

# Neutrons in soft matter

Lecture 2 – Reflectometry & Dynamics

João T. Cabral & Julia S Higgins  
Department of Chemical Engineering  
Imperial College London

David G Bucknall  
Georgia Tech, USA

# Outline

## Lecture 1 – Structure & kinetics – SANS

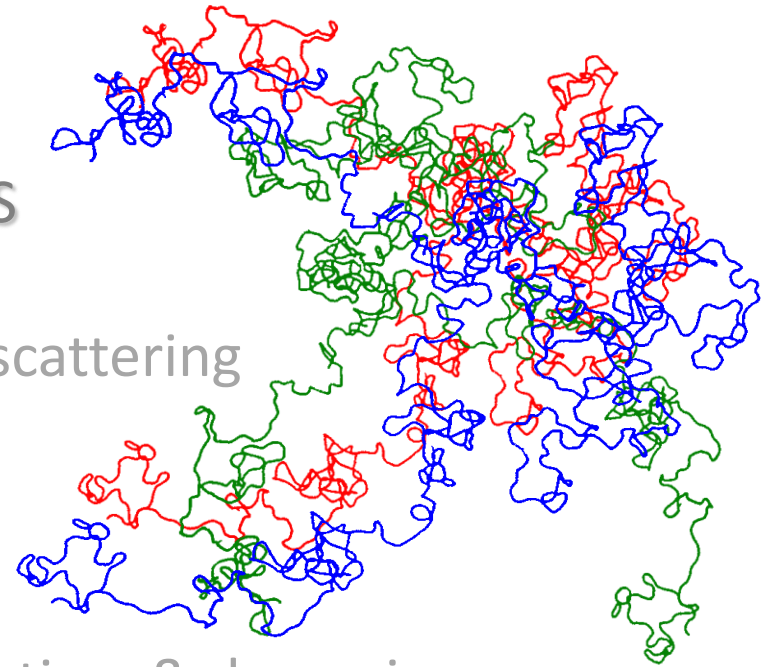
### Introduction

soft matter & relevance of neutron scattering

Single objects: spheres, coils, rods...

Single chain polymer conformation  
(solution and blends)

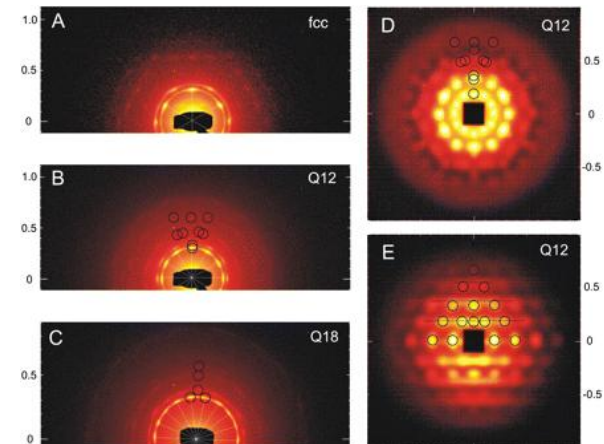
Polymer blends: interactions, conformation & dynamics  
(equilibrium and phase separation)



## Lecture 2 – Interfaces and dynamics

Reflectivity and diffusion

Dynamics in soft matter, QENS, BS, Spin-echo

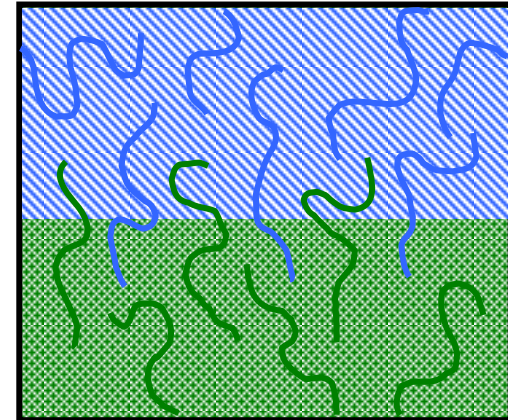


Forster et al (2011)

# Reflectometry: study of interfaces

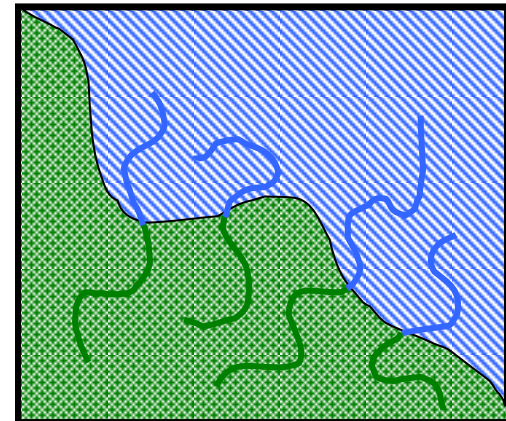
## Miscible systems

- Interdiffusion, e.g., welding

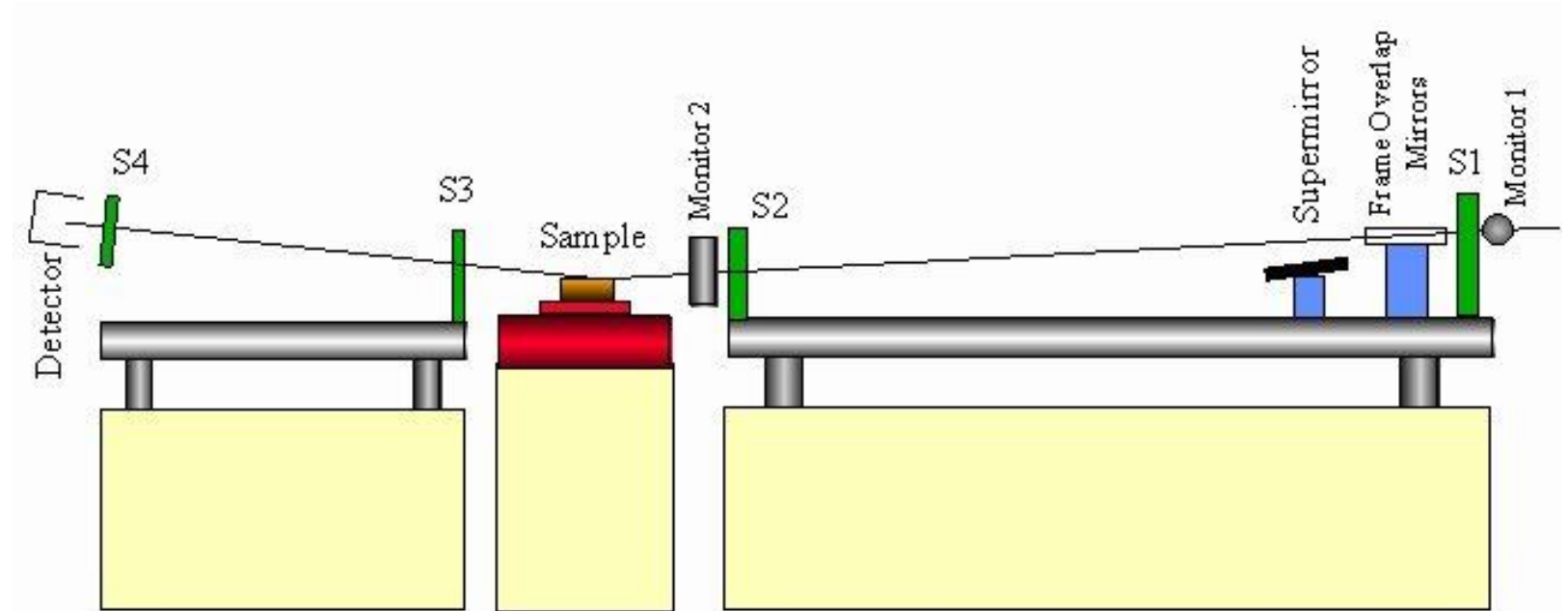


## Immiscible systems

- Copolymers, e.g., di-blocks
- Reduce interfacial tension  
→ smaller dispersed phase
- Entangle with homopolymers  
→ **increase strength**



# Reflectometry



CRISP (ISIS)



# Significance of the interfacial width

Theoretical width

- Infinite molecular weight limit

$$w_t = \frac{2a}{(6\chi)^{0.5}}$$

E Helfand & AM Sapse  
*J Chem Phys* 62 (1975) 1327

where  $a$  (statistical segment length)

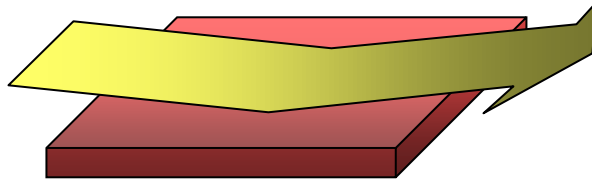
- Finite molecular weight limit

$$w_t = \frac{2a}{\sqrt{6}} \left( \chi - \frac{\pi^2}{6} (N_1^{-1} + N_2^{-1}) \right)^{-1/2}$$

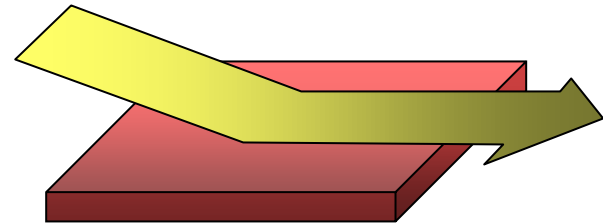
M Stamm & DW Schubert  
*Ann Rev Mater Sci*  
25 (1995) 325

⇒ Measure interfacial width to find  $\chi$

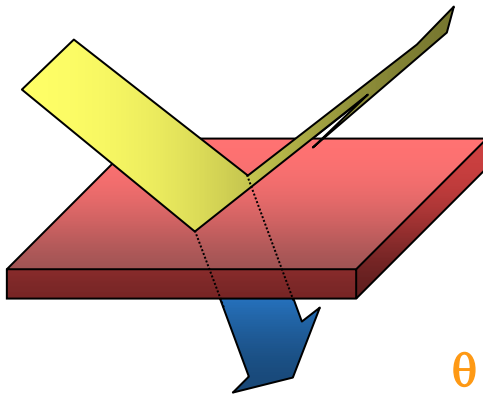
# Basics of Reflectivity



$\theta < \theta_{\text{crit}}$   
only reflection



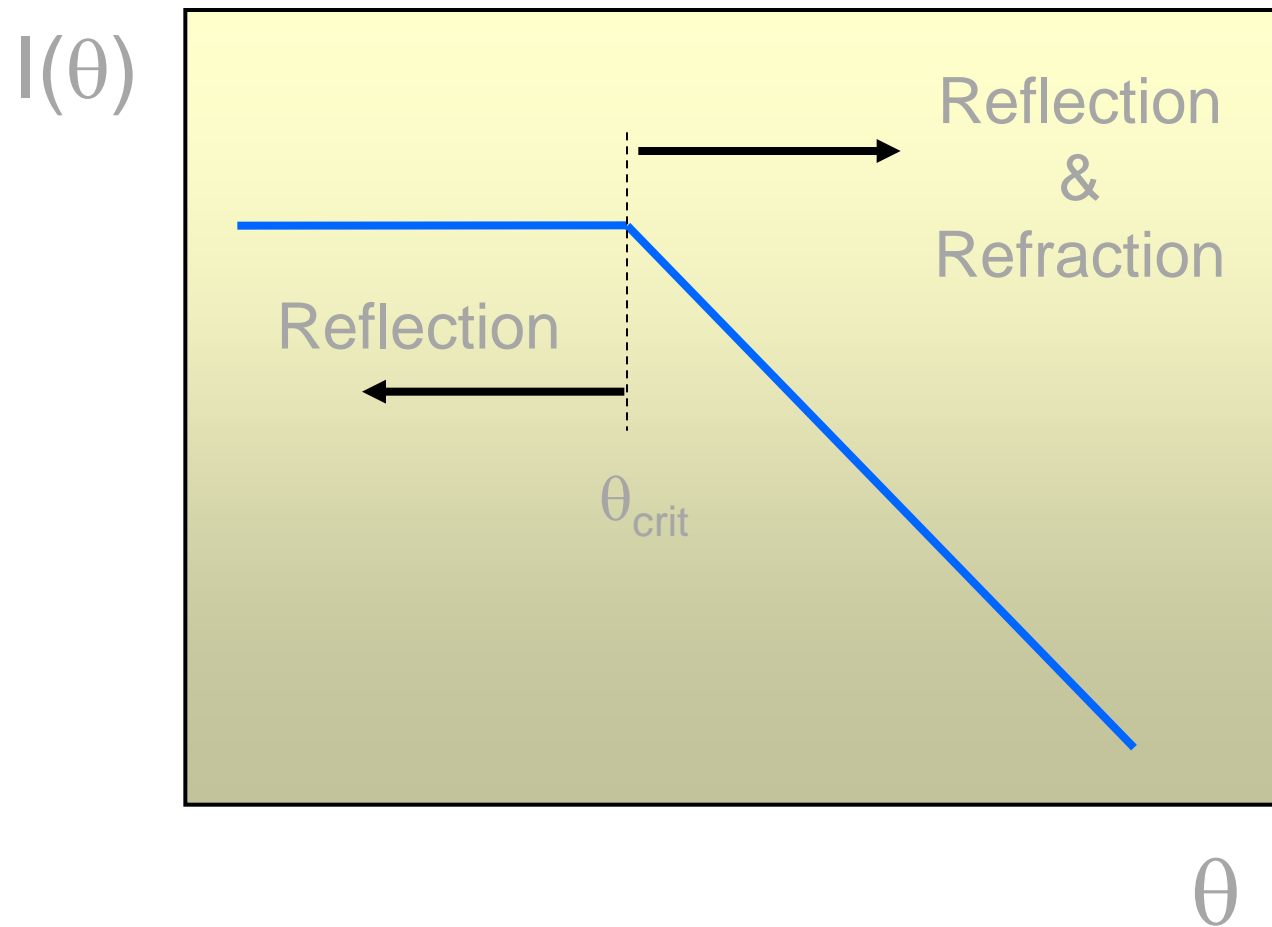
critical angle  $\theta = \theta_{\text{crit}}$



$\theta > \theta_{\text{crit}}$   
reflection and refraction

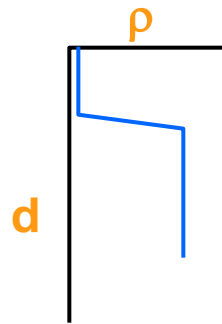
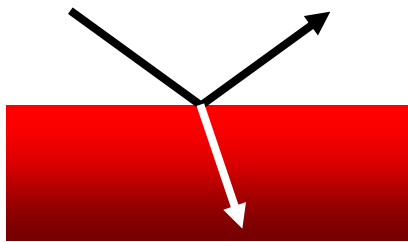


# The Reflectivity Profile

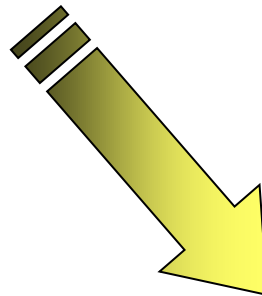




# Simplest Case



# Information Content

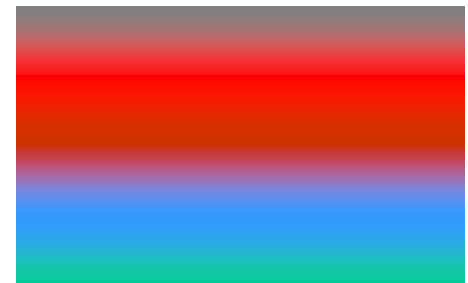
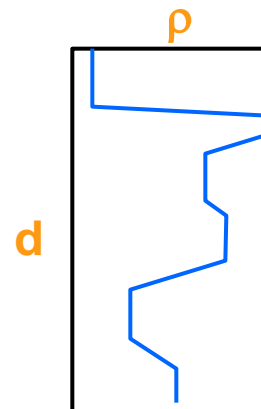


What about lateral information?

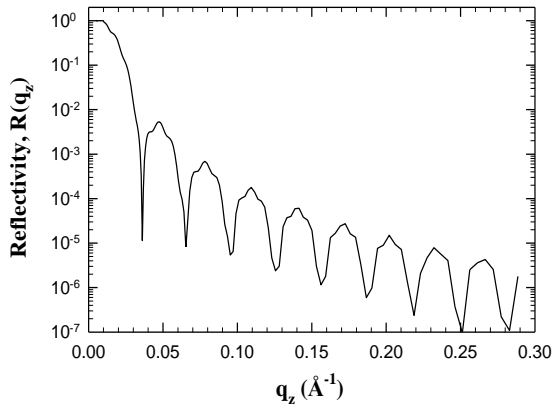
# Complex Case



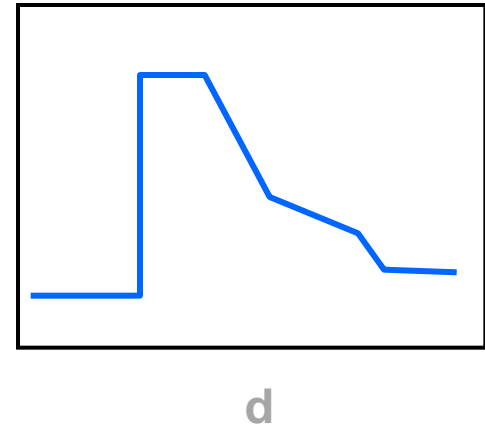
**Off-specular !**



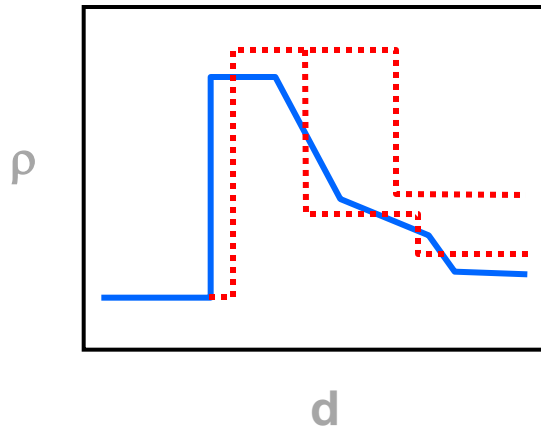
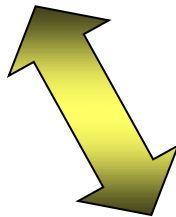
# Evaluating Reflectivity Data



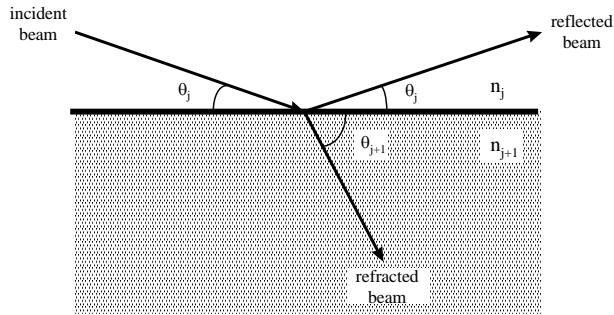
*fft*  
➔  
***Ideal***



***Real world***



# Single layers and bilayers



$$n_j = 1 - \frac{\lambda^2 N_d b}{2\pi} = 1 - \frac{\lambda^2 \rho_z}{2\pi}$$

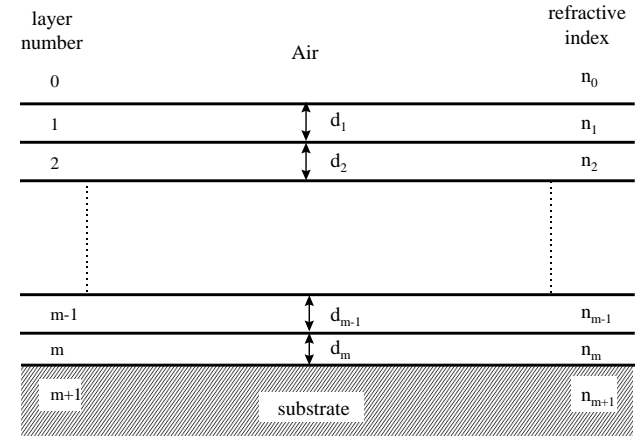
$$n_j \cos \theta_j = n_{j+1} \cos \theta_{j+1} \quad \text{Snell's Law}$$

$$r_{j,j+1} = \frac{n_j \sin \theta - n_{j+1} \sin \theta_{j+1}}{n_j \sin \theta + n_{j+1} \sin \theta_{j+1}} \quad \text{Fresnel's law}$$

$$q = 2k = \frac{4\pi}{\lambda} \sin \theta$$

$$r_{j,j+1} = \left( \frac{q_{z,j} - q_{z,j+1}}{q_{z,j} + q_{z,j+1}} \right)$$

$$R = r_{j,j+1} r_{j,j+1}^*$$



$$r'_{m-1,m} = \frac{r_{m-1,m} - r_{m,m+1} \exp(2i\beta_m)}{1 + r_{m-1,m} r_{m,m+1} \exp(2i\beta_m)}$$

