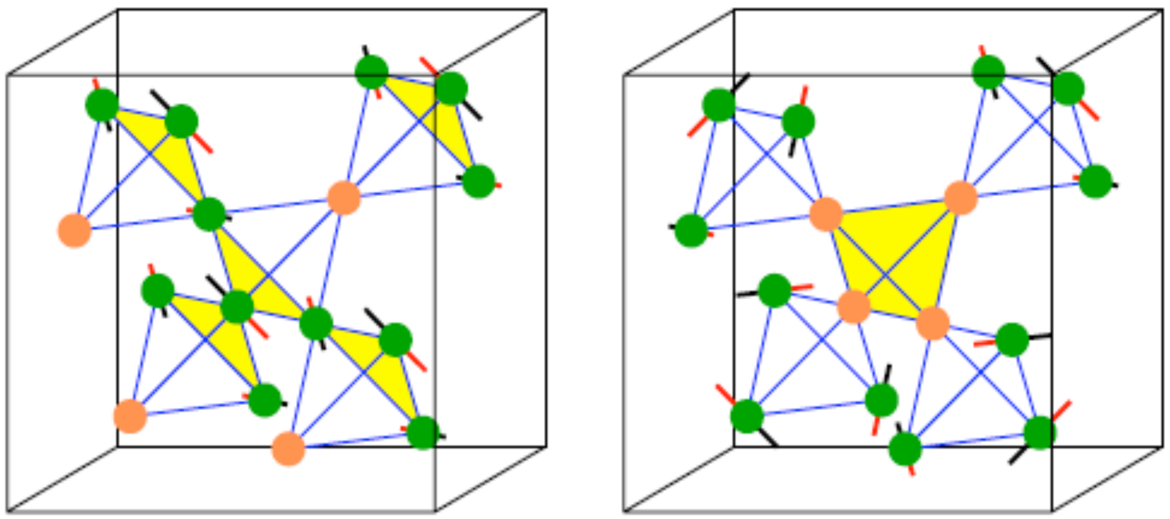
D7, ILL



Neutron Guide
Double focusing monchromator
Fermi chopper
Guide fields
Analysers
Detectors
Mezei flipper
Polarizer
Monitor 1
XYZ coils
Sample
Collimator
Be filter
Analysers
Detectors
Analyser
Monitor 2

Diffuse scattering examples

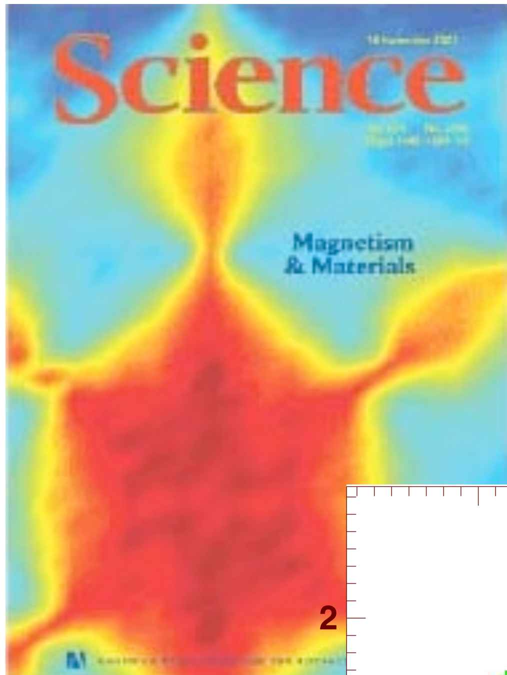




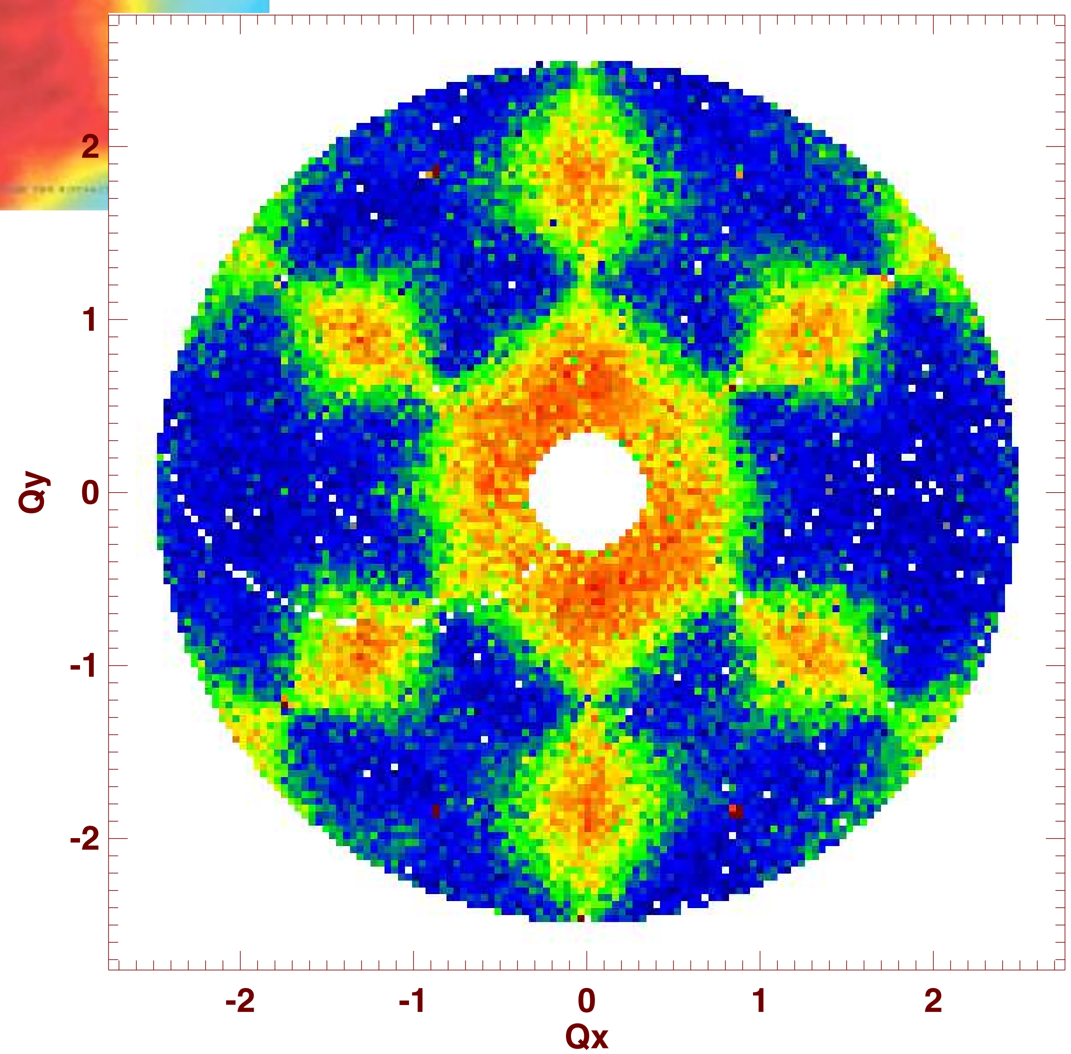
Magnetic short-range order in $Gd_2Ti_2O_7$ - due to geometrical frustration of anti-ferromagnetic exchange interactions

