Imperial College London

Neutrons in soft matter

Lecture 1 - Structure

João T. Cabral Department of Chemical Engineering Imperial College London

Outline

Lecture 1 – Structure & kinetics – SANS Introduction

soft matter & relevance of neutron scattering Single objects: spheres, coils, rods...

Single chain polymer conformation (solution and blends)

Polymer blends: interactions, conformation & dynamics (equilibrium and phase separation)

Lecture 2 – Interfaces and dynamics

Reflectivity and diffusion

Dynamics in soft matter, QENS, BS, Spin-echo



Soft Matter

"molecular systems giving a strong response to very weak command signal"

Condensed matter: states are easily deformed by small external fields, including thermal stresses and thermal fluctuations.

Relevant energy scale comparable with room temperature thermal energy.







Complex fluids: including colloids, polymers, surfactants, foams, gels, liquid crystals, granular and biological materials.





deGennes (1991)

Movie: complex fluids are generally non-Newtonian... and structured



Viscoelasticity?



Neutron scattering is key in soft condensed matter



Neutron scattering is *key* in soft condensed matter





Common soft matter





Common polymers

PE poly(ethylene) $+CH_2-CH_2+n$

PMMA poly(methylmethacrylate)



PVC poly(vinylchloride)



PS poly(styrene) $(-CH_2-CH)_n$

PB poly(butadiene)

$$(-CH_2 - CH - CH_2 -)_n$$

BPA-PC bisphenol-A polycarbonate



Polymer key properties



Glass transition (solid-liquid)

Crystallinity

Interaction parameter χ

 Combine properties to make new materials!

Soft matter: DNA



-0-

Hydrogen Bonding





Soft Matter: membranes, photovoltaics (BHJ)



Scattering theory reminder

∫dω



Scattering cross section

$$\frac{d^{2}\sigma}{d\Omega dE} = \left(\frac{d^{2}\sigma}{d\Omega dE}\right)_{coh} + \left(\frac{d^{2}\sigma}{d\Omega dE}\right)_{inc}$$

$$\left(\frac{d^2\sigma}{d\Omega dE}\right)_{coh} = \frac{1}{2\pi\hbar} \frac{k_1}{k_0} \frac{\sigma_{coh}}{4\pi} \int_{-\infty}^{+\infty} \sum_{i,j} \left\langle e^{-i\mathbf{q}\cdot\mathbf{R}_i(0)} e^{i\mathbf{q}\cdot\mathbf{R}_j(t)} \right\rangle e^{-i\omega t} dt$$
$$\left(\frac{d^2\sigma}{d\Omega dE}\right)_{inc} = \frac{1}{2\pi\hbar} \frac{k_1}{k_0} \frac{\sigma_{inc}}{4\pi} \int_{-\infty}^{+\infty} \sum_i \left\langle e^{-i\mathbf{q}\cdot\mathbf{R}_i(0)} e^{i\mathbf{q}\cdot\mathbf{R}_i(t)} \right\rangle e^{-i\omega t} dt$$

Intermediate scattering function

FT
$$(\mathbf{\Gamma}, \mathbf{Q})$$
 $\mathbf{I}_{s}(\mathbf{q}, t) = \frac{1}{N} \sum_{i} \left\langle e^{-i\mathbf{q}\cdot\mathbf{R}_{\mathbf{f}}(0)} e^{i\mathbf{q}\cdot\mathbf{R}_{\mathbf{f}}(t)} \right\rangle e^{-i\omega t}.$

Pair correlation function $G(\mathbf{r},t) = \frac{1}{(2\pi)^3} \int I(\mathbf{q},t) e^{-i\mathbf{q}\cdot\mathbf{r}} d\mathbf{q}.$ **Elastic structure factor**

$$\int_{-\infty}^{+\infty} S(\mathbf{q},\omega)|_{\mathbf{q}=Const.} \, d\omega = S(\mathbf{q})$$

 $S(\mathbf{q}) \qquad S(q) = Nz^2 P(q) + N^2 z^2 Q(q)$ Form factor $P(q) = \frac{1}{z^2} \sum_{i=1}^{z} \sum_{j=1}^{z} \langle e^{-i\mathbf{q}\cdot\mathbf{r}_{ij}} \rangle$ Structure factor $Q(q) = \frac{1}{z^2} \sum_{i_p=1}^{z} \sum_{j_q=1}^{z} \langle e^{-i\mathbf{q}\cdot\mathbf{r}_{i_pj_q}} \rangle$