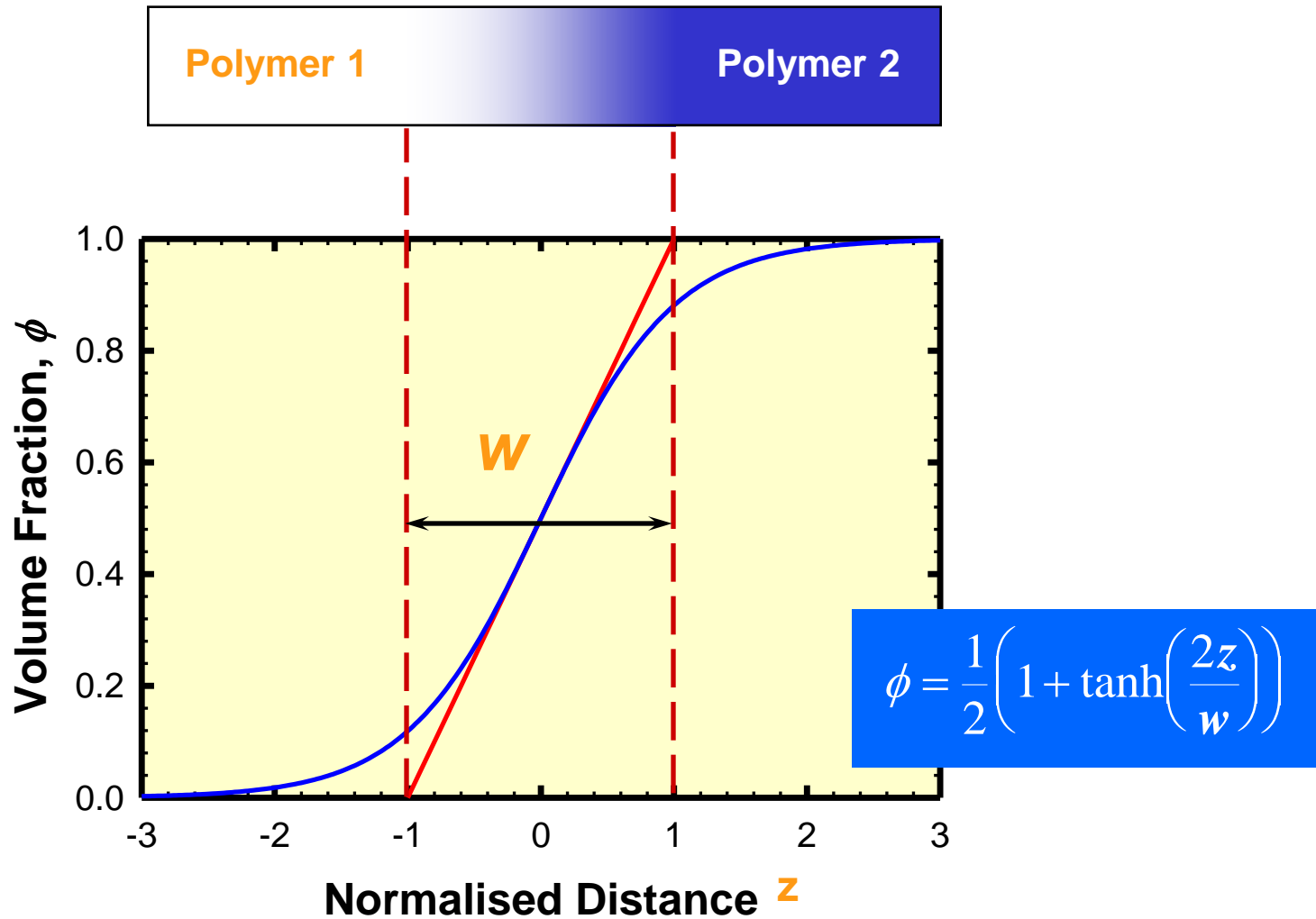
$$\beta_m = (2\pi/\lambda) n_m d_m \sin \theta$$

$$c_m = \begin{bmatrix} \cos \beta_m & -(i/\kappa_m) \sin \beta_m \\ -i\kappa_m \sin \beta_m & \cos \beta_m \end{bmatrix}$$

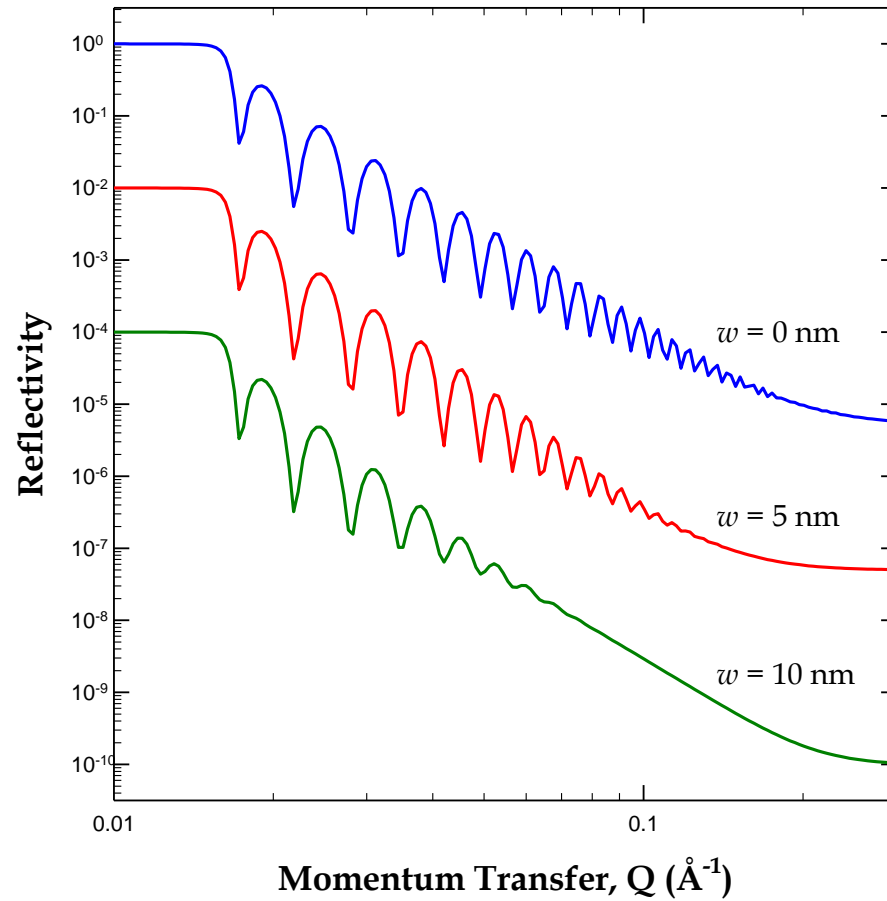
$$M = \prod_{m=0}^m c_m = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

$$R = \left| \frac{(M_{11} + M_{12} \kappa_{m+1}) \kappa_0 - (M_{21} + M_{22}) \kappa_{m+1}}{(M_{11} + M_{12} \kappa_{m+1}) \kappa_0 + (M_{21} + M_{22}) \kappa_{m+1}} \right|^2$$

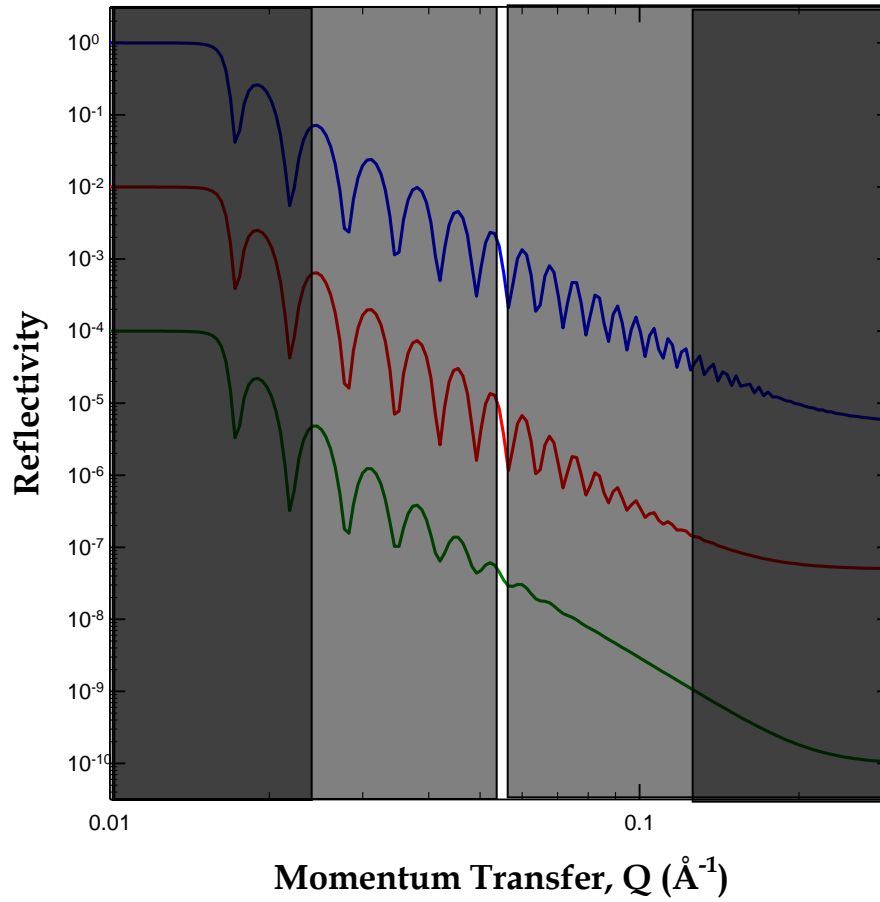
# Interfacial Width - Definition



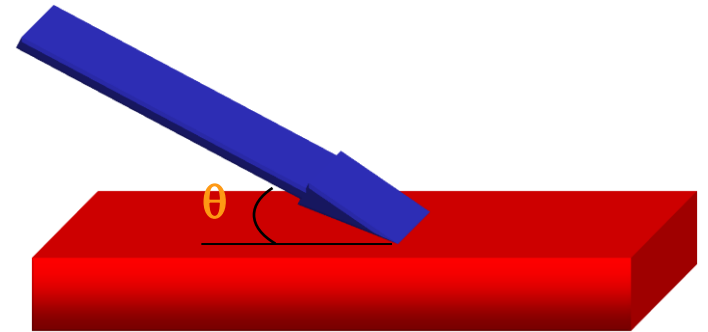
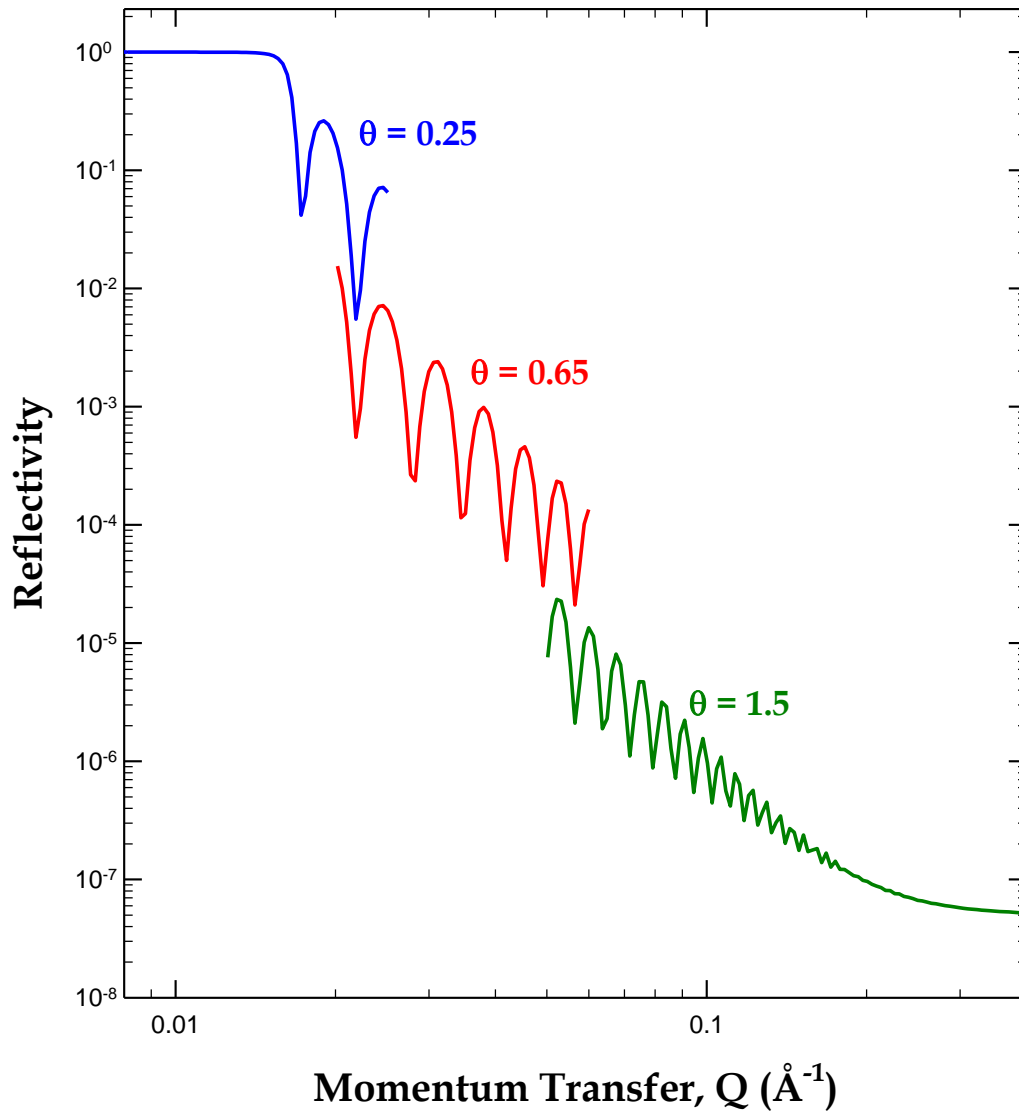
# Effect of Interdiffusion on Reflectivity Profiles



# Effect of Limiting Q range on Observation Window



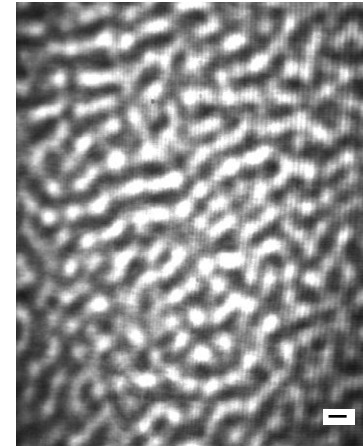
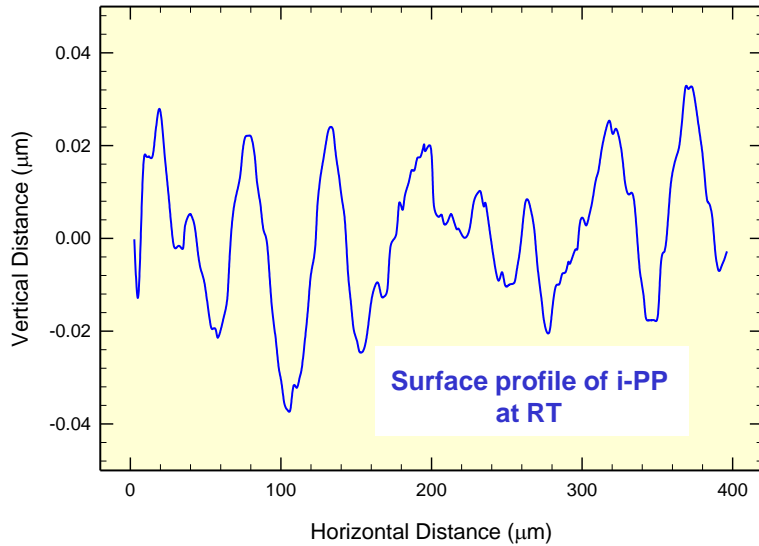
# Effect of Angle on the Q Range



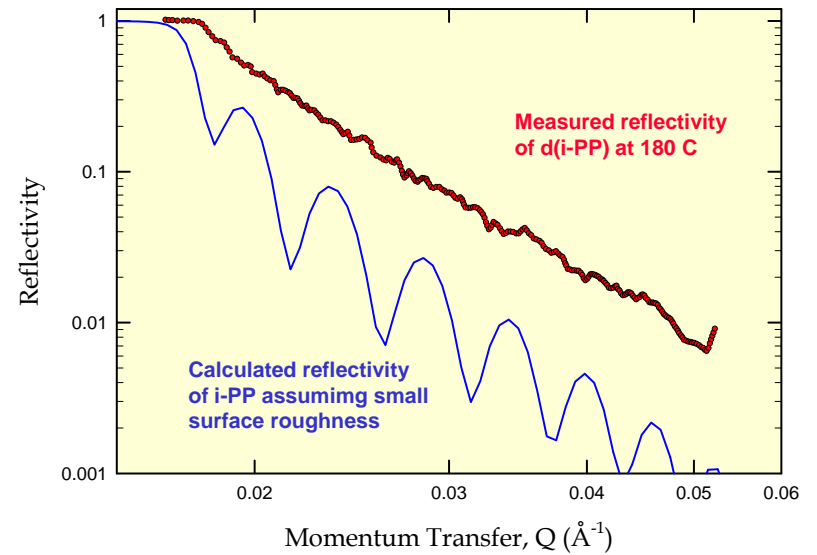
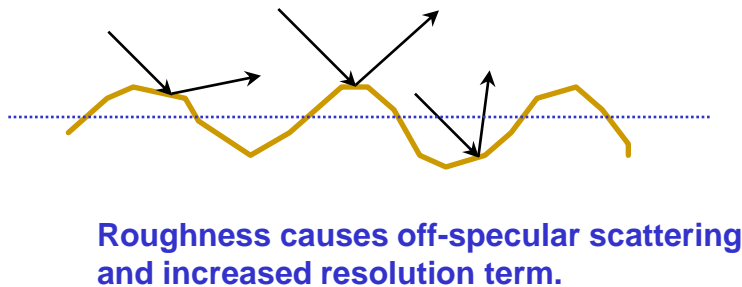
$$Q = \frac{4\pi}{\lambda} \sin \theta$$

$$0.05 < \lambda \text{ (nm)} < 0.65$$

# Effect of Crystallinity on reflectivity

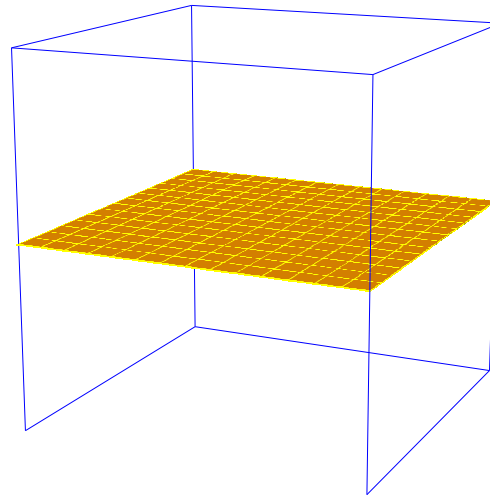


Brewster angle micrograph of surface of i-PP (bar 20 $\mu\text{m}$ )



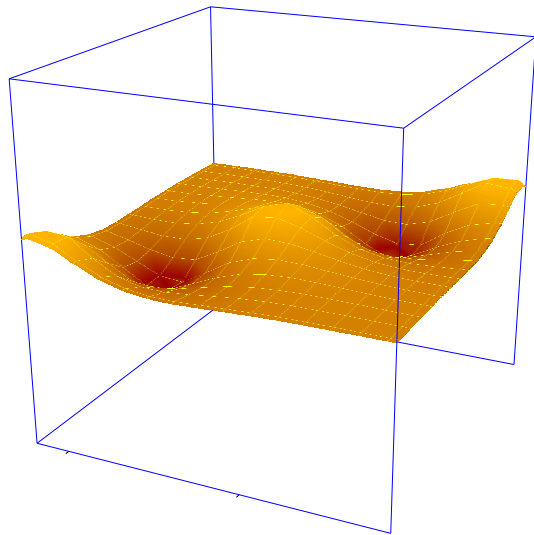


# Thermally Excited Capillary Waves

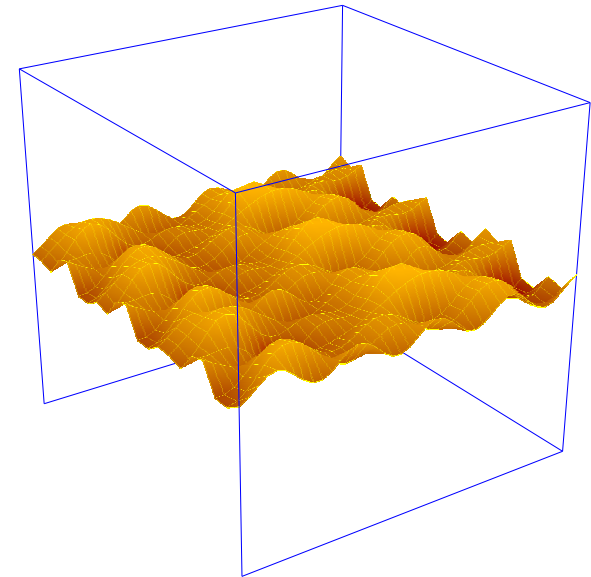


Mean field theory assumes that the interface is flat.