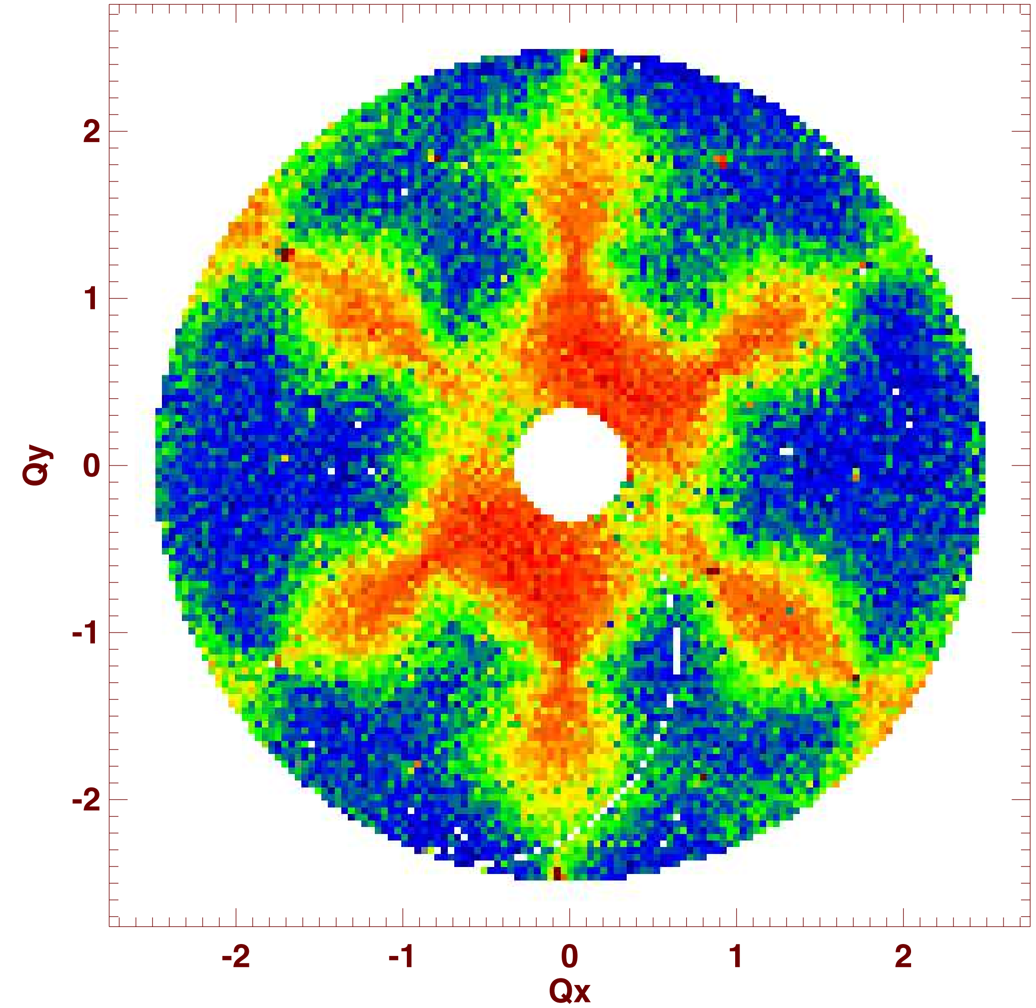
Stewart et al, J Phys: Condensed Matter 16, L321 (2004)

Diffuse scattering examples



$Ho_2Ti_2O_7$

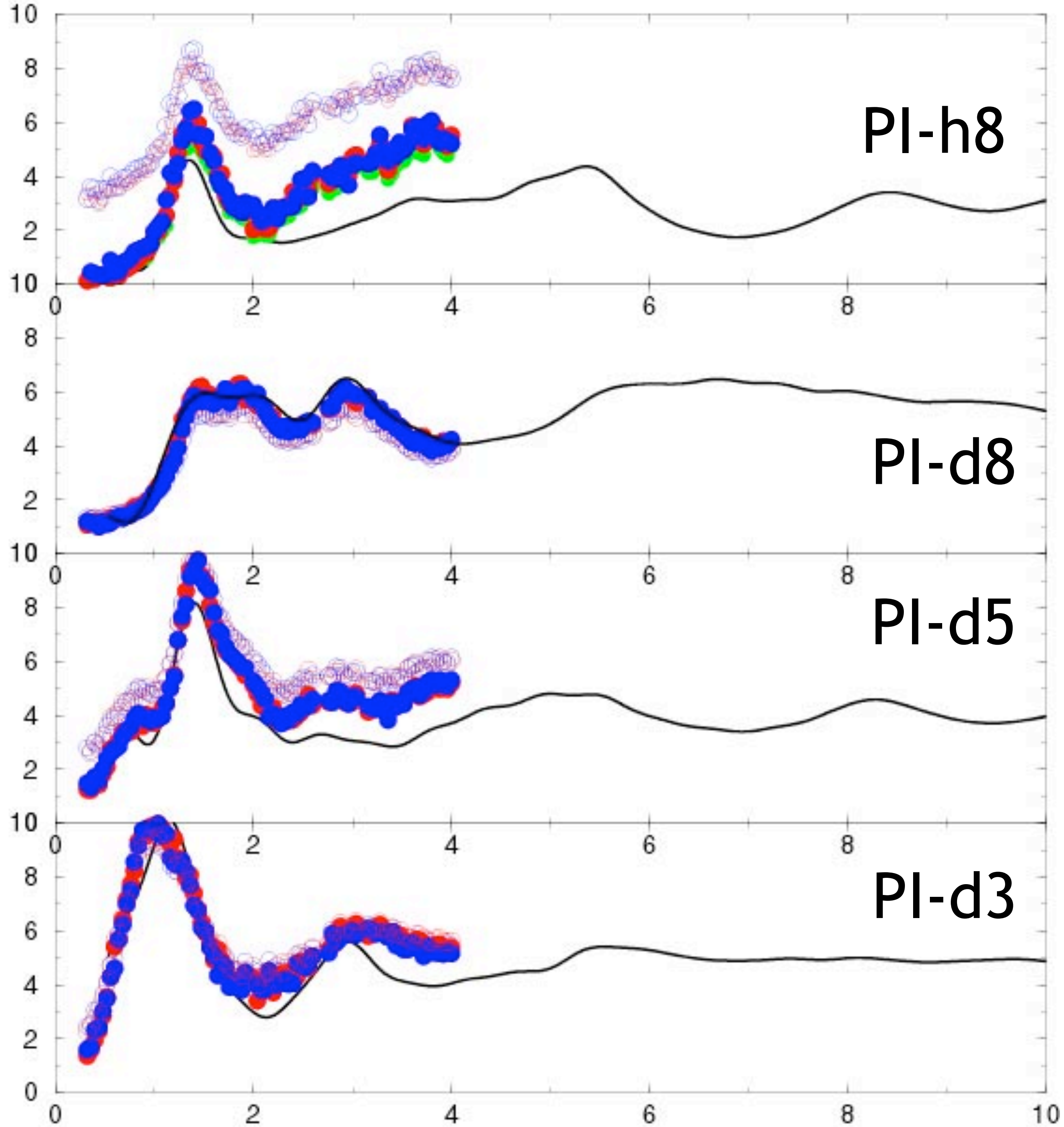




“Stuffed”
$Ho_{2.3}Ti_{1.7}O_7$

T Fennell and P Deen (2007)

Polymer diffraction



Polyisoprene: $(CH_2CH = C(CH_3)CH_2)_n$
Alvarez, et. al.