Reminder: Fourier Transforms

$$H(f) = \int_{-\infty}^{\infty} h(t) e^{2\pi i f t} dt$$

$$h(t) = \int_{-\infty}^{\infty} H(t) e^{-2\pi i f t} df$$

Fourier
transform:

$$\Phi(H(f),t) = h(t)$$

$$\Phi^{-1}(h(t),f) = H(f)$$



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Reminder: Fourier Transforms

Real space



Reciprocal space

SMALL-ANGLE NEUTRON SCATTERING



Absolute scattering intensity [cm⁻¹]

$$\frac{\partial \sigma}{\partial \Omega}(Q) = N_{p} V_{p}^{2} (\Delta \delta)^{2} P(Q) S(Q) + B_{inc} \text{ incoherent background}$$

$$\int V_{p} V_{p}^{2} (\Delta \delta)^{2} P(Q) S(Q) + B_{inc} \text{ incoherent background}$$

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$$\int V_{p} V_{p}$$

Relationship between $q \lambda \theta$ and d



small q ~ large d[large q ~ small d]small λ ~ large q ~ small d[large λ ~ small q ~ large d]

Radiation	Wavelength	
light	~ 500 nm	Ångstrom:
X-rays	~ 1 Å	$1 \text{ Å} = 10^{-10} \text{ m}$
neutrons	~ 5 Å	1 nm = 10 Å

Bottom line:

radiation of small wavelength λ can 'see' smaller sample features d (provided that contrast is sufficient).

Example: crystalline structure of polymer



$$q = \frac{4\pi}{\lambda} \sin(\theta/2)$$

Radiation	Wavelength	Scattering angle
X-rays	~ 1 Å	θ ~ 0.27 degrees
neutrons	~ 5 Å	sin(θ/2) ~ 0.013, θ ~ 0.7 degrees
light	~ 500 nm	sin(θ/2) > 1 impossible!

the smallest dimension probed by wavelength λ corresponds to largest angle θ =180 degrees (backscattering). For light d_{min} ~ 0.25 µm, for X-rays or neutrons d_{min} ~ 0.5 to 2.5 Å. In typical experiments, scattering angles range from 0.1 < θ < 70 degrees

A brief history of polymers



<u>1920</u> Staudinger proposed plastics are long molecules with covalent bonds.

<u>1940</u> Kuhn established they are usually flexible

<u>1953</u> Flory described the shape and size of individual polymer molecules in solutions and melts.

1967 Edwards – polymer molecules in rubbers and glasses are trapped by neighbours in a "tube". Chain confirmation ↔ trajectories quantum particles.

<u>1971</u> de Gennes – molecules in tube move like snakes and eventually escape

Main factors governing polymer physical properties :

- **1.** Number of monomer units in the chain, N is large N >> 1. synthetic polymers $N \approx 10^2 - 10^4$ DNA $N \approx 10^9 - 10^{10}$
- 2. Monomer units are connected in the chain. ⇒ no freedom of independent motion (unlike systems of disconnected particles, e.g. low molecular gases and liquids). ⇒ Polymer systems are poor in entropy.
- 3. Polymer chains are generally *flexible*.







Form factor: P

Self-correlations



(analytical solutions for common shapes)

Multiple lengthscales



2π/q₃ **2π/q**₂



scattering spectrum corresponds to different "magnifications", thus several approximations may be relevant

Scattering from a sphere



Scattering from a (tiny) sphere



Scattering from a random coil









Scattering from a random coil



Debye form factor
$$g_D(x) = \frac{2}{x^2} \left(x - 1 + e^{-x}\right)$$
$$x \equiv q^2 R_g^2$$
$$R_g^2 = \frac{Na^2}{6}$$



Polydisperse random coils



Polydisperse debye form factor

$$\begin{split} g_D(x) &= \frac{2}{(1+1/h)x^2} \left[\left(1 + \frac{x}{h} \right)^{-h} - 1 + x \right] \\ x &\equiv q^2 \langle R_g^2 \rangle_n \equiv \frac{q^2 \langle R_g^2 \rangle_z}{1+2/h} \\ h &= \left(\frac{M_w}{M_n} - 1 \right)^{-1} \end{split}$$

(normalised to PDI)

a=10 Å N=100 → Rg≈4nm N_w=100, N_w/N_n=5

A polydispersity model

(Schultz-Zimm)

$$\begin{split} p(M) &= \frac{h^h}{\Gamma(h)} \left(\frac{M}{M_n}\right)^h e^{-h(\frac{M}{M_n})} \\ h &= (\frac{M_w}{M_n} - 1)^{-1} \end{split}$$



Number average $M_n = \int p(M) M dM$

Weight average $M_w = \frac{\int p(M)M^2 dM}{\int p(M)M dM} \equiv \frac{\langle M \rangle_2}{\langle M \rangle_1}$

z-average
$$M_z = \frac{\int p(M)M^3 dM}{\int p(M)M^2 dM} \equiv \frac{\langle M \rangle_3}{\langle M \rangle_2}$$