At equilibrium capillary waves are thermally excited.

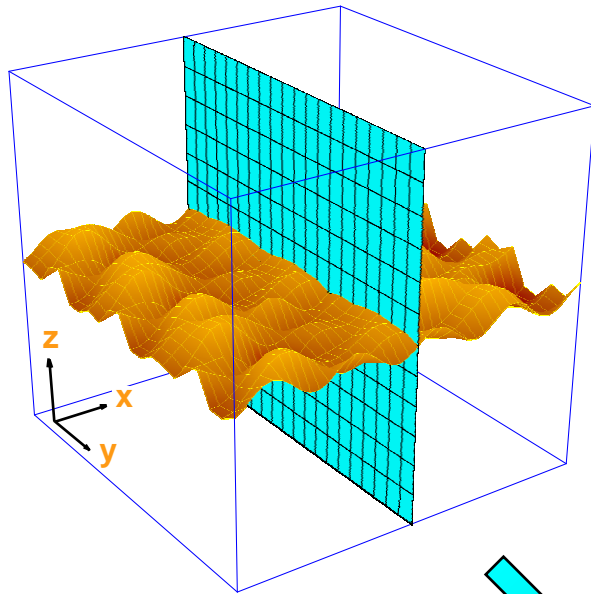


According to the equipartition theorem each mode increases the surface energy by  $0.5 kT$ .



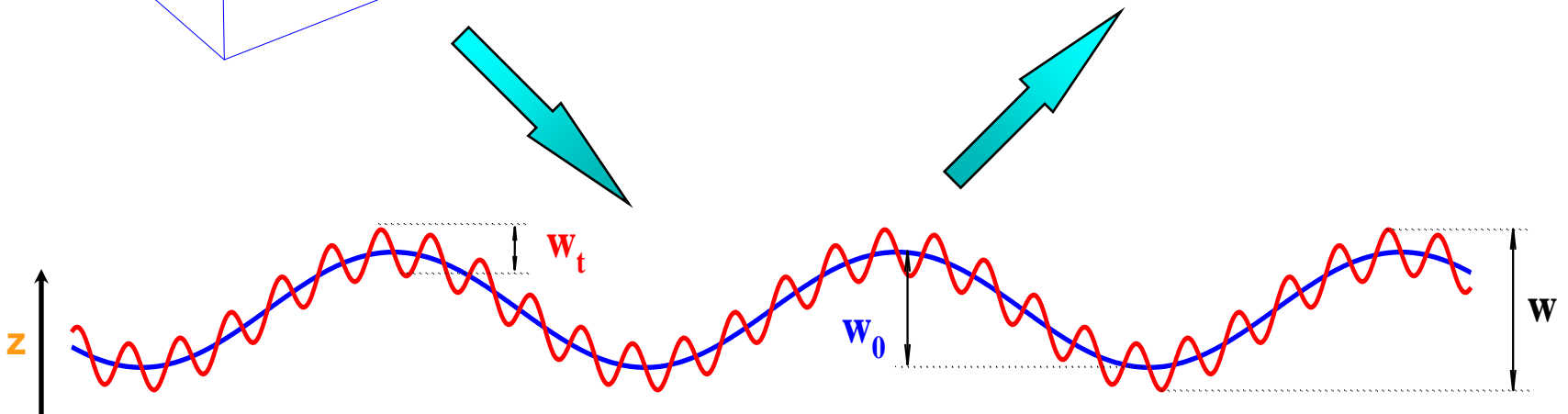
The actual surface is roughened by a superposition of all possible capillary wave modes.

# NR Measured Interfacial Width



$$w = (w_t^2 + w_0^2)^{0.5}$$

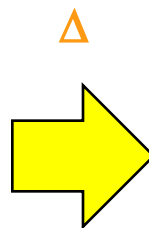
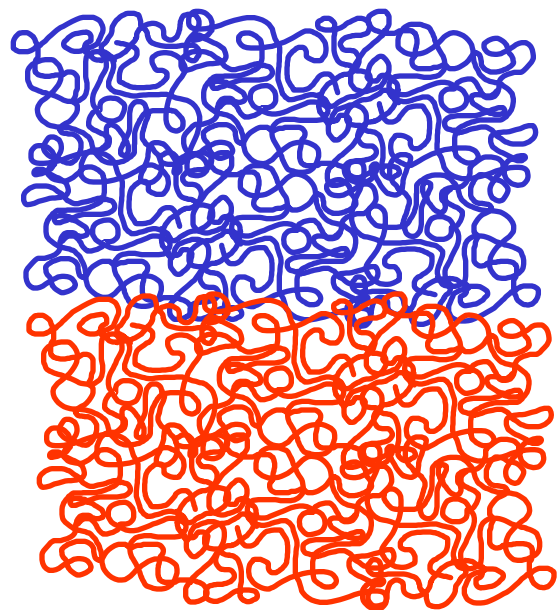
Definition of  $w_0$  dominates derivation of  $w_t$



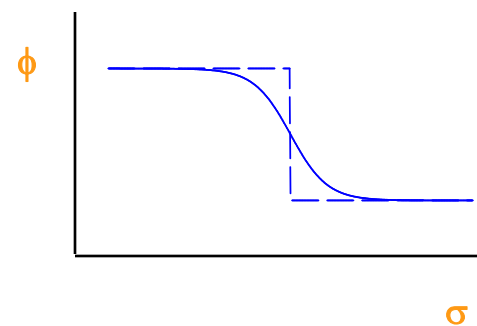
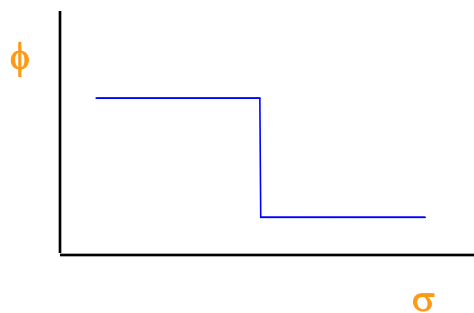
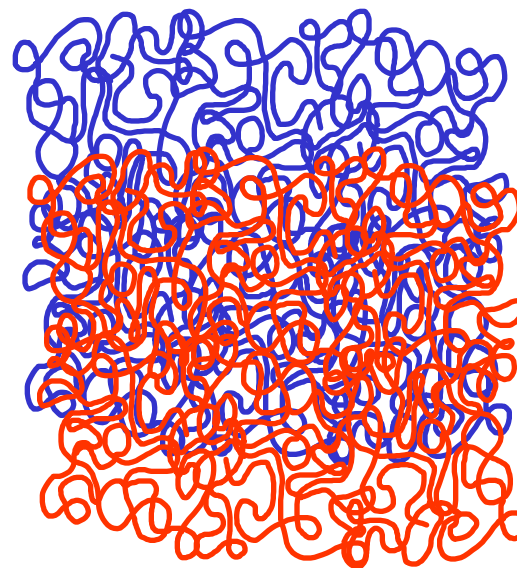
Projection onto z-y plan

# Polymer Interdiffusion

As made  
 $t = 0$



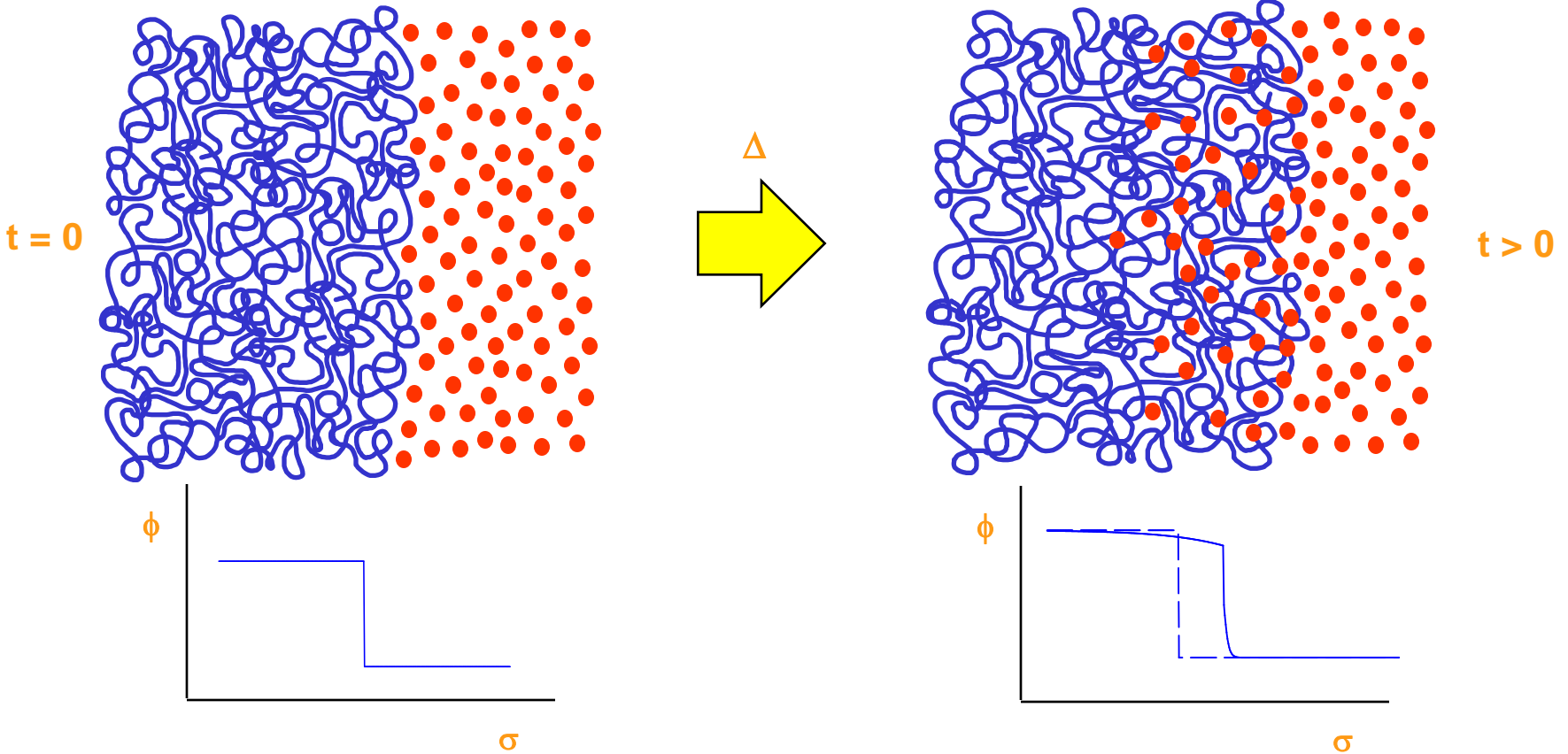
Annealed  
 $t > 0$



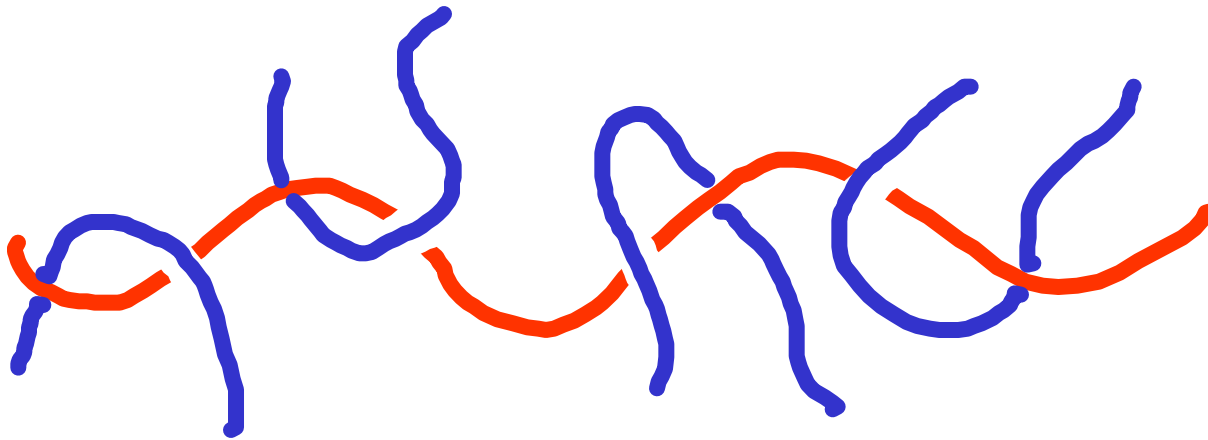
# Non-Fickian Diffusion - Case II Diffusion

$$\sigma \propto t^n \quad n \neq \frac{1}{2}$$

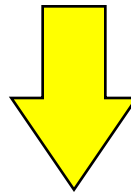
Non-Fickian Diffusion



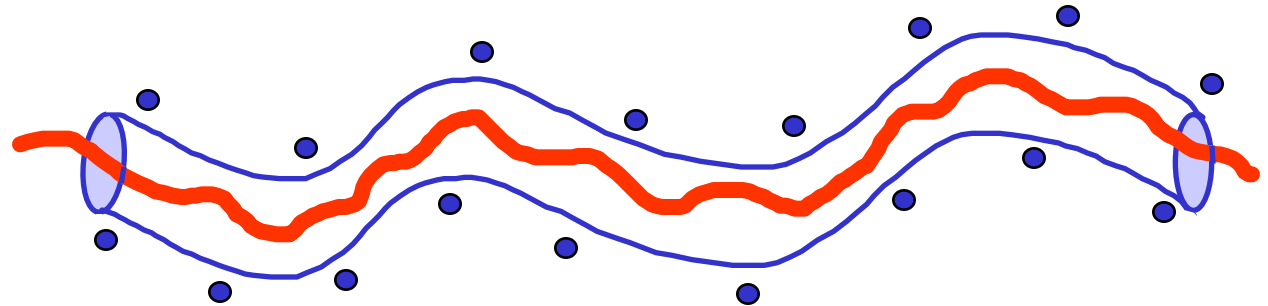
# The Tube Model



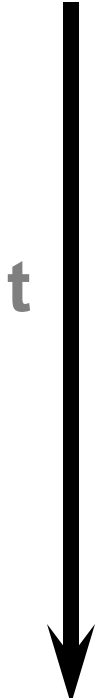
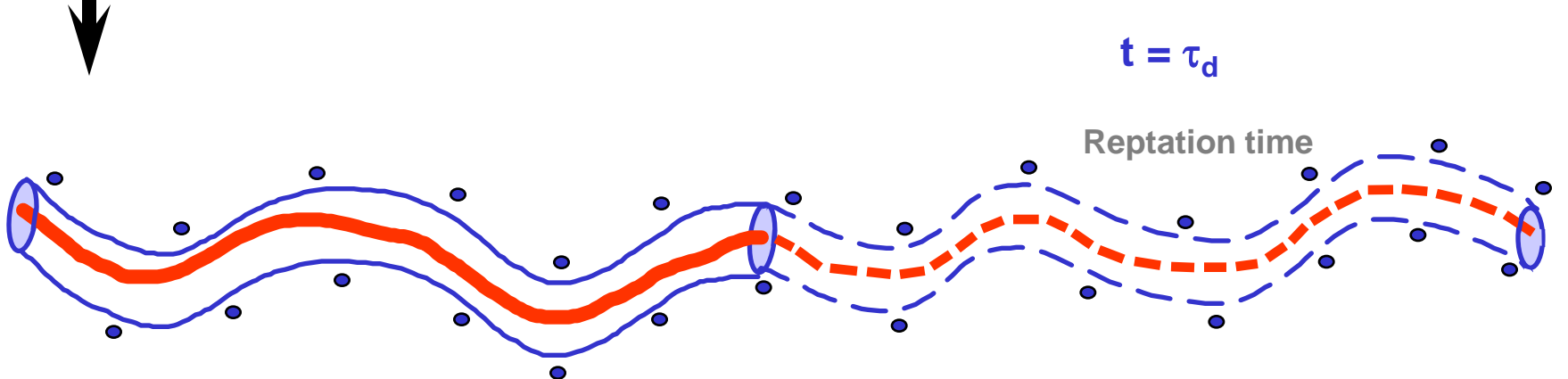
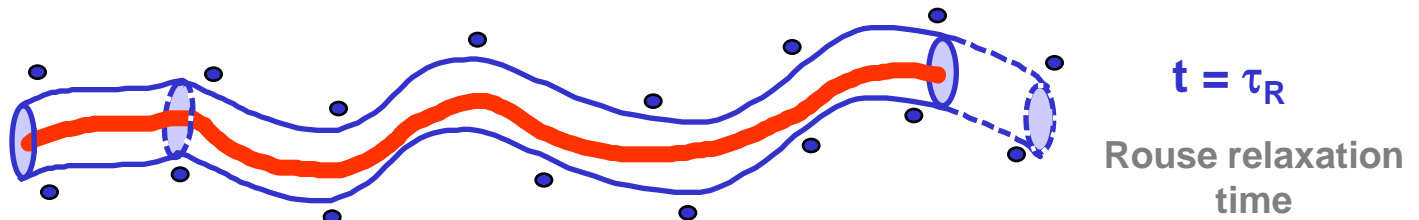
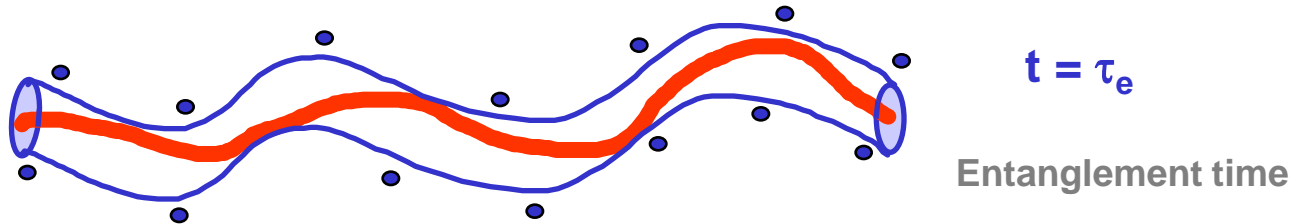
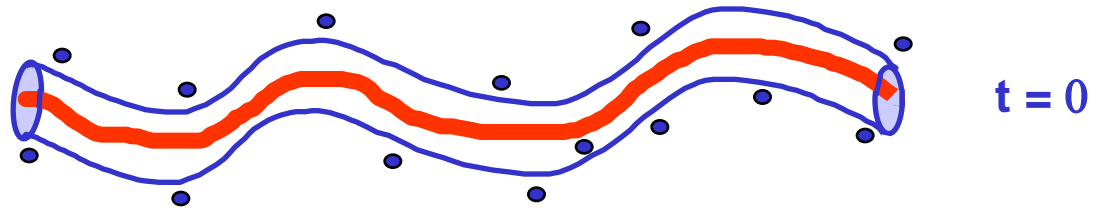
Polymer chains in the melt