- Complete separation of SI scattering
- Internal normalisation (inc. D-W factor)
- Careful analysis of multiple scattering
- Close comparison with MD simulations

Alvarez, et.al., Macromolecules **36** (2003) 238

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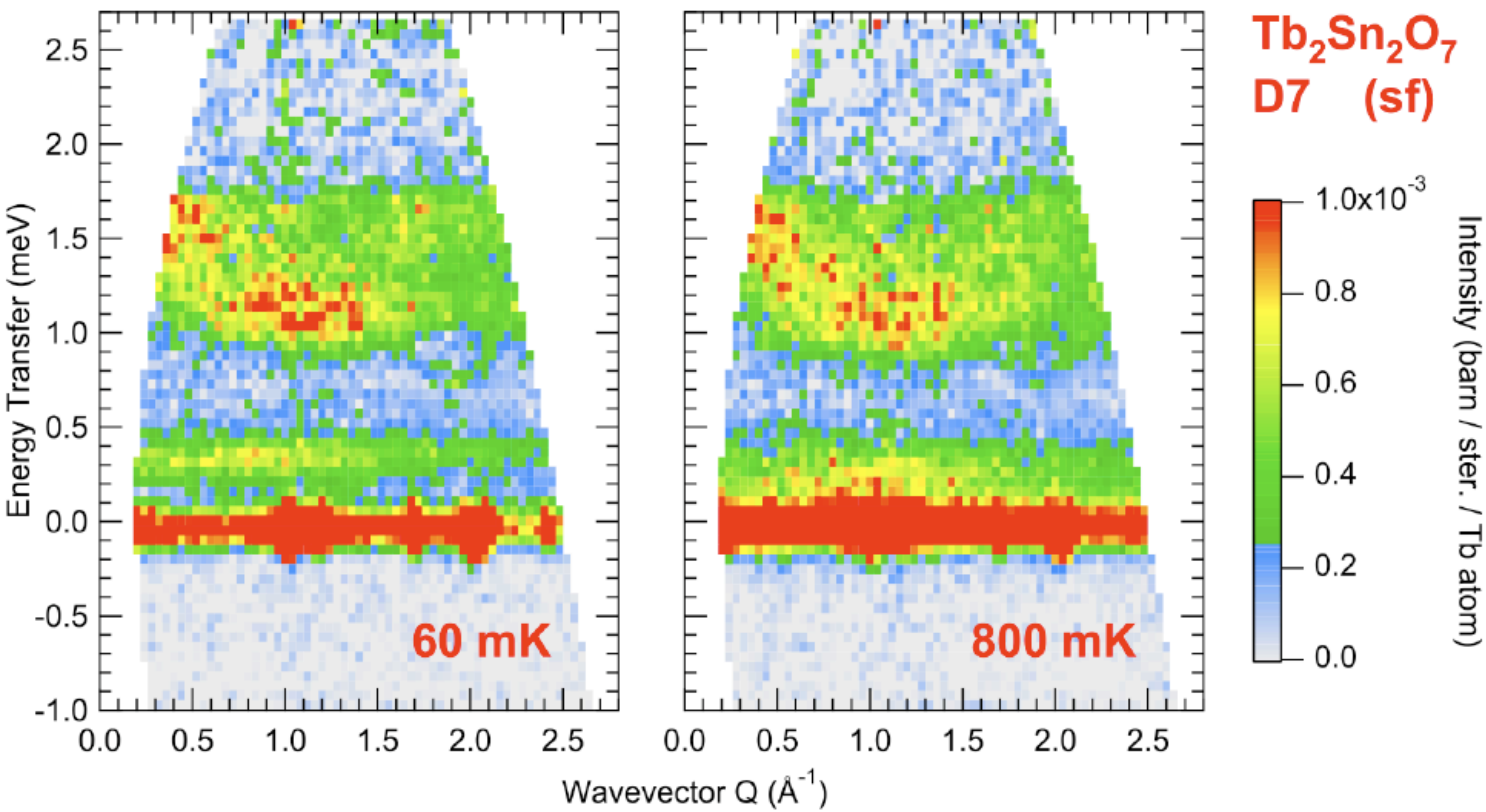
Inelastic magnetic scattering



Pyrochlore - $Tb_2Sn_2O_7$

Rule, et. al., Phys. Rev. B **76** 212405 (2007)

Neutron Polarimetry

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Neutron Polarimetry

To this point we have only been concerned with applying uniaxial polarization analysis - i.e. with measuring the scattered intensity associated with a scalar change of polarization along a particular axis.

This is "uniaxial (longitudinal) polarization analysis"

For a full description of the scattering processes we must perform "polarization analysis" in its true sense, i.e. we must measure all components of the polarization vector

This is "neutron polarimetry"

The general equations are (and I'm just going to quote them!) can found by repeating our previous uniaxial analysis in three dimensions

$$\sigma = \begin{cases} NN^* & \text{Nuclear} \\ \mathbf{M}_\perp \cdot \mathbf{M}_\perp{}^* + i\mathbf{P}_i \cdot \left(\mathbf{M}_\perp{}^* \times \mathbf{M}_\perp\right) & \text{Magnetic} \\ \mathbf{P}_i \cdot \left(\mathbf{M}_\perp N^* + \mathbf{M}_\perp{}^* N\right) & \text{NM Interference} \end{cases}$$

and

$$\mathbf{P}_f \sigma = \begin{cases} \mathbf{P}_i NN^* & \text{Nuclear} \\ -\mathbf{P}_i\left(\mathbf{M}_\perp \cdot \mathbf{M}_\perp{}^*\right) + \mathbf{M}_\perp\left(\mathbf{P}_i \cdot \mathbf{M}_\perp{}^*\right) + \mathbf{M}_\perp{}^*\left(\mathbf{P}_i \cdot \mathbf{M}_\perp\right) - i\left(\mathbf{M}_\perp{}^* \times \mathbf{M}_\perp\right) & \text{Magnetic} \\ \mathbf{M}_\perp N^* + \mathbf{M}_\perp{}^* N - i\left(N\mathbf{M}_\perp{}^* - N^*\mathbf{M}_\perp\right) \times \mathbf{P}_i & \text{NM Interference} \end{cases}$$

Blume (*Phys Rev* <u>*130*</u>*, 1670, 1963, Physica B* <u>*267-268*</u>*, 211, 1999*)

or: Hicks (*Advances in physics,* <u>*45*</u>*, 243, 1996*)

Neutron Polarimetry

$$\sigma = \begin{cases} NN^* & \text{Nuclear} \\ \mathbf{M}_\perp \cdot \mathbf{M}_\perp^* + i\mathbf{P}_i \cdot \left(\mathbf{M}_\perp^* \times \mathbf{M}_\perp\right) & \text{Magnetic} \\ \mathbf{P}_i \cdot \left(\mathbf{M}_\perp N^* + \mathbf{M}_\perp^* N\right) & \text{NM Interference} \end{cases}$$

$$\mathbf{P}_f \sigma = \begin{cases} \mathbf{P}_i NN^* & \text{Nuclear} \\ -\mathbf{P}_i\left(\mathbf{M}_\perp \cdot \mathbf{M}_\perp^*\right) + \mathbf{M}_\perp\left(\mathbf{P}_i \cdot \mathbf{M}_\perp^*\right) + \mathbf{M}_\perp^*\left(\mathbf{P}_i \cdot \mathbf{M}_\perp\right) - i\left(\mathbf{M}_\perp^* \times \mathbf{M}_\perp\right) & \text{Magnetic} \\ \mathbf{M}_\perp N^* + \mathbf{M}_\perp^* N - i\left(N\mathbf{M}_\perp^* - N^*\mathbf{M}_\perp\right) \times \mathbf{P}_i & \text{NM Interference} \end{cases}$$

Points to note are:

1) Pure nuclear scattering does not effect the neutron polarization
2) The cross-terms are non zero only for non-collinear (e.g. spiral) structures where $M_\perp^*$ and $M_\perp$ are not parallel.
3) Scattering by NM interference will only occur when the nuclear and magnetic contributions occur with the same wavevector.

