NEUTRONS FOR SCIENCE

Neutron Magnetic Scattering

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Lecture 1: The plan

- How does the neutron interact with magnetism?
- The fundamental rule of neutron magnetic scattering
- Elastic scattering, and how to understand it
- Magnetic form factors

Conclusions:

- Learn your Fourier transforms
- Get used to using vectors
- Neutrons only ever see the components of the magnetization, **M**, that are *perpendicular* to the scattering vector, **Q**
- Magnetization is distributed in space, therefore the magnetic scattering has a *form factor*

How does the neutron interact with magnetism?

Neutrons have no charge, but they do have a magnetic moment.

The magnetic moment is given by the neutron's spin angular momentum:

$$-\gamma \mu_N \hat{\boldsymbol{\sigma}}$$

where:

- γ is a constant (=1.913)
- μ_N is the nuclear magneton
- $\hat{\boldsymbol{\sigma}}$ is the quantum mechanical Pauli spin operator

Normally refer to it as a spin-1/2 particle

How does the neutron interact with magnetism?

Through the cross-section!

$$\frac{d^2\sigma}{d\Omega \cdot dE} = \frac{k'}{k}\left(\frac{m_n}{2\pi\hbar^2}\right)^2 \sum_{\zeta,s} p_\zeta p_s \sum_{\zeta',s'} \left|\langle \mathbf{k}',s',\zeta' | \hat{V}(\mathbf{r}) | \mathbf{k},s,\zeta \rangle\right|^2 \delta\left(\hbar\omega + E_\zeta - E_{\zeta'}\right)$$

Probabilities of initial target state and neutron spin

Conservation of energy

The *matrix element*, which contains all the physics.

$\hat{V}(\mathbf{r})$ is the *pseudopotential*, which for magnetism is given by:

$$\hat{V}_m(\mathbf{r}) = -\gamma\mu_N \hat{\boldsymbol{\sigma}} \cdot \mathbf{B}(\mathbf{r})$$

where $\mathbf{B}(\mathbf{r})$ is the magnetic induction.

G. L. Squires, *Introduction to the theory of thermal neutron scattering*, Dover Publications, New York, 1978

W. Marshall and S. W. Lovesey, *Theory of thermal neutron scattering*, Oxford University Press, Oxford, 1971

S. W. Lovesey, *Theory of neutron scattering from condensed matter*, Oxford University Press, Oxford, 1986

Elastic scattering

If the incident neutron energy = the final neutron energy, the scattering is *elastic*.

$$\frac{d\sigma}{d\Omega} = \left(\frac{m_n}{2\pi\hbar^2}\right)^2 \sum_{s'} p_s \left| \langle \boldsymbol{k}', s' | \hat{V}(\mathbf{r}) | \boldsymbol{k}, s \rangle \right|^2$$

Forget about the spins for the moment (*unpolarized* neutron scattering) and integrate over all **r**:

$$\langle \boldsymbol{k}' | \hat{V}(\mathbf{r}) | \boldsymbol{k} \rangle = \int \hat{V}(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r}$$

Momentum transfer $\mathbf{Q} = \boldsymbol{k} - \boldsymbol{k}'$

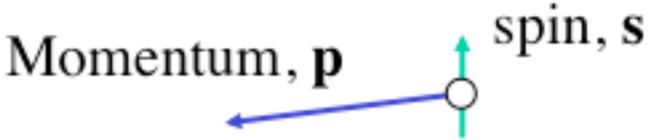
The elastic cross-section is then directly proportional to the *Fourier transform squared* of the potential. Neutron scattering thus works in Fourier space, otherwise called *reciprocal space*.

Learn about Fourier transforms!

Evaluating $\langle \boldsymbol{k}' | \hat{V}_m(\mathbf{r}) | \boldsymbol{k} \rangle$

(Squires, chapter 7)

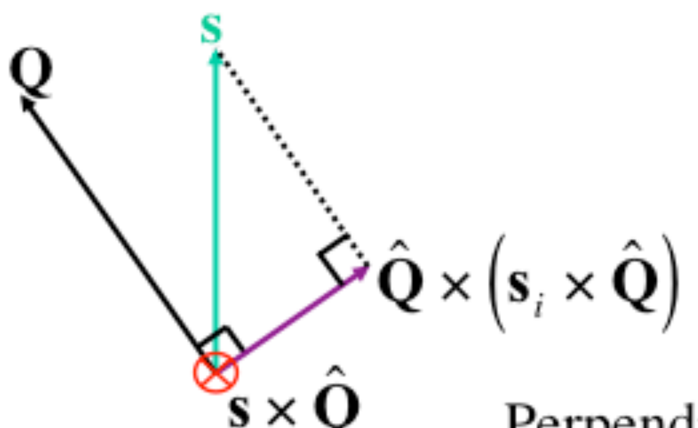
Magnetism is caused by unpaired electrons or movement of charge.