Each chain can be considered to be constrained within a tube



# Polymer Motion



# Polymer Diffusion

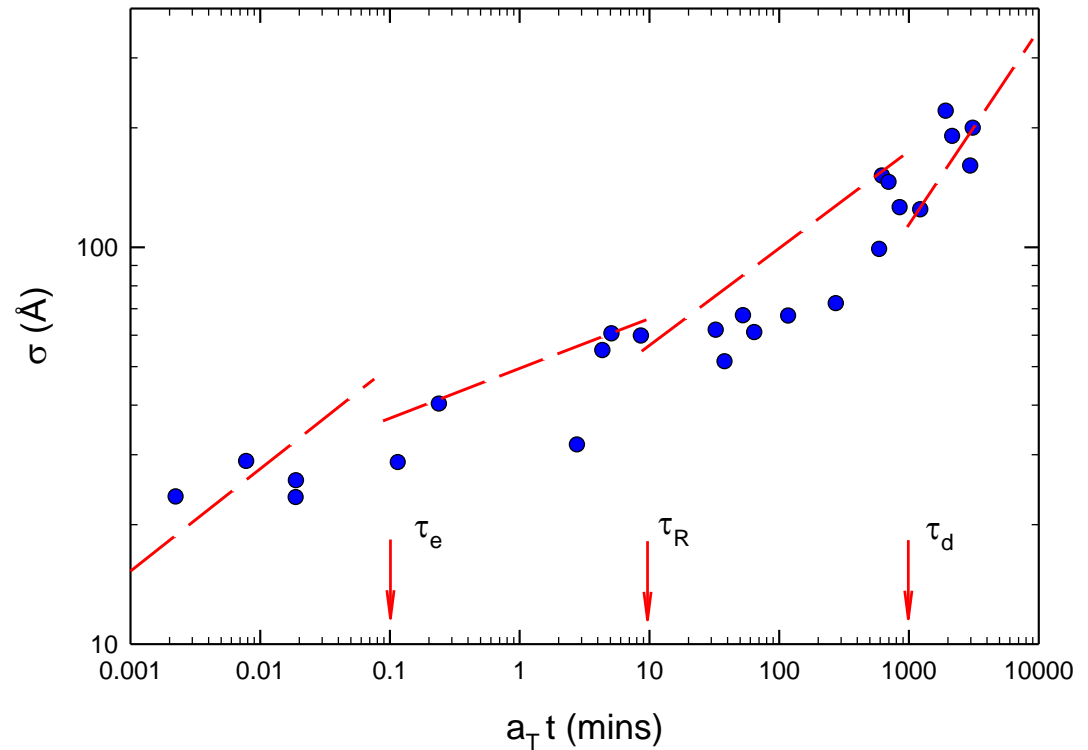
NR Results

$t < \tau_e$   $\longrightarrow$   $\sigma \propto t^{1/4}$

$\tau_e < t < \tau_R$   $\longrightarrow$   $\sigma \propto t^{1/8}$

$\tau_R < t < \tau_d$   $\longrightarrow$   $\sigma \propto t^{1/4}$

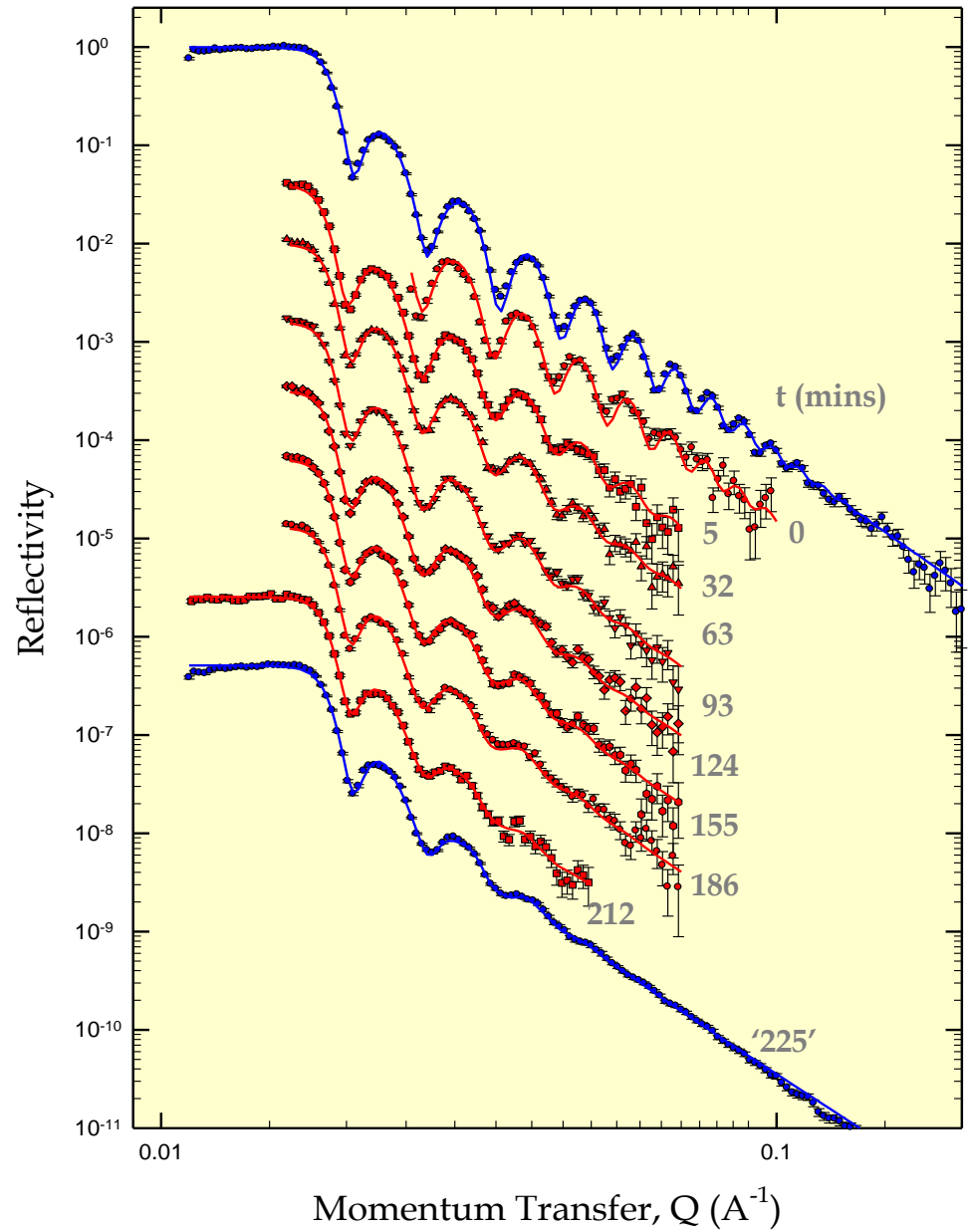
$t > \tau_d$   $\longrightarrow$   $\sigma \propto t^{1/2}$



A Karim et al, Phys Rev B 42 (1990) 6846

# Real Time Reflectivity Measurements

Si / PS (50k) / dPS (40k) @ 115 C





# Calculating a Diffusion Coefficient

$$w = \sqrt{4Dt}$$

For dPS-PS system:

$$D = (1.7 \pm 0.2) \times 10^{-17} \text{ cm}^2\text{s}^{-1}$$

$$D = \frac{k_B T d_T^2}{3N^2 \zeta b^2}$$

M Doi and SF Edwards  
*The Theory of Polymer Dynamics* (1986)

$$D = 2.81 \times 10^{-17} \text{ cm}^2\text{s}^{-1}$$

When  $\zeta$  (115C) = 0.199 dyne.s.cm<sup>-1</sup>  
and  $d_T = 5.7$  nm

Reptation time:

$$\tau_r = \frac{Nb^2}{3\pi^2 D}$$

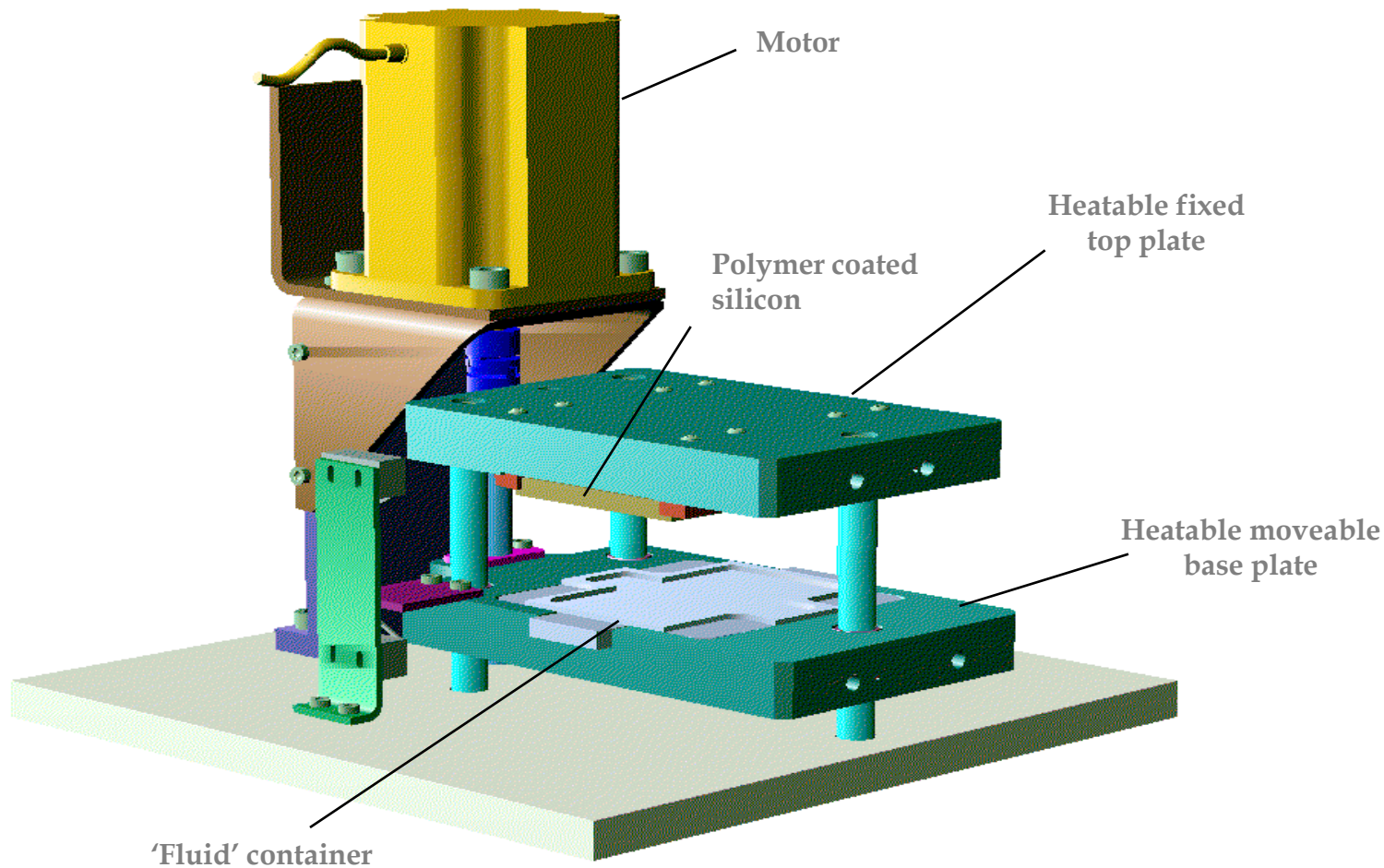
$$\begin{aligned} \tau_r &= 3223 \pm 363 \text{ s (dPS)} \\ &= 4333 \pm 489 \text{ s (hPS)} \end{aligned}$$

Rouse time:

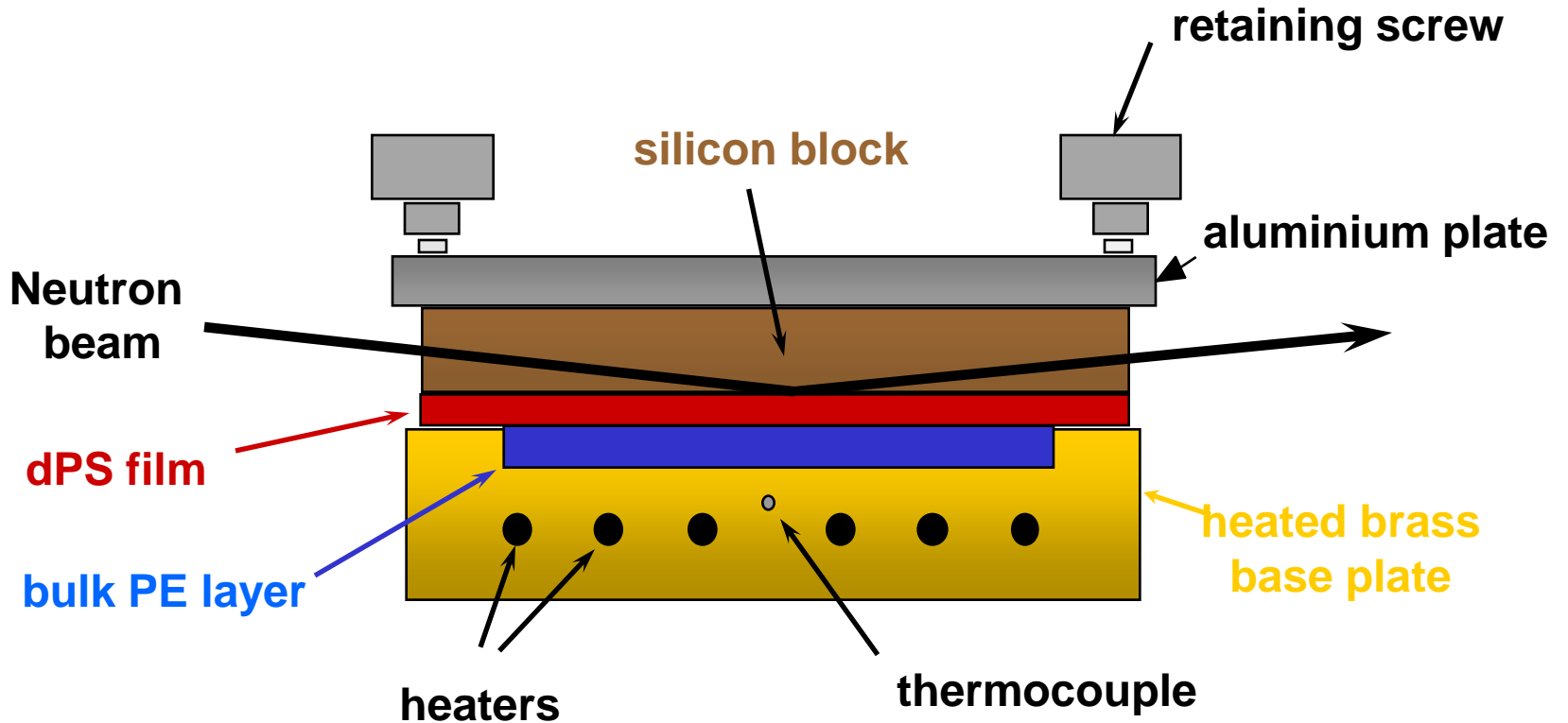
$$\tau_R = \frac{d_T^2}{9\pi^2 D}$$

$$\tau_R = 215 \pm 23 \text{ s}$$

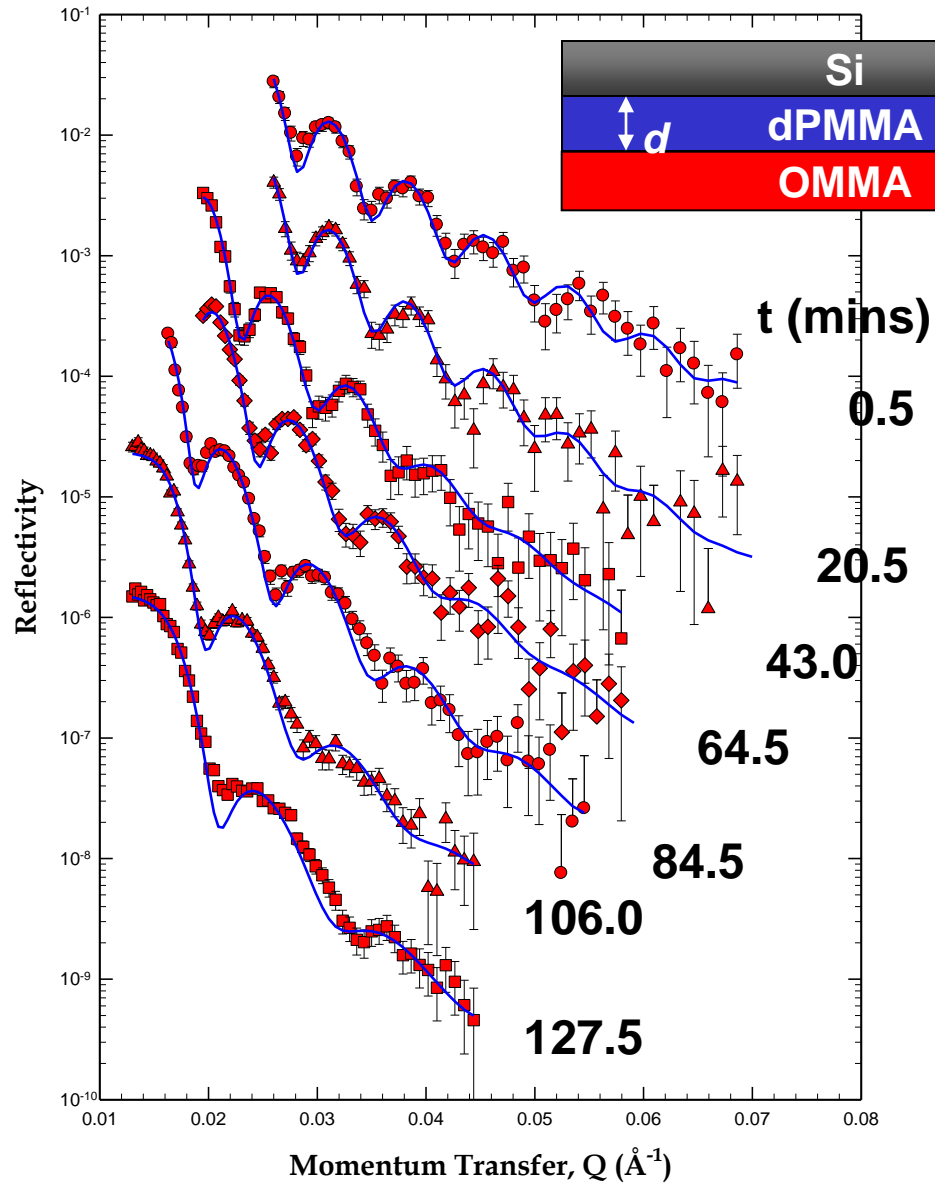
# Polymer-Oligomer Interdiffusion Reflectivity Cell