Where there is no NM interference and no chiral terms (which is generally true for paramagnets and glassy systems) the above equations reduce to the uniaxial equations.

Flipping-ratios revisited

$$\sigma = \begin{cases} NN^* & \text{Nuclear} \\ \mathbf{M}_\perp \cdot \mathbf{M}_\perp^* + i\mathbf{P}_i \cancel{\cdot \left(\mathbf{M}_\perp^* \times \mathbf{M}_\perp\right)} & \text{Magnetic} \\ \mathbf{P}_i \cdot \left(\mathbf{M}_\perp N^* + \mathbf{M}_\perp^* N\right) & \text{NM Interference} \end{cases}$$

$$\mathbf{P}_f \sigma = \begin{cases} \mathbf{P}_i NN^* & \text{Nuclear} \\ -\mathbf{P}_i\left(\mathbf{M}_\perp \cdot \mathbf{M}_\perp^*\right) + \mathbf{M}_\perp\left(\mathbf{P}_i \cdot \mathbf{M}_\perp^*\right) + \mathbf{M}_\perp^*\left(\mathbf{P}_i \cdot \mathbf{M}_\perp\right) - \cancel{i\left(\mathbf{M}_\perp^* \times \mathbf{M}_\perp\right)} & \text{Magnetic} \\ \mathbf{M}_\perp N^* + \mathbf{M}_\perp^* N - i\left(\cancel{N\mathbf{M}_\perp^* - N^*\mathbf{M}_\perp}\right) \times \mathbf{P}_i & \text{NM Interference} \end{cases}$$

The imaginary part of $M_\perp$ contains the phase information on the magnetic order. For non-chiral structures,

$$\mathbf{M}_\perp = \mathbf{M}_\perp^*$$

$$\Rightarrow \mathbf{M}_\perp^* \times \mathbf{M}_\perp = 0$$

Interaction vector

$$\left.\begin{array}{c} \mathbf{M}_\perp \propto \mathbf{Q} \times \mathbf{M} \times \mathbf{Q} \\ \mathbf{M} \perp \mathbf{Q} \\ \text{Sample is magnetised} \end{array}\right\} \Rightarrow \mathbf{M}_\perp \| \mathbf{M} \Rightarrow \mathbf{M}_\perp \| \mathbf{P}_i$$

$$\Rightarrow \mathbf{P}_i = \mathbf{P}_f$$

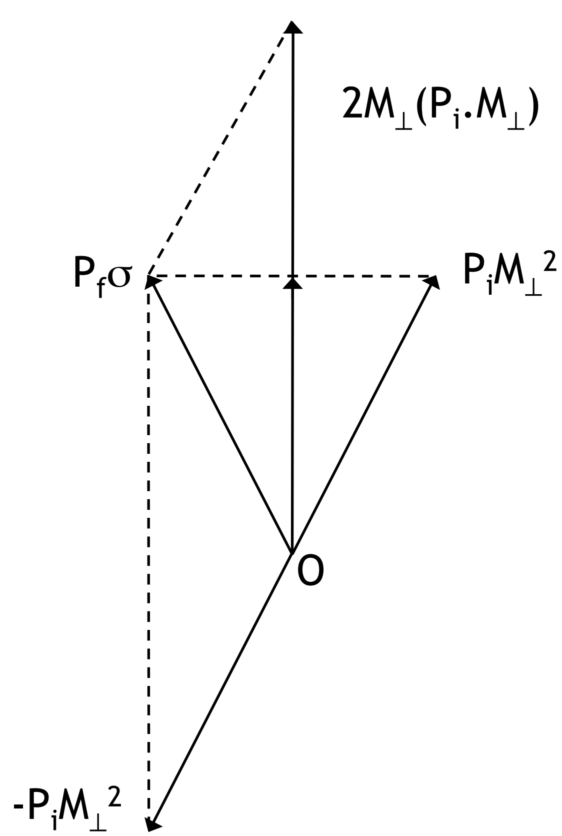
So we recover the form of the cross-section for magnetic diffraction - and the observation that there is no spin-flip scattering

Polarimetry examples

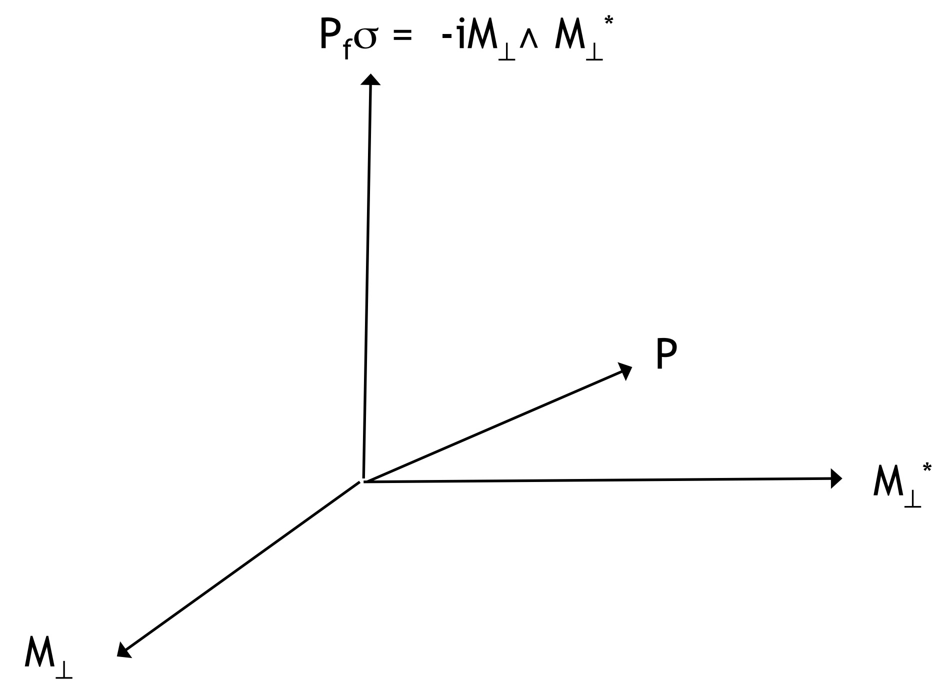
$$\mathbf{P}_f\sigma = \begin{cases} \mathbf{P}_i N N^* \\ -\mathbf{P}_i\left(\mathbf{M}_\perp\cdot\mathbf{M}_\perp^{\,*}\right)+\mathbf{M}_\perp\left(\mathbf{P}_i\cdot\mathbf{M}_\perp^{\,*}\right)+\mathbf{M}_\perp^{\,*}\left(\mathbf{P}_i\cdot\mathbf{M}_\perp\right)-i\left(\mathbf{M}_\perp^{\,*}\times\mathbf{M}_\perp\right) \\ \mathbf{M}_\perp N^* + \mathbf{M}_\perp^{\,*} N - i\left(N\mathbf{M}_\perp^{\,*} - N^*\mathbf{M}_\perp\right)\times\mathbf{P}_i \end{cases}$$

- e.g. $M_\perp$ and $M_\perp^*$ are parallel -
- no nuclear scattering

$$\mathbf{P}_f\sigma = -\mathbf{P}_i\mathbf{M}_\perp^2 + 2\mathbf{M}_\perp\left(\mathbf{P}_i\cdot\mathbf{M}_\perp\right)$$



- e.g. $M_\perp$ and $M_\perp^*$ are perpendicular -
- no nuclear scattering - *chiral systems*