$$\langle \boldsymbol{k}' | \hat{V}_m(\mathbf{r}_i) | \boldsymbol{k} \rangle =$$

Spin:

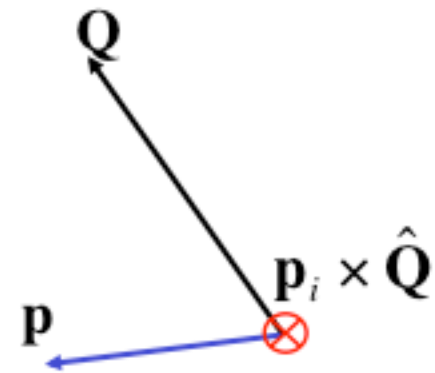
$$4\pi \exp(i\mathbf{Q}\cdot \mathbf{r}_i)\left\{\hat{\mathbf{Q}} \times \left(\mathbf{s}_i \times \hat{\mathbf{Q}}\right)\right\}$$



Perpendicular to **Q**

Movement / Orbital

$$\frac{4\pi i}{\hbar Q} \exp(i\mathbf{Q}\cdot \mathbf{r}_i)\left(\mathbf{p}_i \times \hat{\mathbf{Q}}\right)$$



The fundamental rule of neutron magnetic scattering

Neutrons *only ever* see the components of the magnetization that are *perpendicular* to the scattering vector!

The fundamental rule of neutron magnetic scattering

Taking elastic scattering again:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{m_n}{2\pi\hbar^2}\right)^2 \left|\langle \boldsymbol{k}' | \hat{V}(\mathbf{r}) | \boldsymbol{k} \rangle\right|^2$$

Neutron scattering therefore probes the components of the sample magnetization that are *perpendicular* to the neutron's momentum transfer, **Q**.

$$\int V_m(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r} = \mathbf{M}_\perp(\mathbf{Q})$$

and $$\frac{d\sigma_{magnetic}}{d\Omega} = \langle \mathbf{M}^*_\perp(\mathbf{Q}) \rangle \langle \mathbf{M}_\perp(\mathbf{Q}) \rangle$$

Neutron scattering measures the *correlations* in magnetization, i.e. the influence a magnetic moment has on its neighbours.

It is capable of doing this over all length scales, limited only by wavelength.

Learn your Fourier transforms!

and

Learn and understand the convolution theorem!

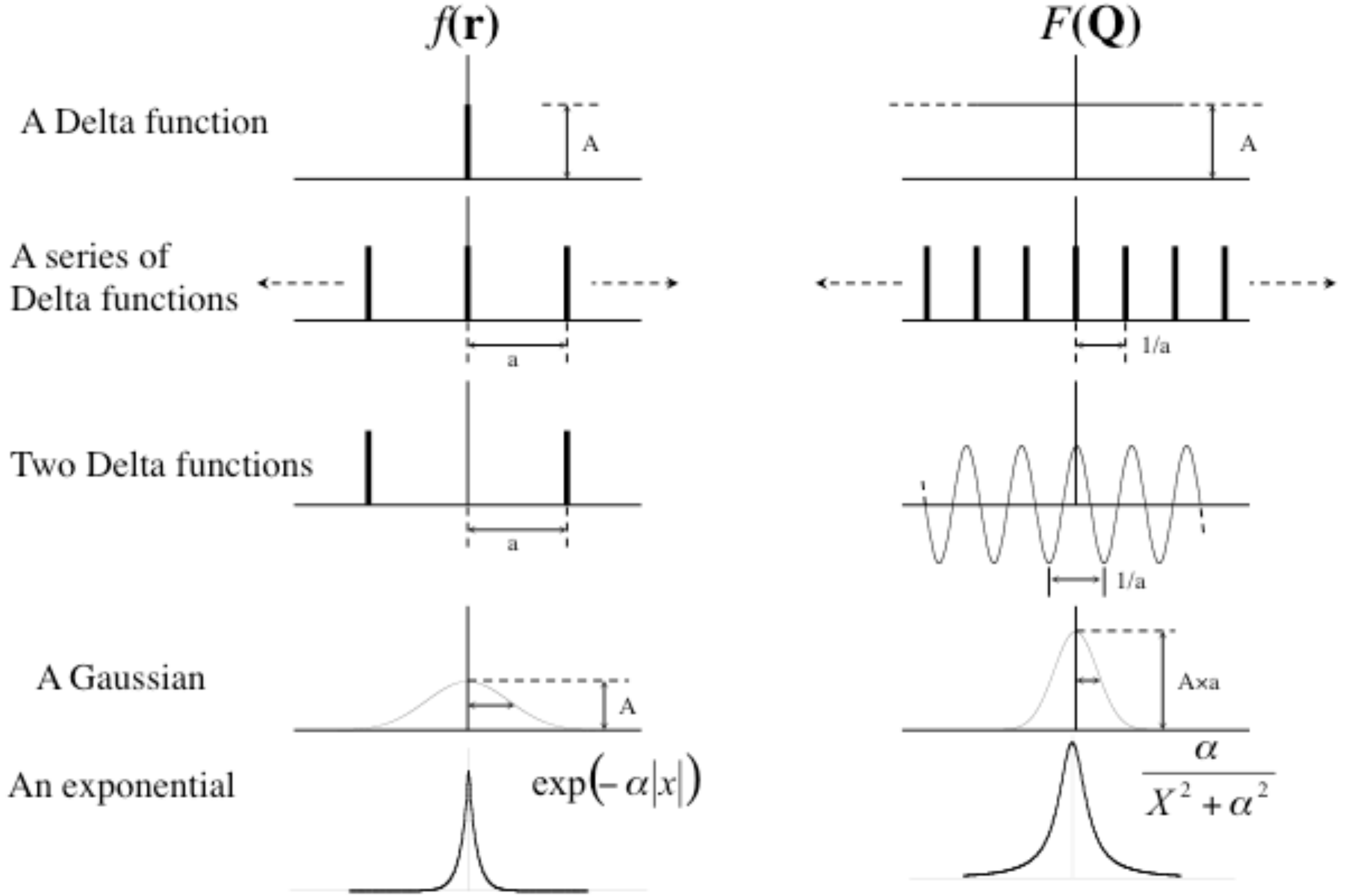
$$f(r) \otimes g(r) = \int f(x) g(r-x) \cdot dx$$