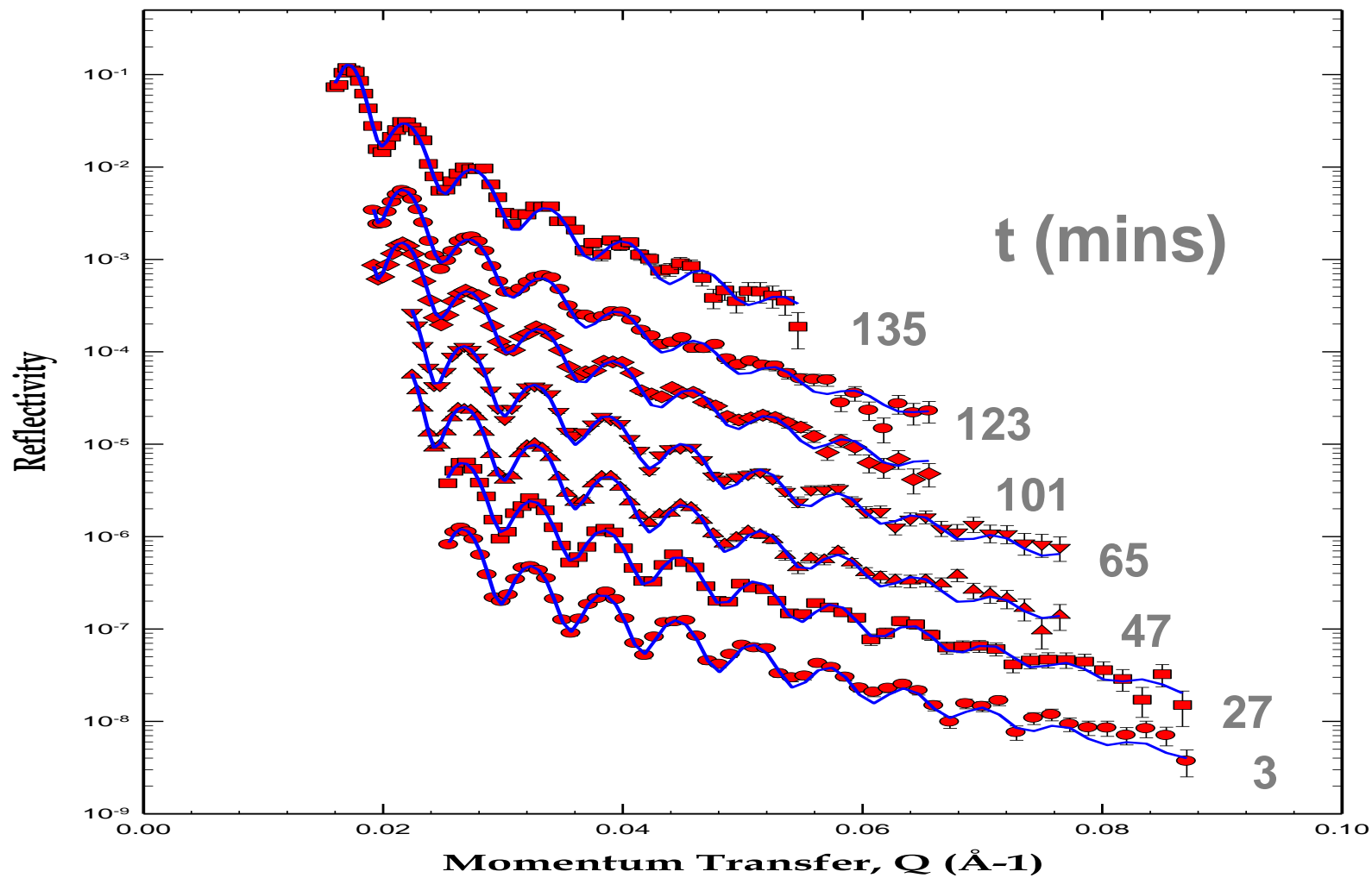
# Neutron Reflectivity Melt Cell



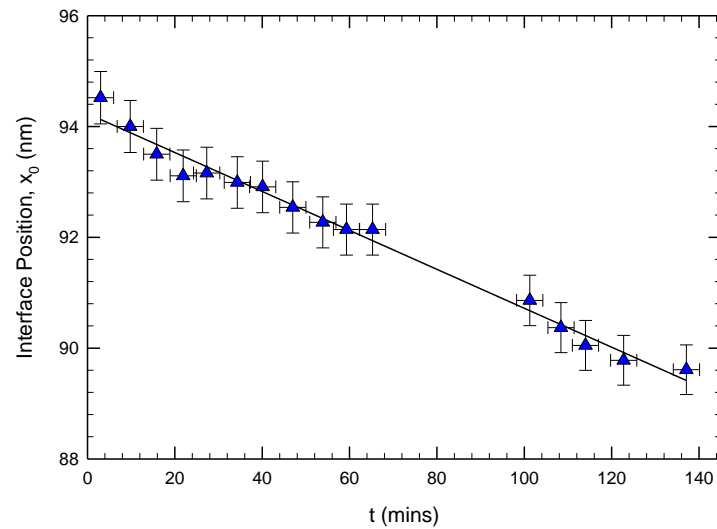
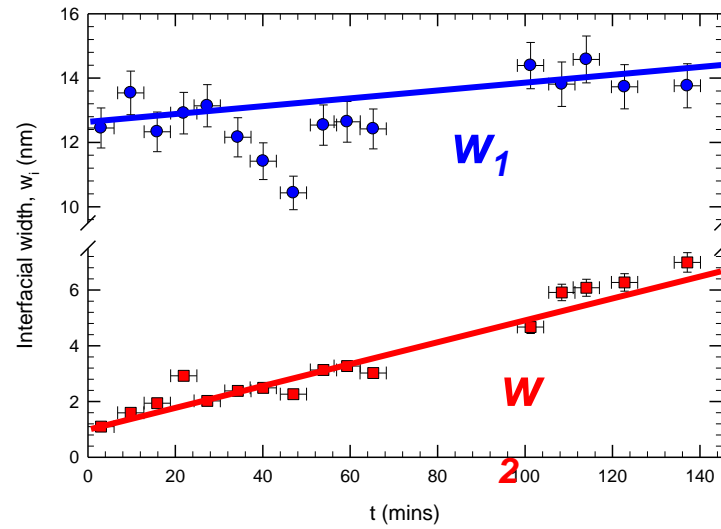
# dPMMA(100k) / OMMA(510) @ 45 C



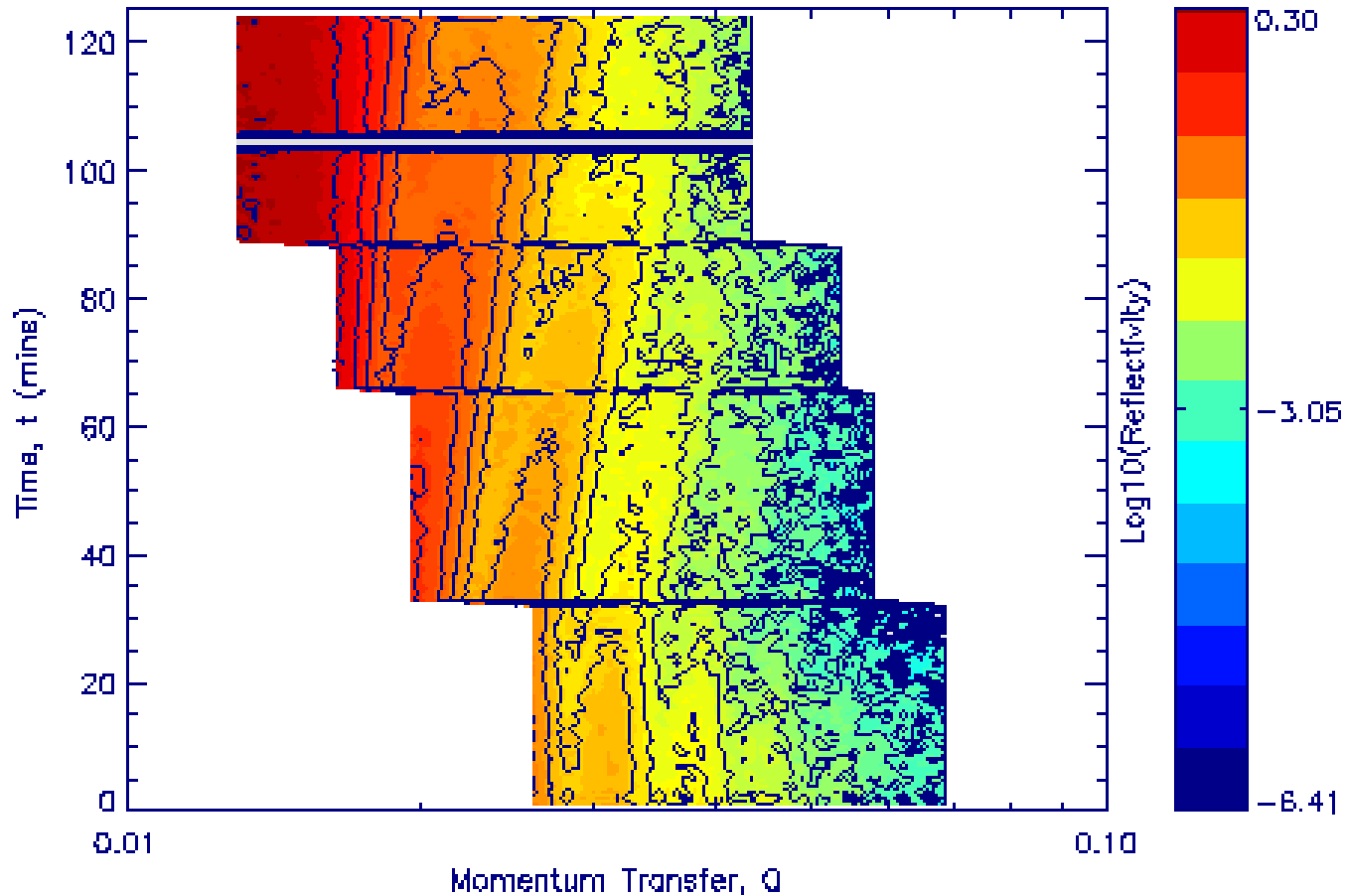
# dPS (101k) / OSt (1100) Interdiffusion @ 65C



# dPS (101k) / OSt (1100) Interdiffusion @ 65C

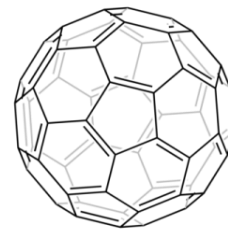


# Off-specular reflection



# (B) Soft Matter application: depth profiling of nanofilled polymer films

Fullerene C60  
0-5% w/w



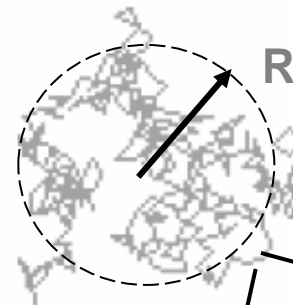
1

nm

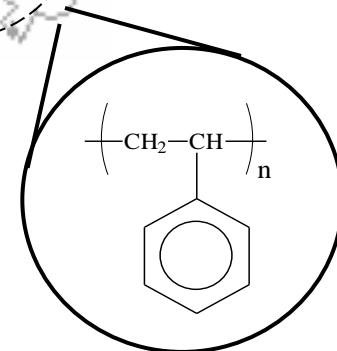
h



$h = 5-500 \text{ nm}$



$R_g = 2-20 \text{ nm}$

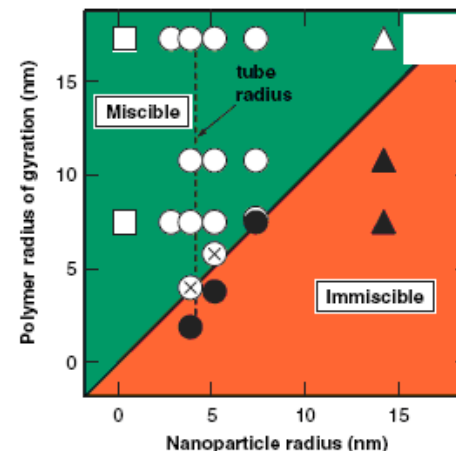
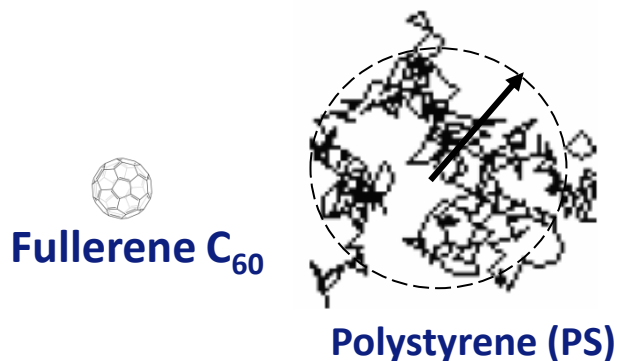


Polystyrene (PS)

$R_g = 0.27 M_w^{1/2}$

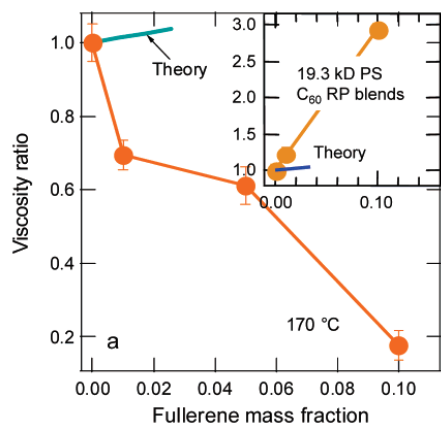


# Polymer-fullerene 'mixtures'



Mackay et al. Science (2006) 311, 1740

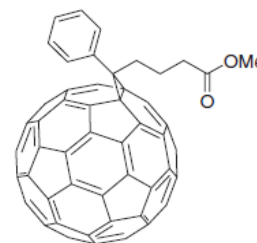
## Transport properties



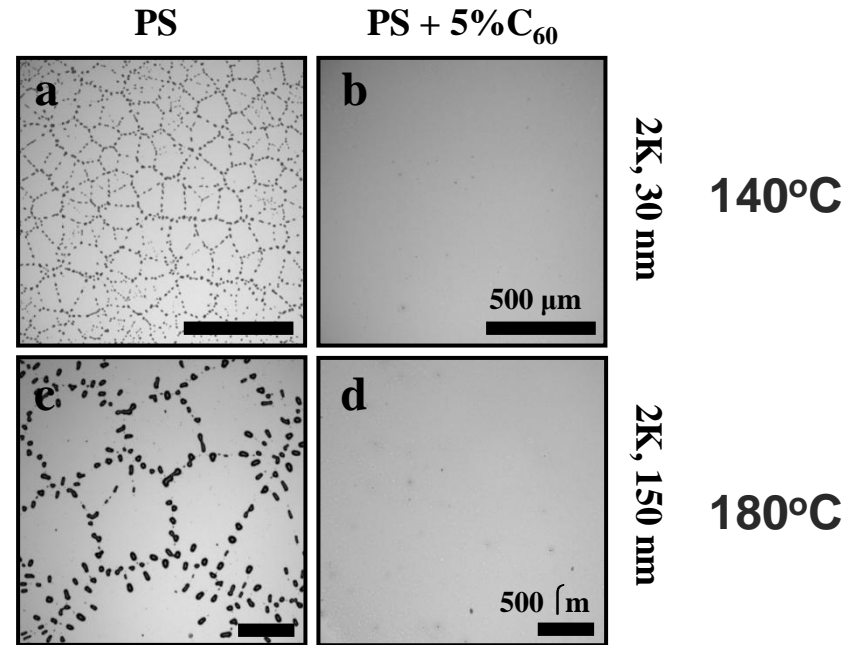
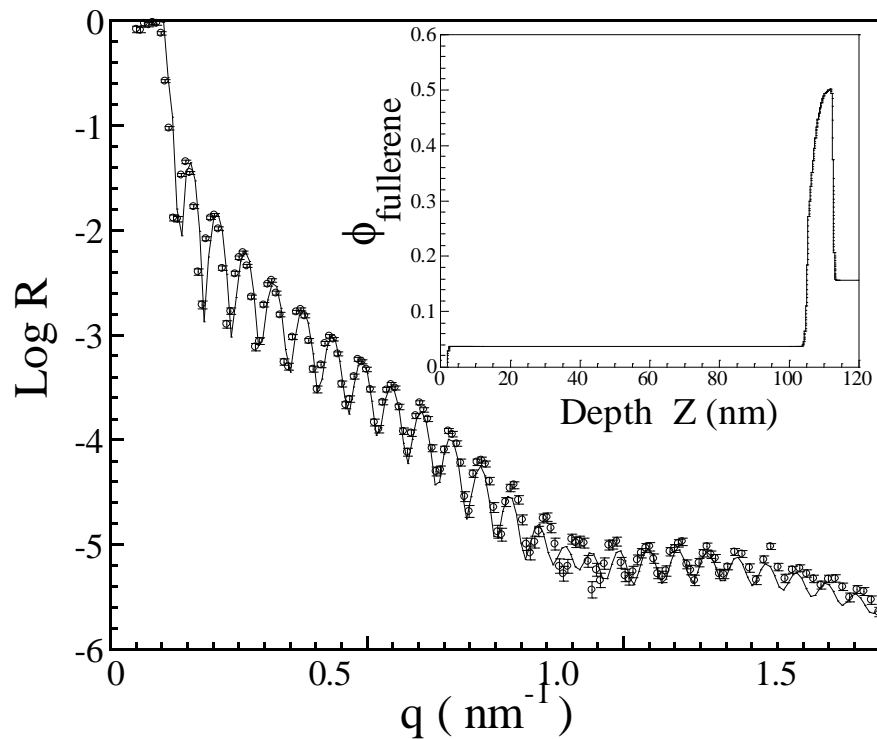
Tuteja et al. Nat. Mat. (2003) 2, 762

## Electrical properties

Fullerene derivative (PCBM)