Creation of polarization - independent of incident $\mathbf{P}_i$

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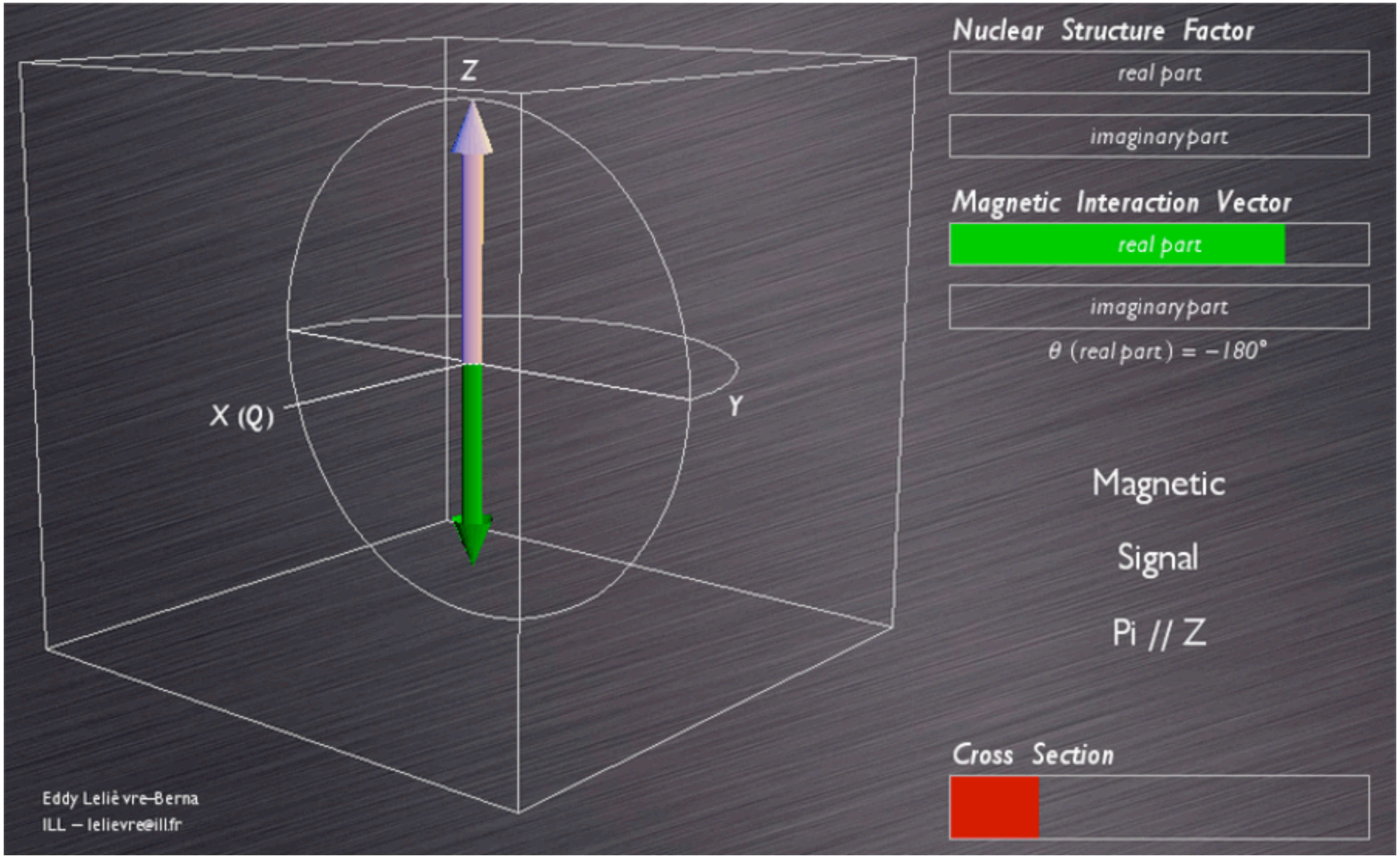
Polarimetry - M real



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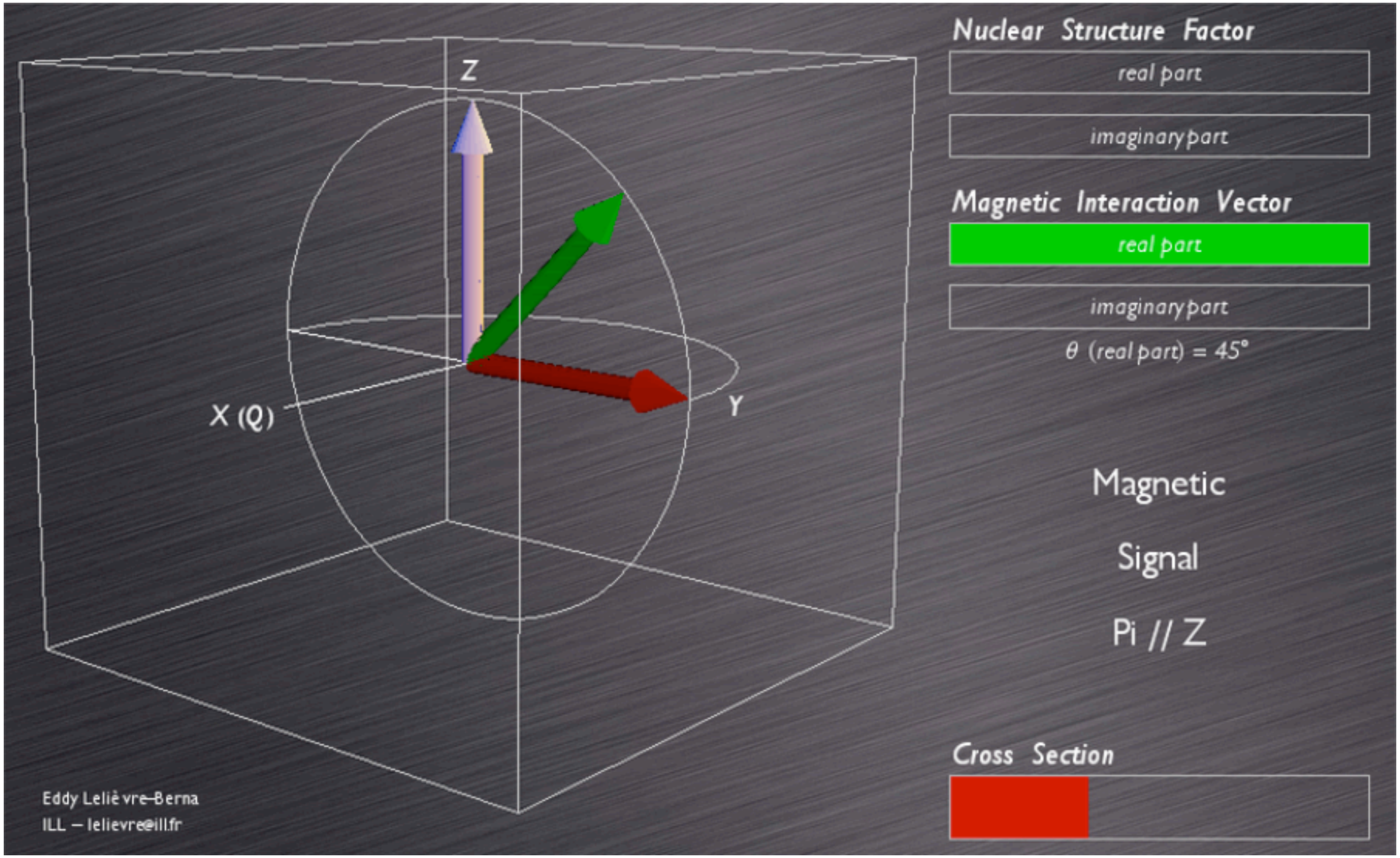
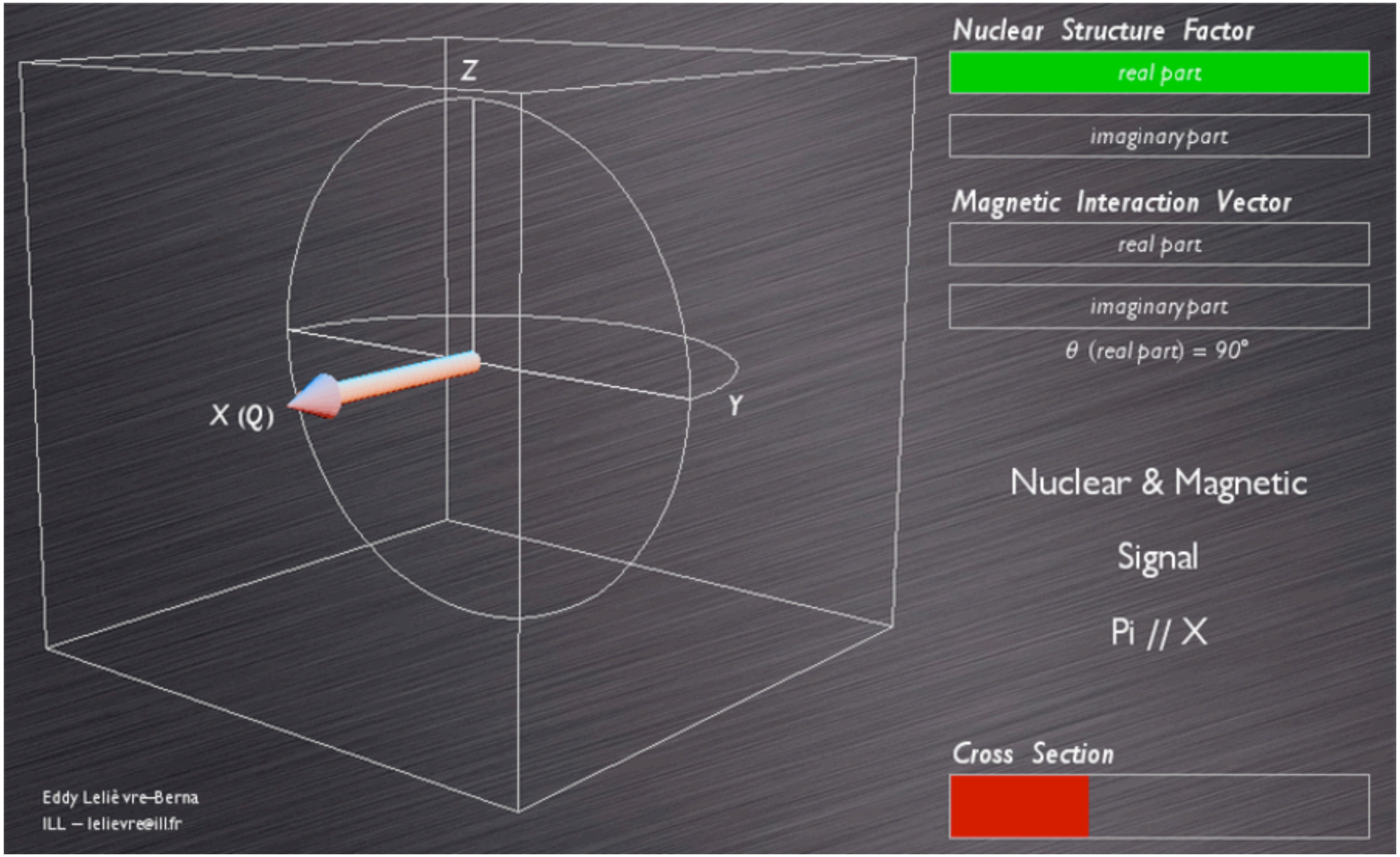
Polarimetry - M complex