$$\Im(f(r)) = F(q)$$

$$\Im(g(r)) = G(q)$$

$$\Im(f(r) \otimes g(r)) = F(q) \times G(q)$$

Fourier Transforms $F(\mathbf{Q}) = \int f(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r}$



Elastic scattering

$$\frac{d\sigma}{d\Omega} = \left(\frac{m_n}{2\pi\hbar^2}\right)^2 \sum_{\zeta',s'} p_s \left| \langle \boldsymbol{k}', s' | \hat{V}(\mathbf{r}) | \boldsymbol{k}, s \rangle \right|^2$$

$$\propto \underbrace{\int \left| \langle \hat{V} \rangle \right|^2 e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r}}_{} + \underbrace{\left(\left| \langle \hat{V}^2 \rangle \right| - \left| \langle \hat{V} \rangle \right|^2 \right) \int z(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r}}_{}$$

The contribution from deviations from the average structure: *Short-range* order

The contribution from the average structure of the sample: *Long-range* order

Crystalline structures

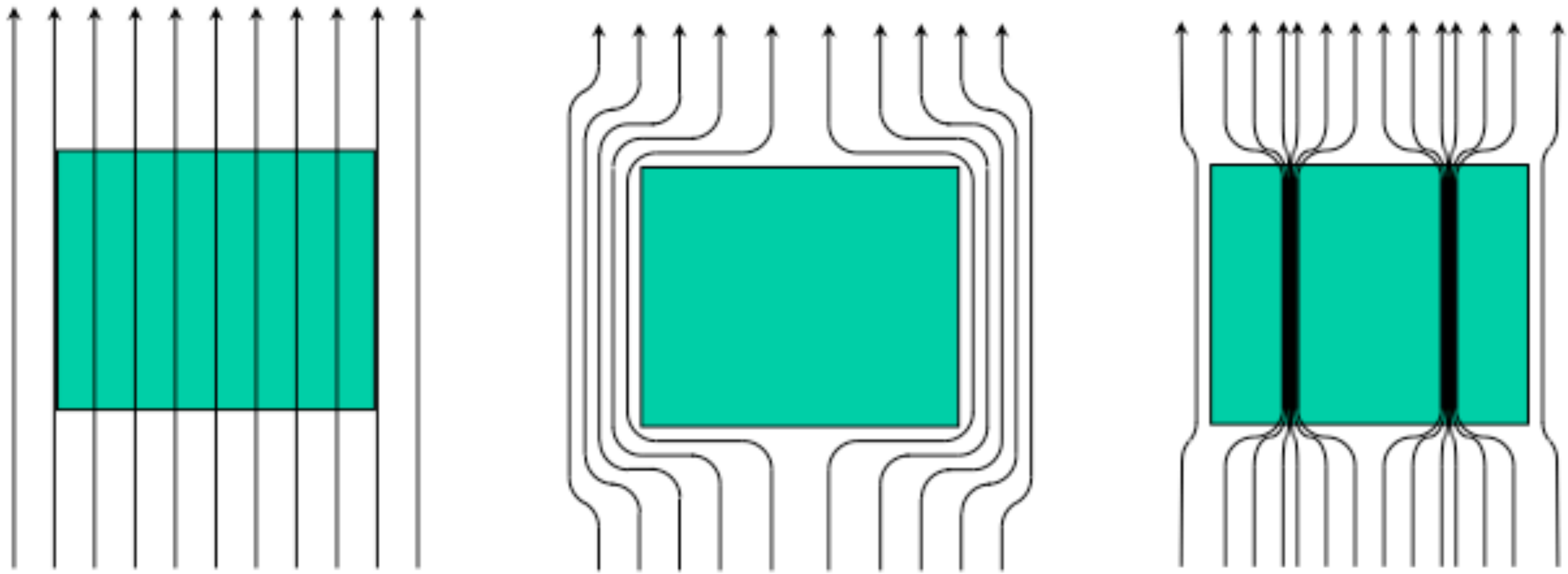
$$\frac{d\sigma}{d\Omega} \propto \int \left| \left\langle \hat{V} \right\rangle \right|^2 e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r}$$