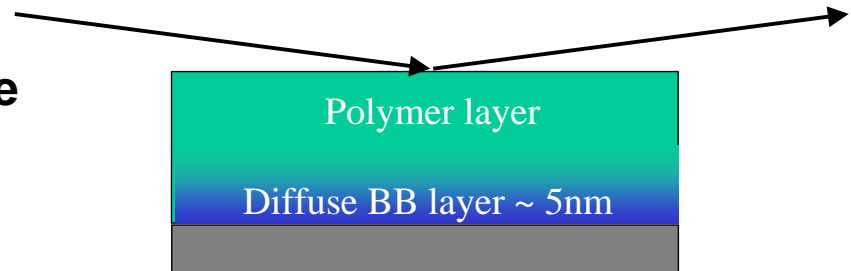


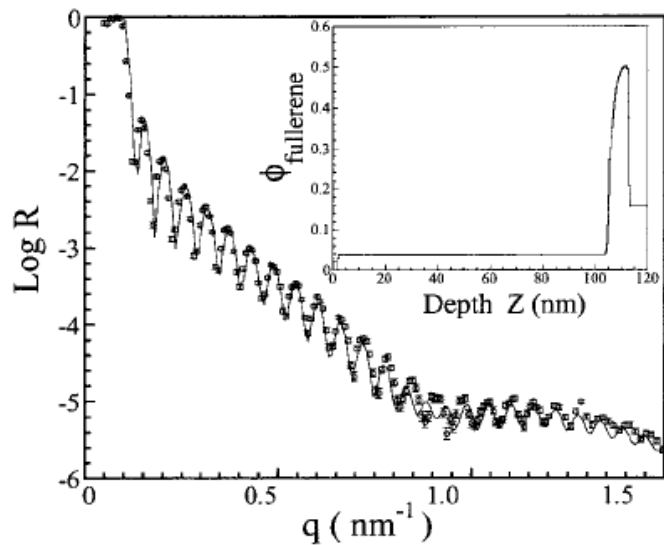
Heegar et al. Adv. Funct. Mater (2005) 15, 1617



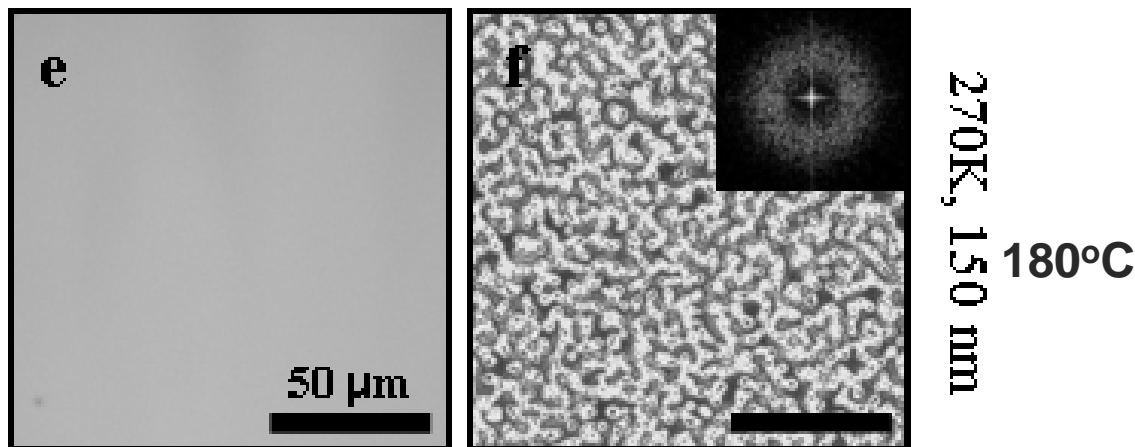
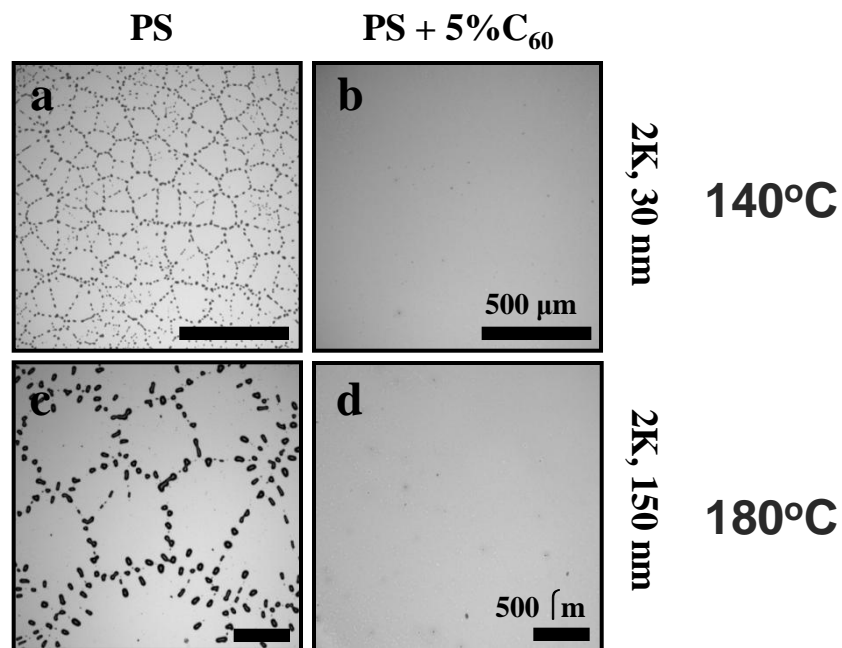
PS+5% C60 h = 100nm

- Neutron reflection from PS containing 5% fullerene on Si
- Fullerenes not present at air surface (confirmed water contact angle)
- Segregation of 2-5 nm thick *diffuse* layer of fullerenes on Si





PS+5% C60 h = 100nm



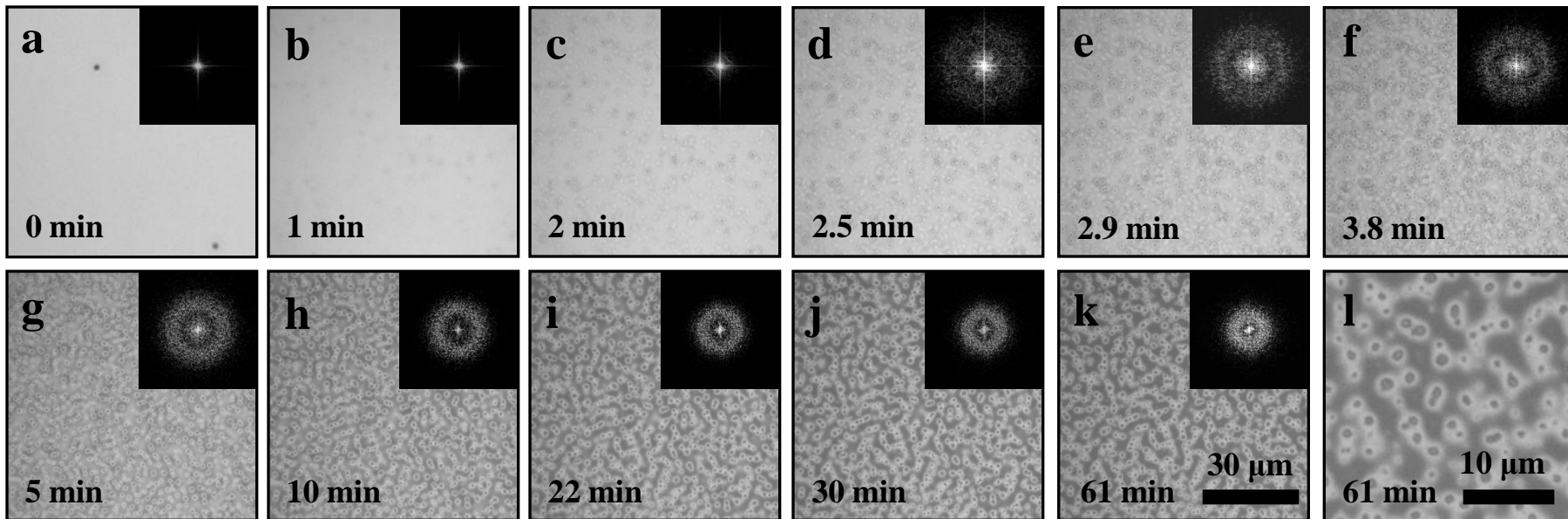
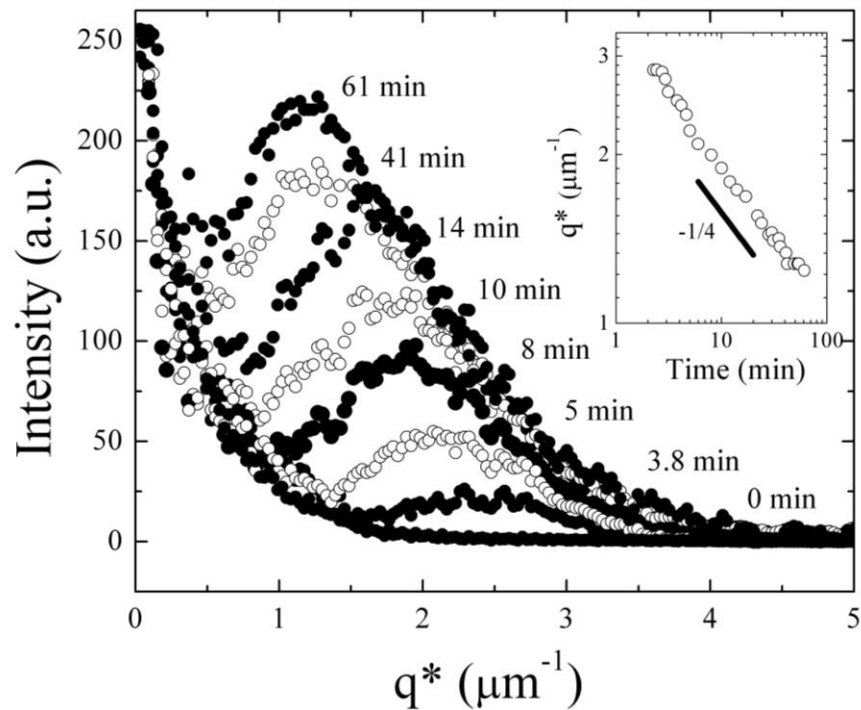
'Spinodal Clustering'

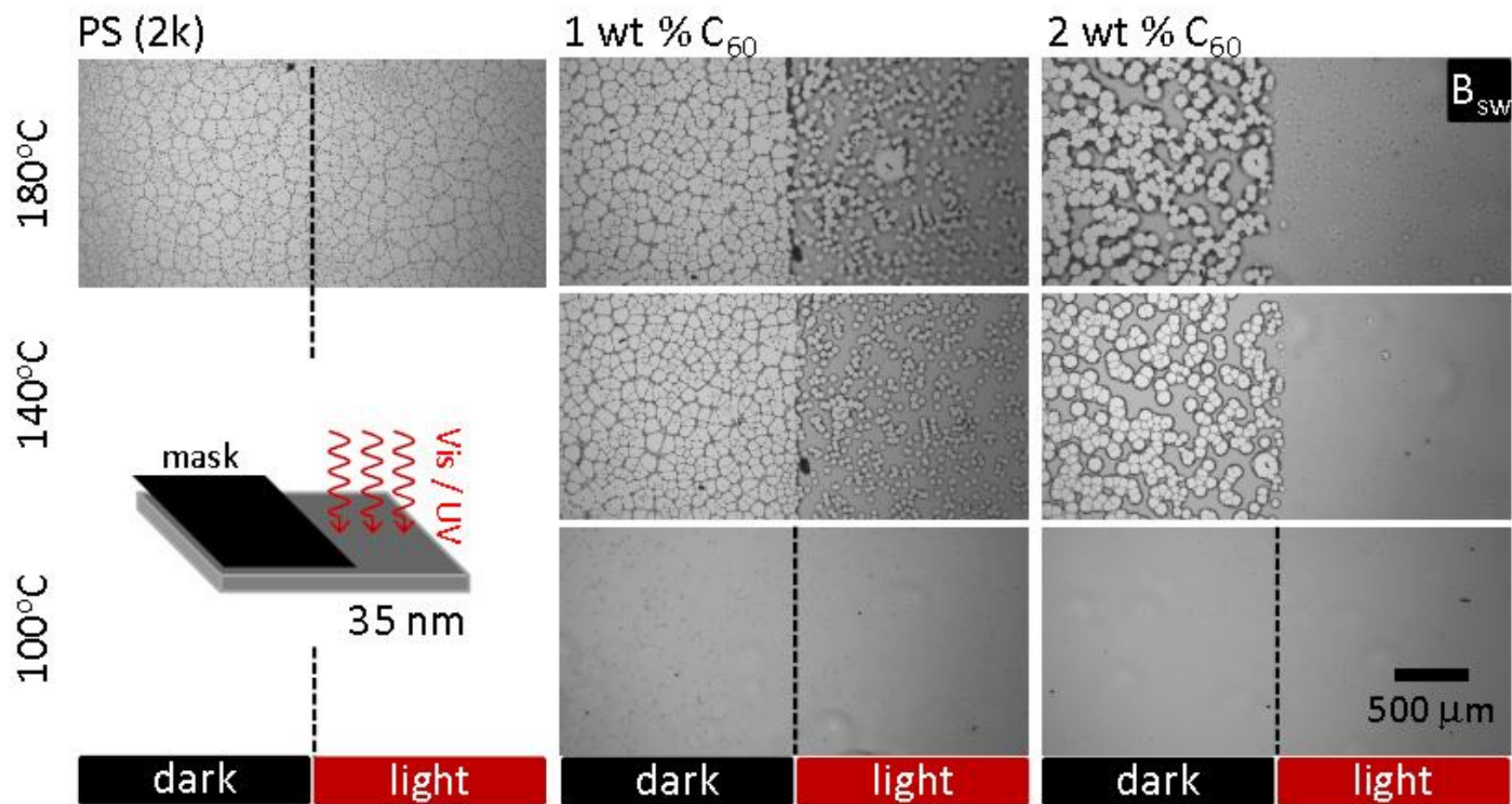
# Coarsening Kinetics

PS+5% $C_{60}$

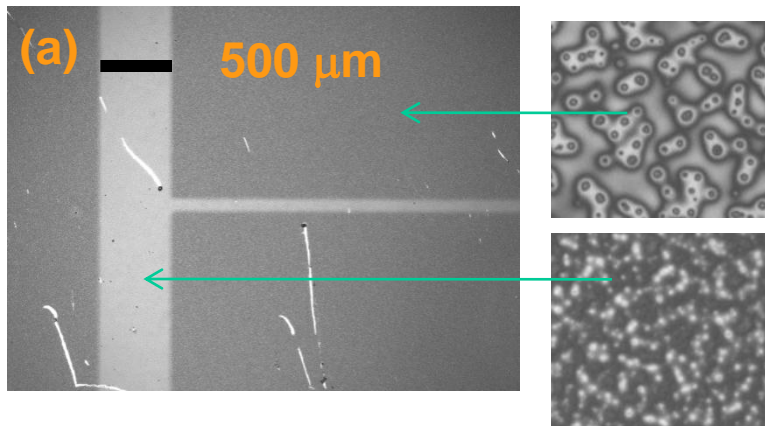
170 °C

$h=160$  nm

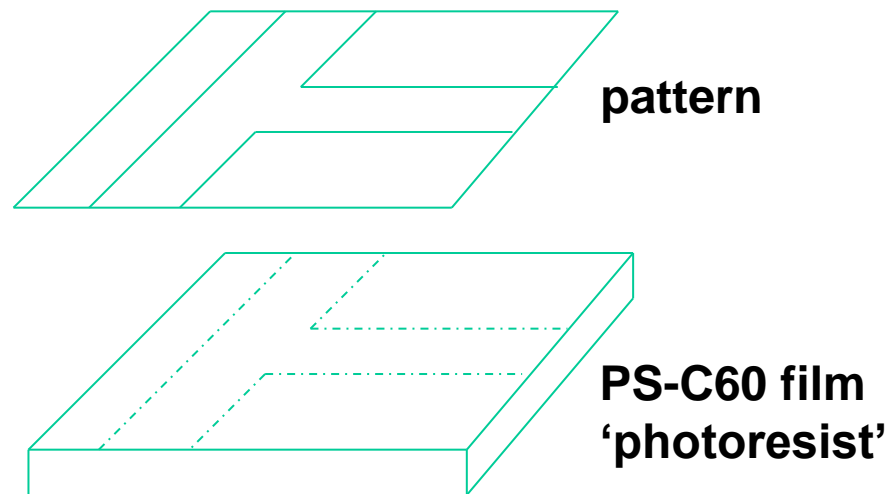
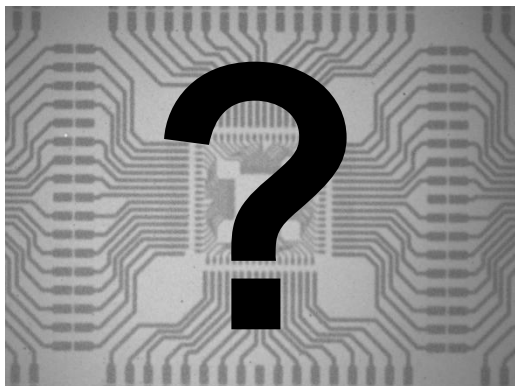
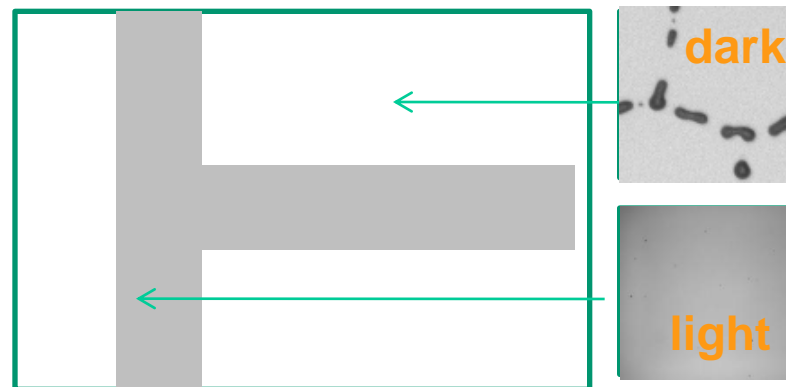