Polarimetry - NM (real/real)



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The Polarization Tensor

The goal is to determine complex magnetic structures. In practice this is done by measuring the polarization tensor **P** which is unambiguously defines all the terms in the Blume equations

$$\mathbf{P}_f = \boldsymbol{P}\mathbf{P}_i \qquad \text{where} \qquad \boldsymbol{P} = \begin{bmatrix} P_{xx} & P_{xy} & P_{xz} \\ P_{yx} & P_{yy} & P_{yz} \\ P_{zx} & P_{zy} & P_{zz} \end{bmatrix}$$

In practice, this is what is measured in a neutron polarimetry measurement

As an illustrative example, in the case of collinear antiferromagnet - aligned in the z-direction, the polarization tensor would be

$$\boldsymbol{P} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The appearance of any non-collinear magnetism would feed through into the off-diagonal components

NB The 3-directional PA method (D7) only measures the diagonal components of this matrix and would therefore miss this information

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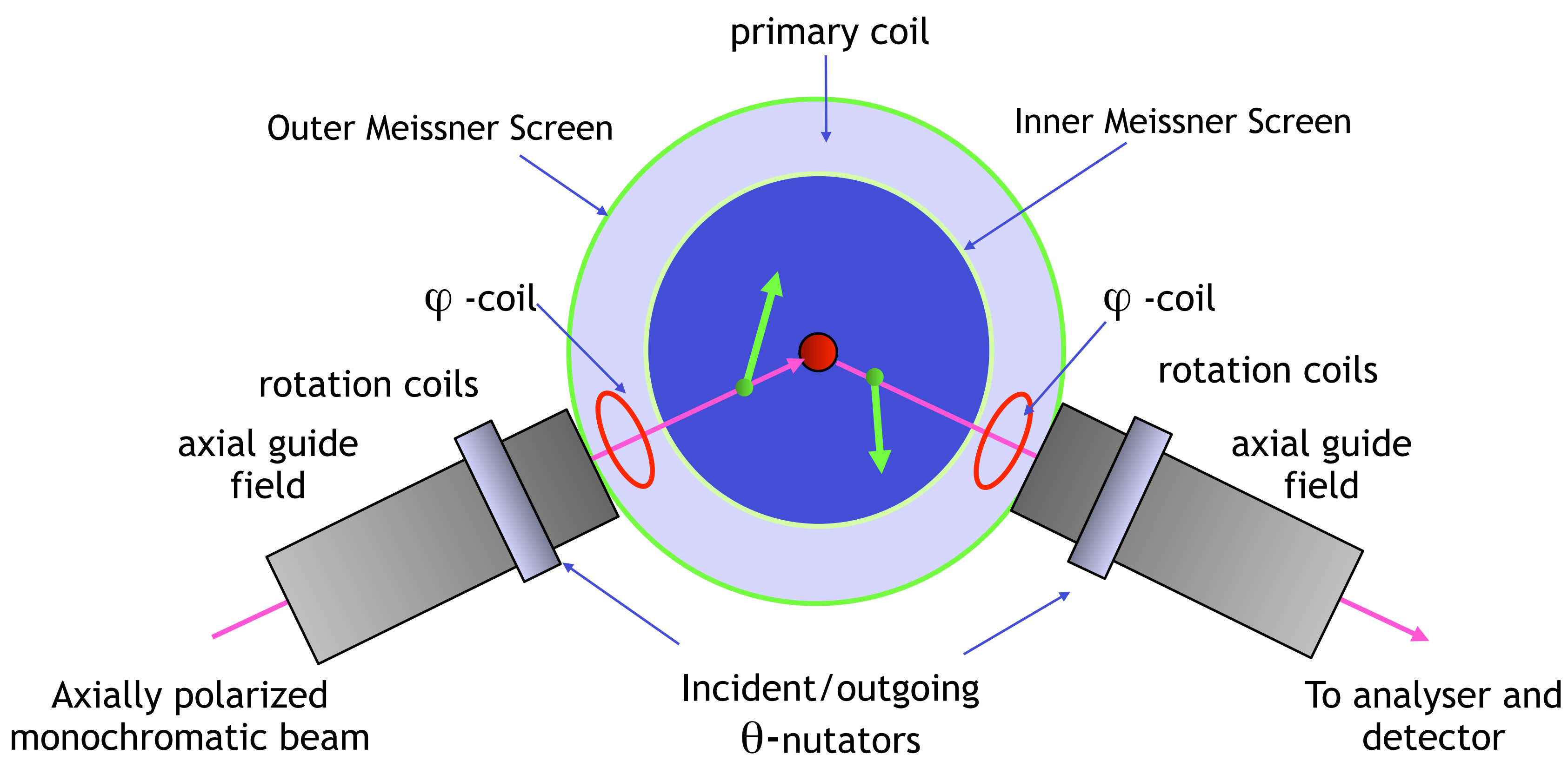
CRYOPAD