Useful form factors



http://www.ncnr.nist.gov/resources/

Structure factor: S



Interference between radiation scattered by distinct objects

$$S(Q) = 1 + \frac{4\pi N_p}{QV} \int_0^\infty [g(r) - 1] r \operatorname{Sin}(Qr) dr$$
$$G(r) = \frac{4\pi N_p r^2}{V} g(r)$$

Radial distribution function, provides information about their relative position



Ordered structure 'crystal'



Disordered structure



Interactions: Polymers in solution and melt

1953 Flory described the shape and size of individual polymer molecules in solutions and melts.





Ideal chains occur in θ -solutions (ie, neutral solvent, $A_2=0$) or in melt. Chains expand or contract depending on interactions: A_2 (Second Virial coeff, for solutions) or χ (for polymer mixtures)

Polymer miscibility (1)

Flory-Huggins lattice



Thermodynamics $\Delta G_m = \Delta H_m - T\Delta S_m$

Combinatorial entropy

$$-\frac{\Delta S}{R} = n_A \ln \phi_A + n_B \ln \phi_B \qquad \Omega \quad \text{Boltzmann law}$$

Enthalpy $\Delta H_m = K_B T \phi_A \phi_B \chi_{AB}$

$$\frac{\Delta G_m}{K_B T} = \frac{\phi_A}{N_A} \ln \phi_A + \frac{\phi_B}{N_B} \ln \phi_B + \phi_A \phi_B \chi_{AB}$$

Combinatorial entropy

Enthalpy

 $\chi < 0$ mixing occurs $\forall T, \phi$ $\chi > 0$ $\Delta G_m = \Delta H_m - T \Delta S_m$ only at high T

Polymer miscibility (2)





Isotopic polymer mixture

$$\frac{\partial \sigma}{\partial \Omega} \langle q \rangle = \langle b_D - b_R \rangle^2 S_{DD} \langle q \rangle = \langle b_D - b_R \rangle^2 \phi \langle 1 - \phi \rangle N \epsilon^2 P \langle q \rangle$$

$$\frac{1}{V} \frac{d\sigma}{d\Omega} \langle q \rangle \Big|_{eqe^{-1}} = \langle b_D - b_R \rangle^2 \phi \langle 1 - \phi \rangle \langle M \rangle_{e_0} \frac{\rho N_A}{m \epsilon^2} P \langle q \rangle$$

$$= \langle b_D - b_R \rangle^2 \phi \langle 1 - \phi \rangle \langle M \rangle_{e_0} \frac{\langle \Delta \rho \rangle^2}{\rho N_A} P \langle q \rangle$$

Approximations: Guinier & Zimm



Interacting polymer mixtures

$$\begin{aligned} \frac{1}{V d\Omega} \left(g \right) \Big|_{z_{\text{PR}} = 4} &= N_A \left(\frac{b_1}{v_1} - \frac{b_2}{v_2} \right)^2 S(g) \\ \frac{1}{S(q)} &= \frac{1}{S_1(g)} + \frac{1}{S_2(g)} - 2\frac{\tilde{\chi}_{12}}{v_0} \\ S_4(g) &= \phi_4 v_4 \langle N_4 \rangle_{\text{re}} \langle g_D(R_g^2 g^2) \rangle_{\text{so}} \end{aligned}$$

$$\begin{aligned} \text{Zimm} \quad S_4(g) \approx \phi_4 v_4 \langle N_4 \rangle_{\text{so}} \left(1 - \frac{1}{3} \langle R_g^2 \rangle_x g^2 \right) \\ &= \frac{1}{S(g)} = \frac{1}{S(0)} \left[1 + \frac{1}{3} R_{egg}^2 g^2 \right] \end{aligned} \text{ where } \quad R_{egg}^2 = \left(\frac{\langle R_{g4}^2 \rangle_x}{\phi_1 v_1 \langle N_1 \rangle_{\text{so}}} + \frac{\langle R_{g2}^2 \rangle_x}{\phi_2 v_2 \langle N_2 \rangle_{\text{so}}} \right) S(0) \\ &= \frac{1}{S(0)} = \frac{1}{\phi_1 v_1 \langle N_1 \rangle_{\text{so}}} + \frac{1}{\phi_2 v_2 \langle N_2 \rangle_{\text{so}}} - 2\frac{\tilde{\chi}_{12}}{v_0} \end{aligned}$$