# High Mw

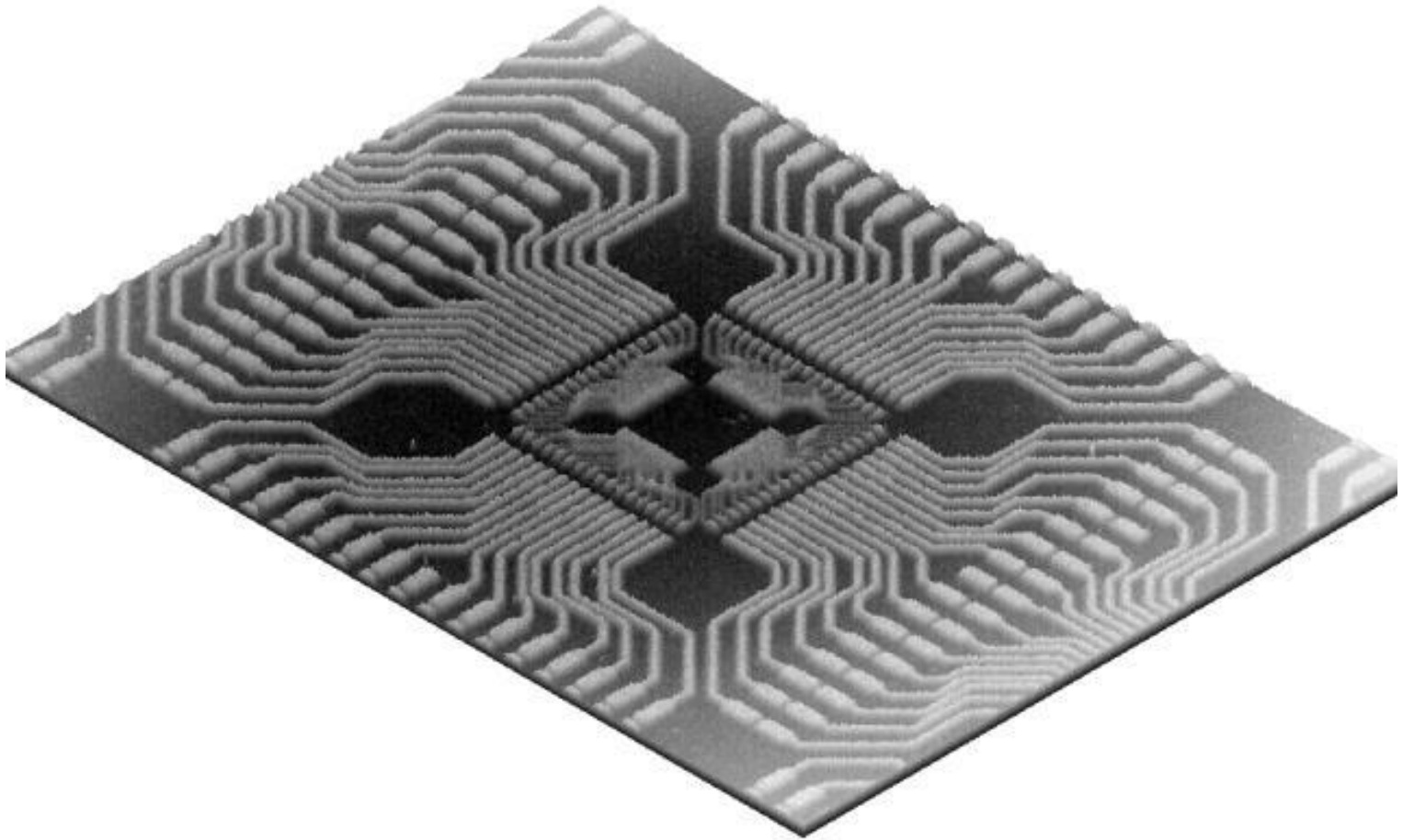


# Low Mw



Annealing above  $T_g$

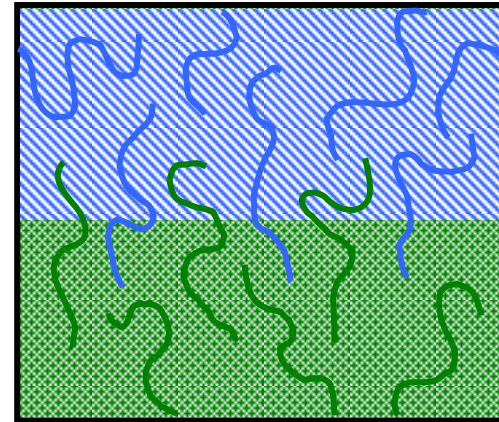
# Coupling of self-assembly & patterning



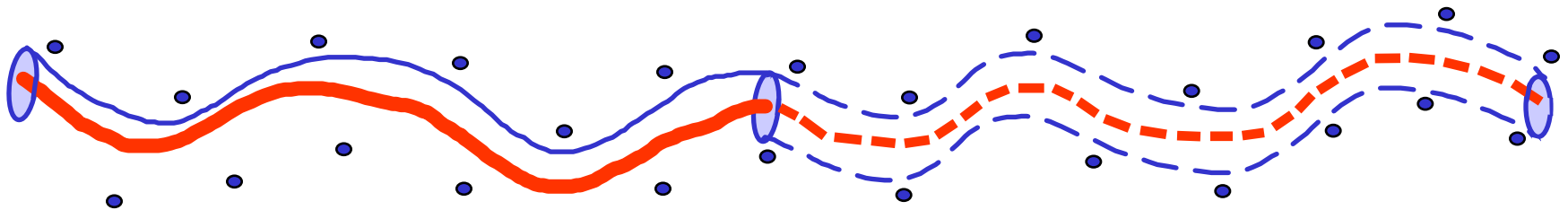
# Summary

## Reflectivity

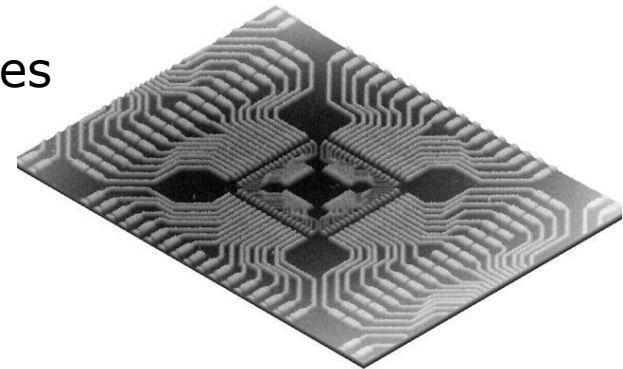
- study and design interfaces



- investigate diffusion mechanisms



- engineer 'functional' surfaces / devices





# Neutrons in soft matter

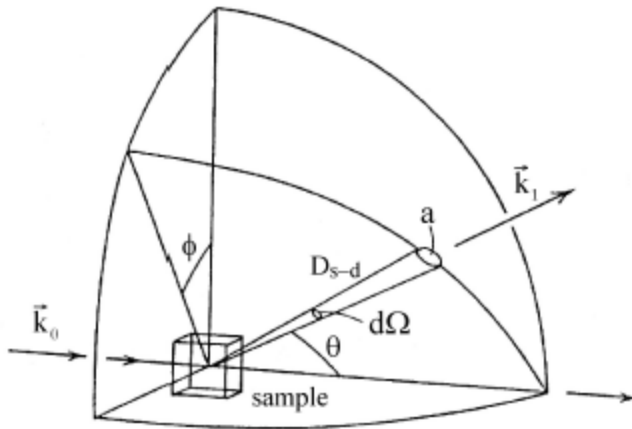
Lecture 2 (II) – Dynamics

João T. Cabral

Department of Chemical Engineering  
Imperial College London

# Scattering theory reminder

## Scattering cross section



$$\frac{d^2\sigma}{d\Omega dE} = \left(\frac{d^2\sigma}{d\Omega dE}\right)_{coh} + \left(\frac{d^2\sigma}{d\Omega dE}\right)_{inc}$$

*coherent incoherent*

$$\left(\frac{d^2\sigma}{d\Omega dE}\right)_{coh} = \frac{1}{2\pi\hbar} \frac{k_1}{k_0} \frac{\sigma_{coh}}{4\pi} \int_{-\infty}^{+\infty} \sum_{i,j} \langle e^{-i\mathbf{q}\cdot\mathbf{R}_i(0)} e^{i\mathbf{q}\cdot\mathbf{R}_j(t)} \rangle e^{-i\omega t} dt$$

$$\left(\frac{d^2\sigma}{d\Omega dE}\right)_{inc} = \frac{1}{2\pi\hbar} \frac{k_1}{k_0} \frac{\sigma_{inc}}{4\pi} \int_{-\infty}^{+\infty} \sum_i \langle e^{-i\mathbf{q}\cdot\mathbf{R}_i(0)} e^{i\mathbf{q}\cdot\mathbf{R}_i(t)} \rangle e^{-i\omega t} dt$$

## Dynamic structure factor

FT  $(t, \omega)$   $\updownarrow$   $S(\mathbf{q}, \omega) = \frac{1}{2\pi\hbar} \int_{-\infty}^{+\infty} I(\mathbf{q}, t) e^{-i\omega t} dt.$

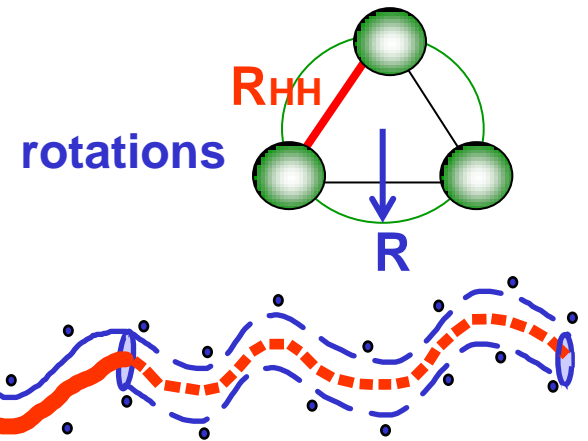
## Intermediate scattering function

FT  $(r, q)$   $\updownarrow$   $I_s(\mathbf{q}, t) = \frac{1}{N} \sum_i \langle e^{-i\mathbf{q}\cdot\mathbf{R}_i(0)} e^{i\mathbf{q}\cdot\mathbf{R}_i(t)} \rangle e^{-i\omega t}.$

## Pair correlation function

$$G(\mathbf{r}, t) = \frac{1}{(2\pi)^3} \int I(\mathbf{q}, t) e^{-i\mathbf{q}\cdot\mathbf{r}} d\mathbf{q}.$$

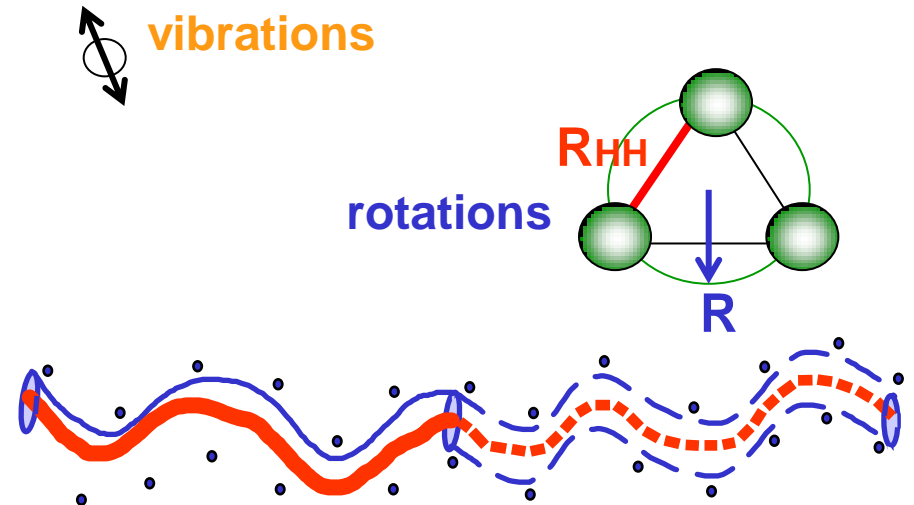
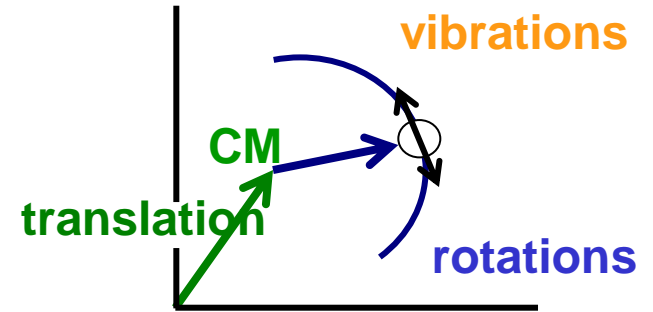
$\updownarrow$  vibrations



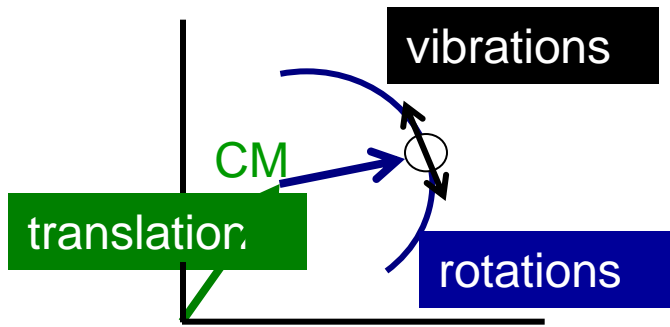
# single-particle dynamics

motion decomposition

$$I_{self}(Q, t) = \frac{1}{N} \sum_i \left\langle e^{iQ \cdot [V(t) - V(0)]} \right\rangle \left\langle e^{iQ \cdot [T(t) - T(0)]} \right\rangle \left\langle e^{iQ \cdot [R(t) - R(0)]} \right\rangle$$



# single-particle tools



motion decomposition

$$I_{self}(Q, t) = \frac{1}{N} \sum_i \langle e^{iQ \cdot [V(t) - V(0)]} \rangle \langle e^{iQ \cdot [T(t) - T(0)]} \rangle \langle e^{iQ \cdot [R(t) - R(0)]} \rangle$$

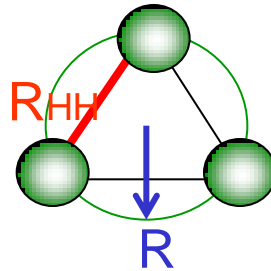
CM translation frozen for polymers  $T \ll T_g$ .

Proton delocalisation DW factor:  $e^{-\frac{1}{3}Q^2 \langle u^2 \rangle}$

relevant proton reorientations: methyl and phenyl rotations about group's axis.

Methyl protons 3-fold jumps

$$R \approx 1.032 \text{ \AA}$$



$$S_{rot}(Q, \omega) = A_0(Q)\delta(\omega) + A_1(Q) \frac{1}{\pi} \frac{3/2\tau}{(3/2\tau)^2 + \omega^2}$$

with

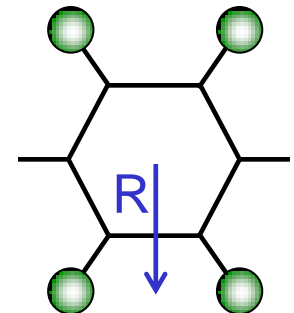
$$\begin{cases} A_0(Q) = \frac{1}{3}[1 + 2j_0(Qr\sqrt{3})] \\ A_1(Q) = 1 - A_0(Q) \end{cases}$$

Phenyl proton 2-fold jumps

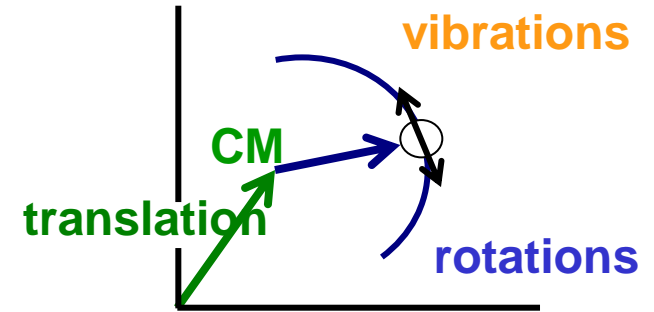
$$R \approx 2.28 \text{ \AA}$$

$$S_{rot}(Q, \omega) = A_0(Q)\delta(\omega) + A_1(Q) \frac{1}{\pi} \frac{2/\tau}{(2/\tau)^2 + \omega^2}$$

with  $A_0(Q) = \frac{1}{2}[1 + j_0(2Qr)]$



# single-particle dynamics



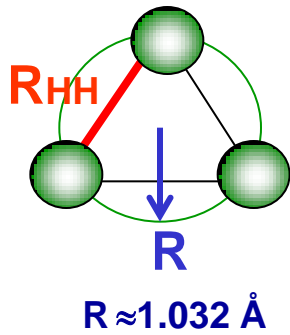
motion decomposition in the glass

CM translation: frozen for polymers  $T \ll T_g$ .

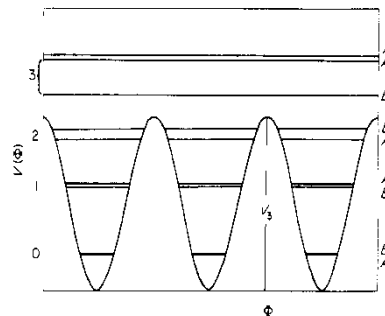
Proton delocalisation: DW factor:  $e^{-\frac{1}{3}Q^2 \langle u^2 \rangle}$

example:

Side group rotations:



3-fold  $\text{CH}_3$  potential



Methyl protons 3-fold jumps

$$S_{\text{rot}}(Q, \omega) = A_0(Q)\delta(\omega) + A_1(Q) \frac{1}{\pi} \frac{3/2\tau}{(3/2\tau)^2 + \omega^2}$$

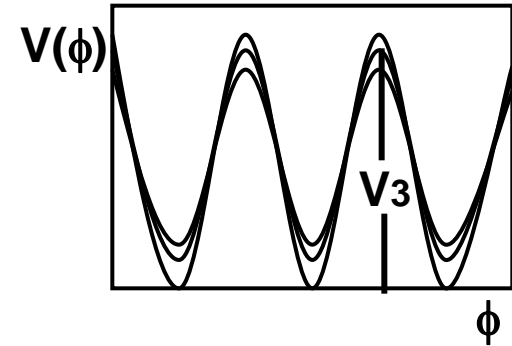
with

$$\begin{cases} A_0(Q) = \frac{1}{3} [1 + 2j_0(Qr\sqrt{3})] \\ A_1(Q) = 1 - A_0(Q) \end{cases}$$

# distribution $\tau_{\text{correlation}}$

glassy polymers: no single relaxation time

variety local environments | intra- molecular  
environments | inter-



(Gaussian) distribution of potential barriers:

$$g(E_i) = \frac{1}{\sigma_E \sqrt{2\pi}} e^{-\frac{(E_i - E_0)^2}{2\sigma_E^2}} \quad \text{if } \Gamma = \Gamma_0 e^{-\frac{E_A}{RT}}$$

(log-Gaussian) distribution of reorientation times:

$$g(\ln \Gamma_i) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{\ln^2(\Gamma_i/\Gamma_0)}{2\sigma^2}} \quad \begin{array}{l} E_0: \text{ average barrier height} \\ \sigma: \text{ distribution width} \end{array}$$

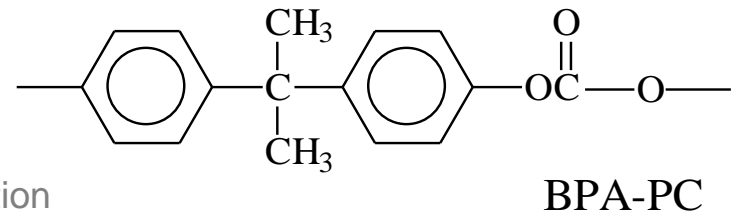
Dynamic structure factor:  $S_{\text{rot}}(Q, \omega) = A_0(Q)\delta(\omega) + A_1(Q) \sum_{i=1}^N g_i L_i(\omega)$

# Case study: Polycarbonates

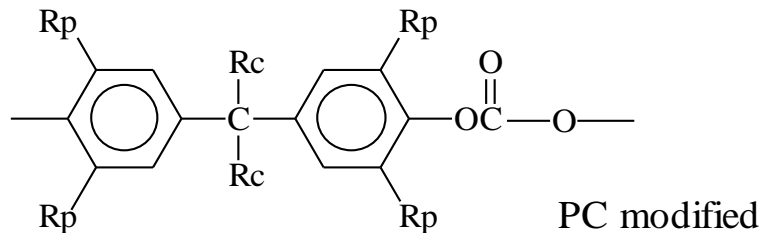
## Bisphenol-A polycarbonate

thermoplastic polymer with remarkable

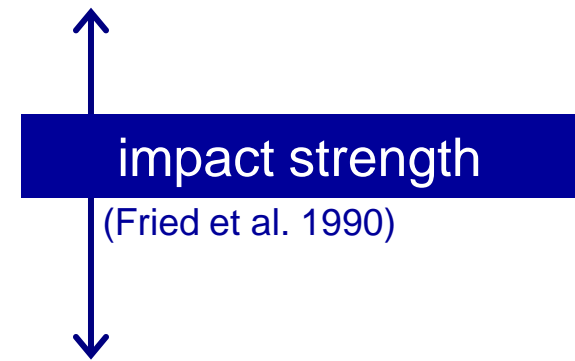
- optical clarity
- **mechanical properties** – high  $T_{\text{glass}}$  transition  
– large impact strength  
– ductility.
- commercial applications



depend strongly on architecture

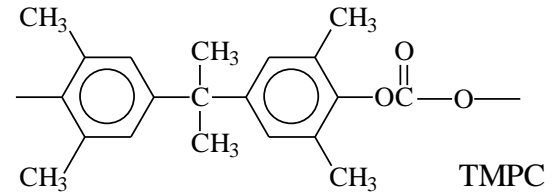
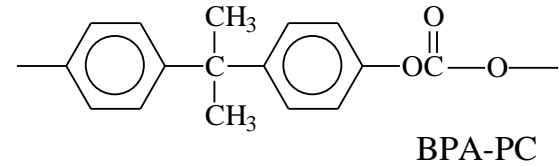
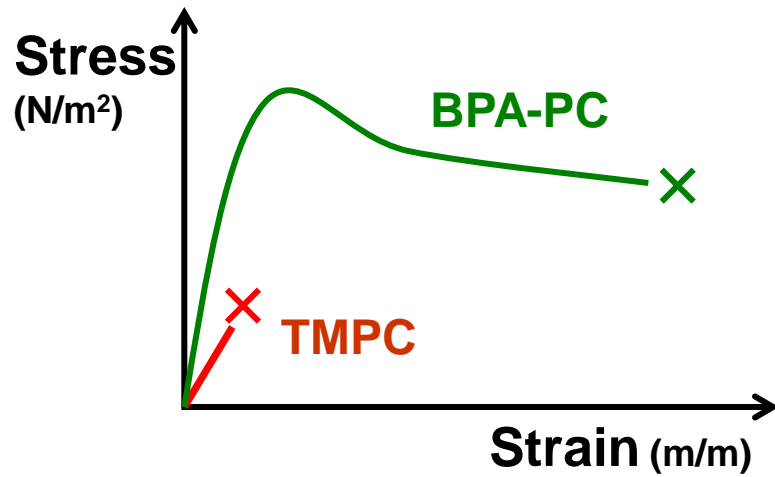