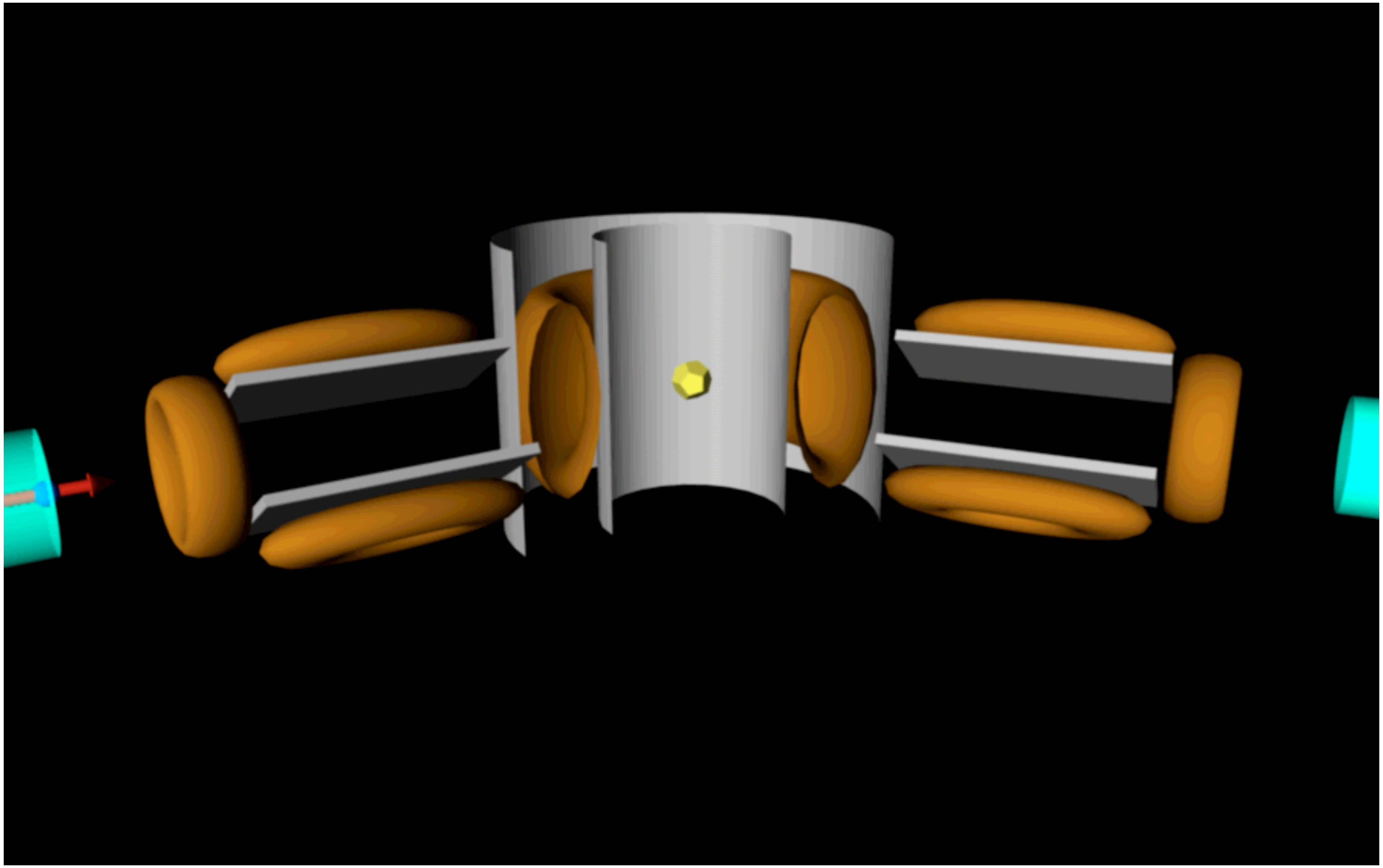
CRYOPAD has been developed by Tasset and co-workers at ILL in order to determine the vector polarization of the scattered beam for any predetermined direction of the vector polarization of the incident beam (i.e. measurement of the polarization tensor)

The field along the whole of the neutron beam is perfectly defined with the help of spin nutators, precession coils, Meissner screens, in order to align and analyse the polarization in any direction in space.

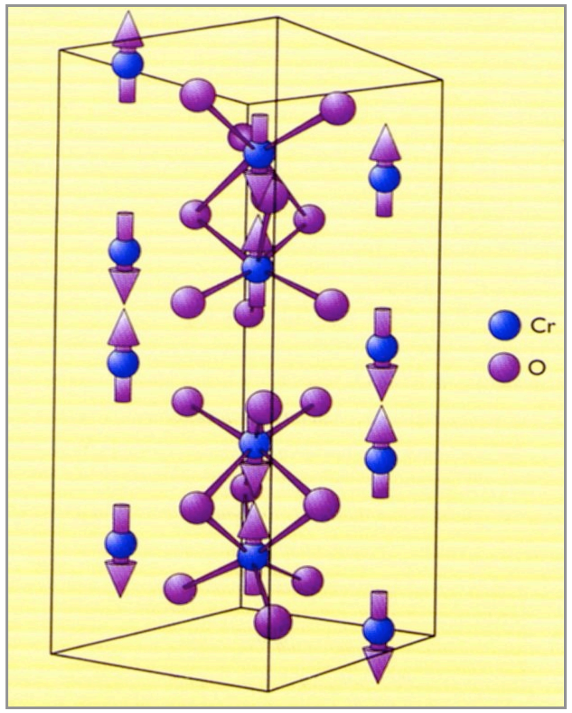


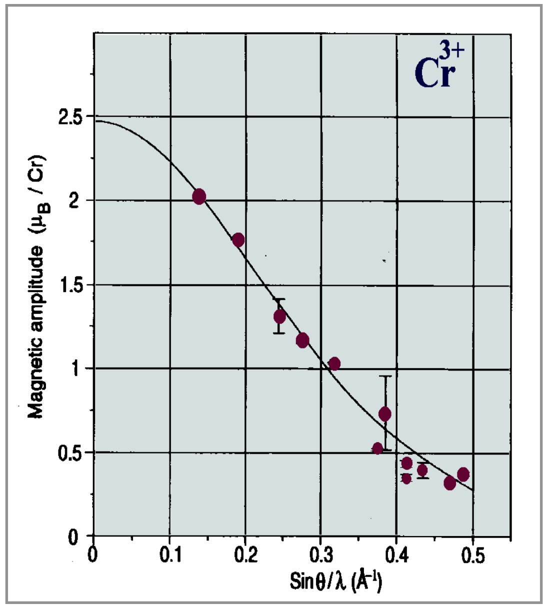
Tasset et al, Physica B 267-8, 69, (1999)