NO

Random Phase Approximation

RPA (de Gennes, 1979):

$$\frac{1}{S(q)} = \frac{1}{S_1(q)} + \frac{1}{S_2(q)} - 2\frac{\tilde{\chi}_{12}}{v_o} \qquad 1/S(0) = \frac{1}{S_1(q)} + \frac{1}{S_2(q)} - \frac{1}{S_1(q)} + \frac{1}{S$$

Ornstein-Zenike:

At low angle (qRg << 1),

$$S(q) = \frac{S(0)}{1 + \xi^2 q^2}$$

G"

Linear in the Zimm representation as:

$$\frac{1/S(q) = 1/S(0) + Aq^{2}}{\frac{1}{S(q)} = 2(\chi_{S} - \chi_{F}) + \frac{\xi^{2}}{S(0)}q^{2}}$$

 T_s determined by extrapolation of G" to 0



Equilibrium: SANS

TMPC/PSd 50/50

1-phase scattering

Orstein-Zernike

Interaction χFH



Random Phase Approximation

$$\frac{1}{S_{AB}(q)} = \frac{1}{\phi_A v_A \langle N_A \rangle_n \langle g_D(q)_A \rangle_w} + \frac{1}{\phi_B v_B \langle N_B \rangle_n \langle g_D(q)_B \rangle_w} - 2\frac{\tilde{\chi}_{AB}}{v_o}$$

$$\frac{1}{\mathrm{S}(\mathbf{q})} = 2(\boldsymbol{\chi}_{\mathrm{S}} - \boldsymbol{\chi}_{\mathrm{F}}) + \frac{\boldsymbol{\xi}^{2}}{\mathrm{S}(\mathbf{0})}\mathbf{q}^{2}$$
$$\boldsymbol{\xi}^{2} = \frac{v_{0}}{36(\boldsymbol{\tilde{\chi}}_{\mathrm{S}} - \boldsymbol{\tilde{\chi}}_{\mathrm{AB}})} \left(\frac{\langle N_{A} \rangle_{z}}{\langle N_{A} \rangle_{w}} \frac{a_{A}^{2}}{\boldsymbol{\phi}_{A} v_{A}} + \frac{\langle N_{B} \rangle_{z}}{\langle N_{B} \rangle_{w}} \frac{a_{B}^{2}}{\boldsymbol{\phi}_{B} v_{B}}\right)$$

Equilibrium: Kratky asymptote





Kratky asymptote and segment length

$$S(q) \approx \frac{12\phi_1\phi_2}{q^2} \frac{v_o}{\hat{a}^2}$$

$$\frac{\hat{a}^2}{v_0} \equiv \phi 1 \phi 2 \left(\frac{a_1^2}{\phi_1 v_1} + \frac{a_2^2}{\phi_2 v_2} \right)$$

Non-equilibrium: Fluctuations & Phase separation



Phase separation: spinodal decomposition



Phase separation



 Λ_m : characteristic length of phase separation ~10s-100s nm





Nucleation & Growth





Comparatively SLOW, since activated process

Nucleation & Growth





Energy balance $\Delta G(r) = -\frac{4\pi}{3}r^{3}\Delta g + 4\pi r^{2}\sigma$ bulk interface

→ Critical droplet

$$\Delta G_{barrier} = \frac{16\pi\sigma^3}{3\Delta g^2}$$

- $r_c = 2\sigma/\Delta g$

Spinodal decomposition



Spinodal decomposition



Cahn-Hilliard Cook theory

$$S(q,t) = S_T(q) + [S(q,0) - S_T(q)] \exp^{-2\tau_q^{-1}t}$$

$$\tau_q^{-1} = q^2 \Lambda(q) S_T^{-1}(q)$$

$$\Lambda(q) = Wa^2 \phi(1-\phi)g_D(q)$$

$$\Lambda(q) = \frac{Wd^2}{Z} \phi(1-\phi)g_D(q)$$

(Rouse Model)

(Reptation Model)





Spatially resolved "rheology"





Opportunities & recent developments



Flow fields (and microfluidics?)

CTAC/ Pentanol/Water

















Optical micrographs: mixture of SDS / Octanol / Brine q=1 ml/h; extension rate of 10^4 s^{-1} .

Conclusion: *Microstructure alignment & orientation*



Spatio-temporal mapping:



(Pollack, Austin, etc)

0.20

Spatio-temporal mapping:

Entangled polymers under flow



Neutron-Mapping Polymer Flow: Scattering, Flow Visualization, and Molecular Theory J. Bent, et al. Science 301, 1691 (2003):

Soft colloids under flow





Forster et al. PNAS 2011



PRL 2010

'Spinodal Clustering'

Summary



③ Form and structure

Mixtures & design