BPA-PC: ~2400 J/m



TMPC: ~70 J/m

# Toughness

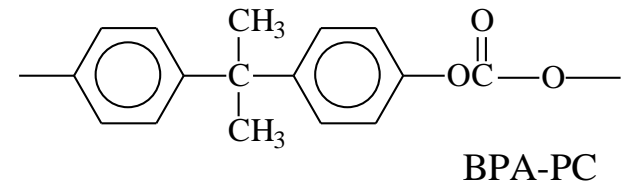




# Polycarbonates

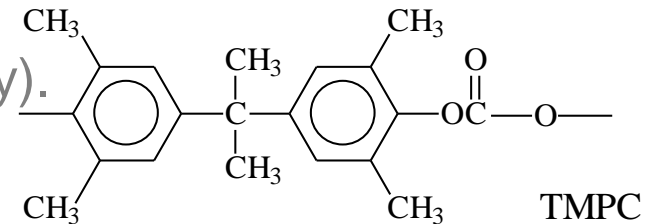
## Glassy BPAPC

**tough** → co-operative phenyl motion,  
involve  $\geq 1$  monomer  
(account for dielectric/mechanical activity).

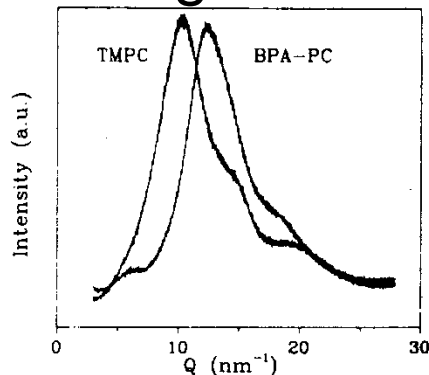


## Glassy TMPC

**most brittle PC** → substituted  $\text{CH}_3$  hinder backbone mobility;  
poor chain packing (large free volume).



## Packing



$$\rho(\text{PC}) = 1.198\text{g/cm}^3$$
$$\rho(\text{TMPC}) = 1.084\text{g/cm}^3$$

QENS:

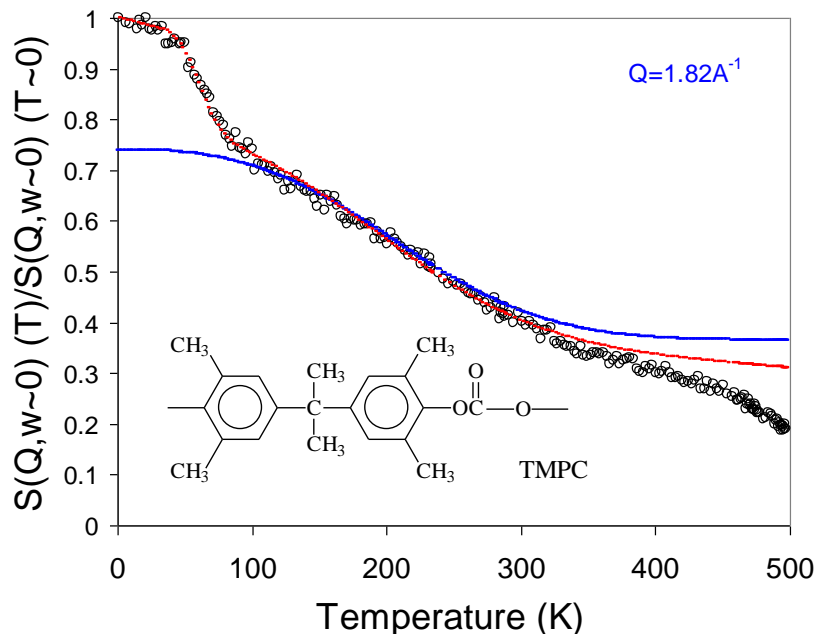
characterise dynamics of local reorientation.

# quantitative window scans

Elastic scans

$$S(Q, \omega \sim 0) = \int_{-\infty}^{+\infty} S(Q, \omega') R(\omega - \omega') d\omega' \Big|_{\omega=0}$$

for a Lorentzian resolution  $S(Q, \omega \sim 0) \approx A_0(Q) + \frac{2}{\pi} [1 - A_0(Q)] \arctan\left(\frac{\Gamma_{\text{res}}}{\Gamma}\right)$



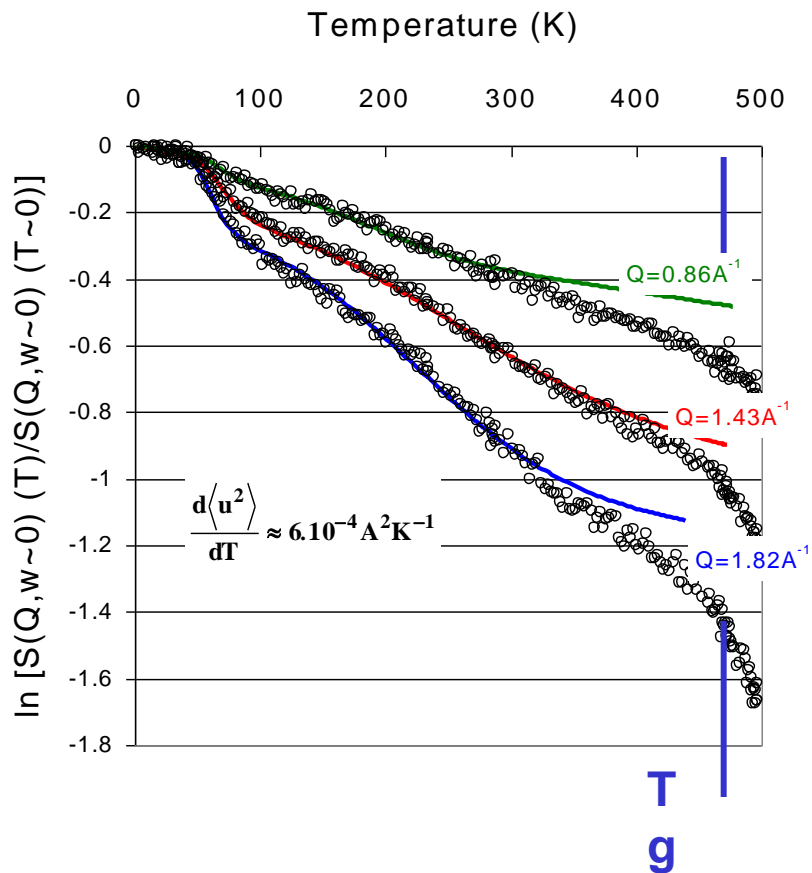
## PARAMETERS

- $\langle u^2 \rangle(T)$  ← initial slope
- distribution:  $E_A$  and  $\sigma$
- $\Gamma_0$

## ASSUMED

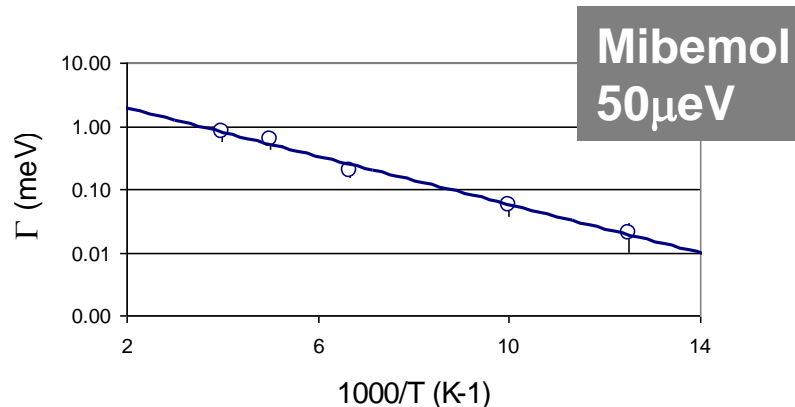
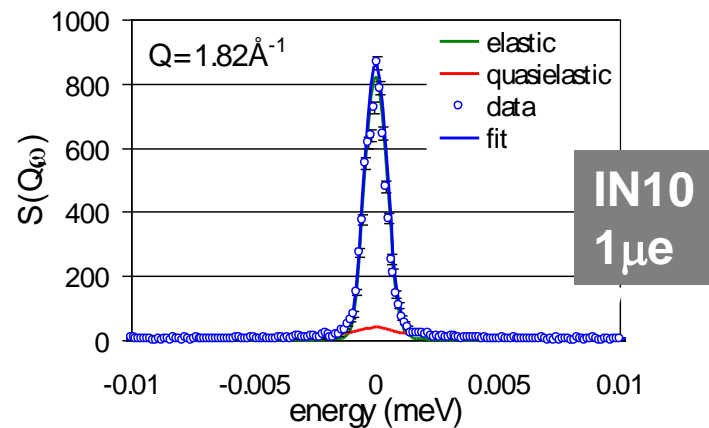
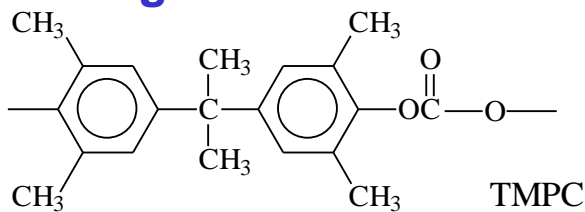
- geometry ← EISF
- activation ansatz:  $\Gamma = \Gamma_0 e^{-\frac{E_A}{RT}}$

# TMPC



Ea1~6  
kJ/mol  $\sigma_1 \sim 1$

Ea2=15  
kJ/mol  $\sigma_1 \sim 5$



# low temperature relaxation

TMPC first relaxation step:

- very low T  $\rightarrow$  low  $E_0$
- rather sharp  $\rightarrow$  narrow

$\rightarrow$  candidate: rotational tunneling

Mathiew equation: inelastic lines

$$S_{\text{rot}}(Q, \omega) = \frac{5 + 4j\omega(Qr)}{9} \delta(\omega) + \frac{2(1 - j\omega(Qr))}{9} [\delta(\omega - \omega_t) + \delta(\omega + \omega_t)]$$

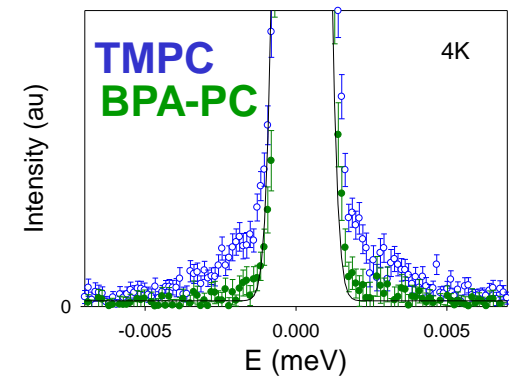
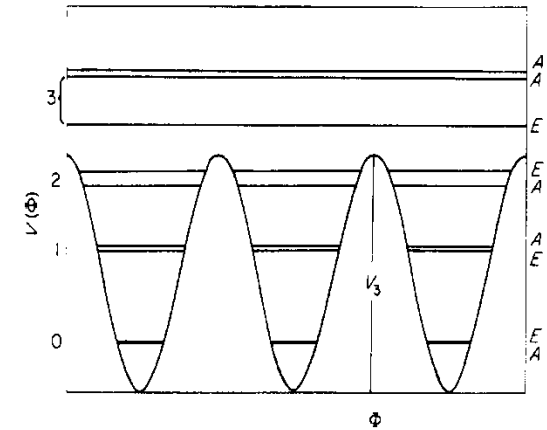
with  $\hbar\omega_t \propto E_A^{3/4} e^{-\sqrt{E_A}}$

Distribution of  $E_A \rightarrow$

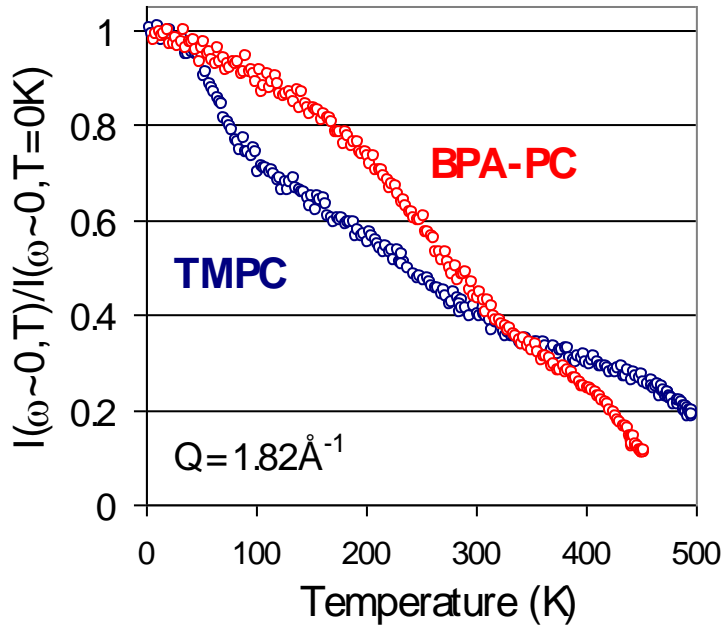
highly asymmetric distribution of  $\omega t$

(Colmenero et al, PRL 1998)

3-fold  $\text{CH}_3$  potential



# BPA-PC

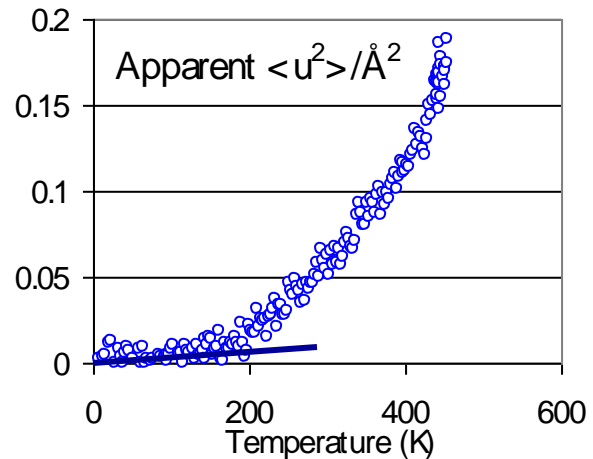
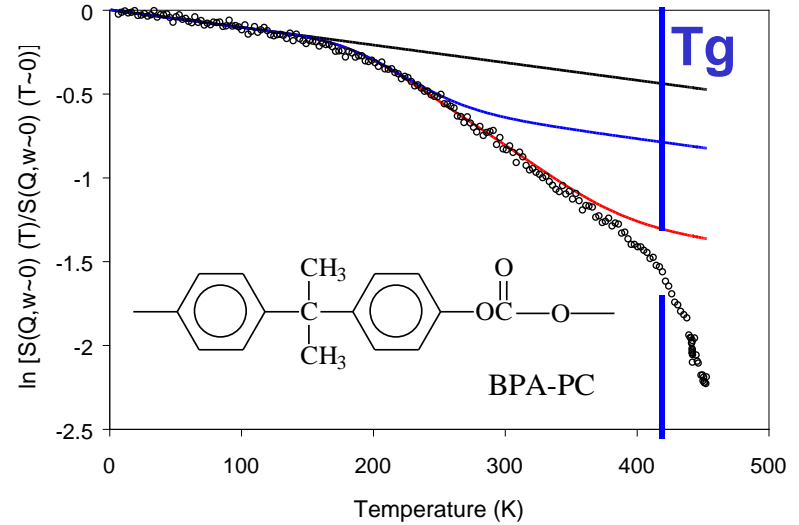


Ephenyl ~ 37  
kJ/mol  $\sigma_1 \sim 6$

Ech3 = 15 kJ/mol  
 $\sigma_1 \sim 3$

compatible with TMPC

(after Spiess et al. 1987)



# Distribution?

Glassy polymers:

backbone chain conformation

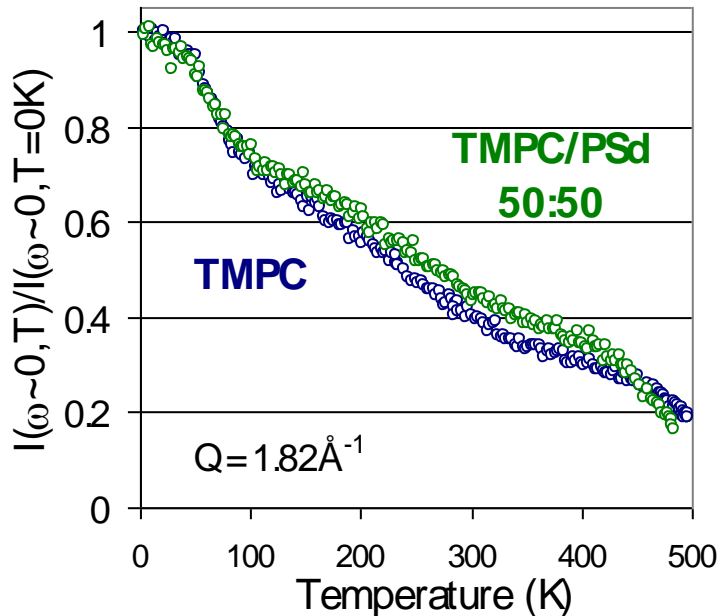
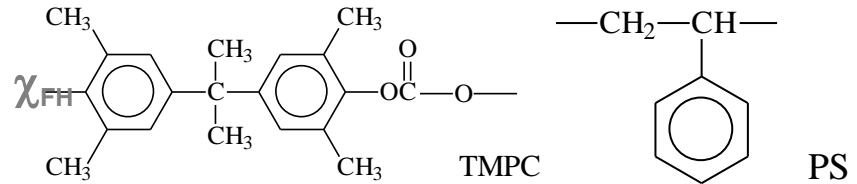
Structural disorder



Blending

≠ inter-molecular potential.  
 ≈ intra-

TMPC: only PC miscible with PS, large  $\chi_{FH}$



1st step: no resolvable perturbation

2nd step: broadened distribution

intramolecular environment

- average  $E_A$
- architectural considerations

intermolecular → limited effect on  $\sigma$

# Conclusions: CASE STUDY

Characterisation local dynamics of PCs:

two architectures → toughest (BPA-PC) & most brittle (TMPC)

**Technique** combined backscattering window scans, inelastic BS & TOF

**TMPC**

exhibits two methyl relaxations of rather different distribution of potentials

**Blending**

affects  $\sigma(E_A)$

**BPA-PC**

Phenyl + methyl