CRYOPAD



Examples from CRYOPAD





Cr_2O_3 *Brown et al, Physica B 267-268, 215, 1999*)

Cr_2O_3 is a collinear antiferromagnet with zero propagation vector for which the magnetic and nuclear scattering are phase shifted by °90

It is anti-centrosymmetric and therefore information about 180º antiferromagnetic domains cannot be obtained by measuring just the cross section or with uniaxial polarized neutron measurements

By cooling under various conditions of electric and magnetic fields an imbalance in domain populations is achieved - the crystal is then measured in zero field

Not only are the magnetic structures for the cooling conditions obtained - but for the first time the zero field magnetic form factor of an antiferromagnet is determined

Other studies include inelastic scattering measurements of, for example, $CuGeO_3$ *(Regnault et al, Physica B 267-268, 227, 1999)* and structural studies of complex magnetic phases e.g. Nd

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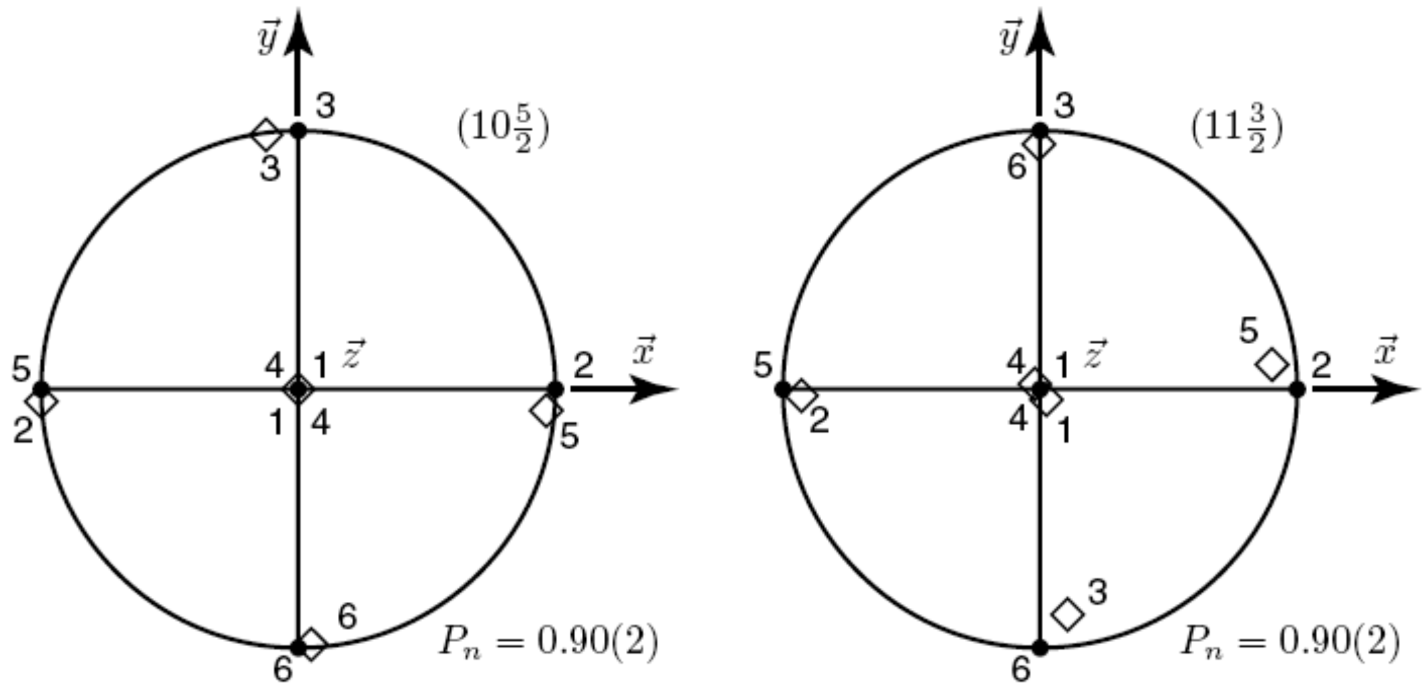
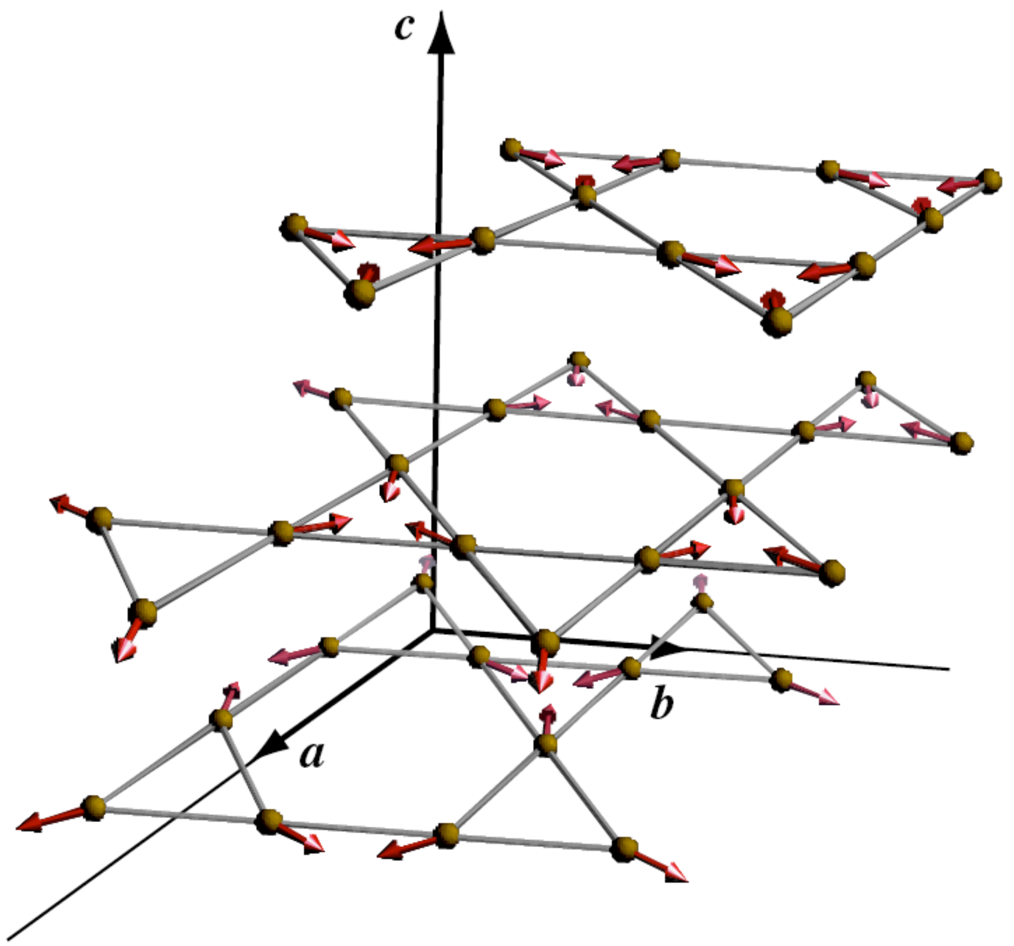
Examples from CRYOPAD

Magnetic order in kagome lattice magnets





Stereograms showing the directions of incident and scattered polarizations for the 1 0 5/2
and 1 1 3/2 reflections. The symbols • and ◊ represent respectively the incident and scattered polarization directions. The numbers are used to identify the corresponding pairs.

Harrison et. al., 2005

The End