The Fourier transform from a series of delta-functions

Bragg's Law: $2d\sin\theta=\lambda$

Leads to Magnetic Crystallography

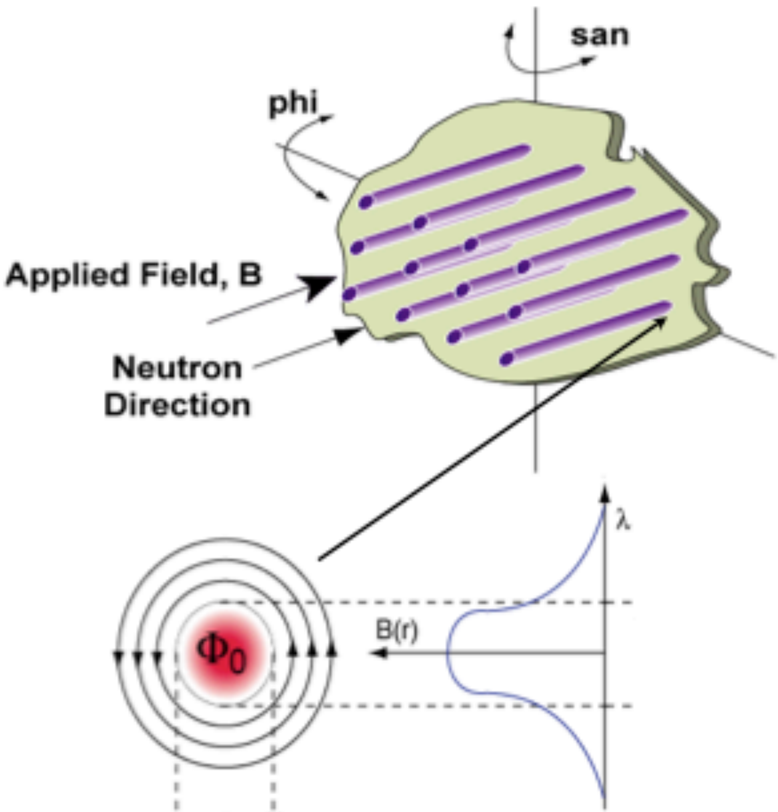
Superconductivity



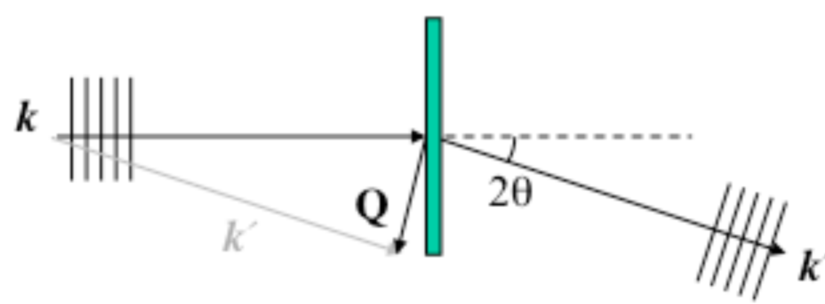
Normal state | Superconducting state | Flux line lattice

A simple example of magnetic elastic scattering

MgB_2 is a superconductor below 39K, and expels all magnetic field lines (Meissner effect). Above a critical field, flux lines penetrate the sample.



Scattering geometry

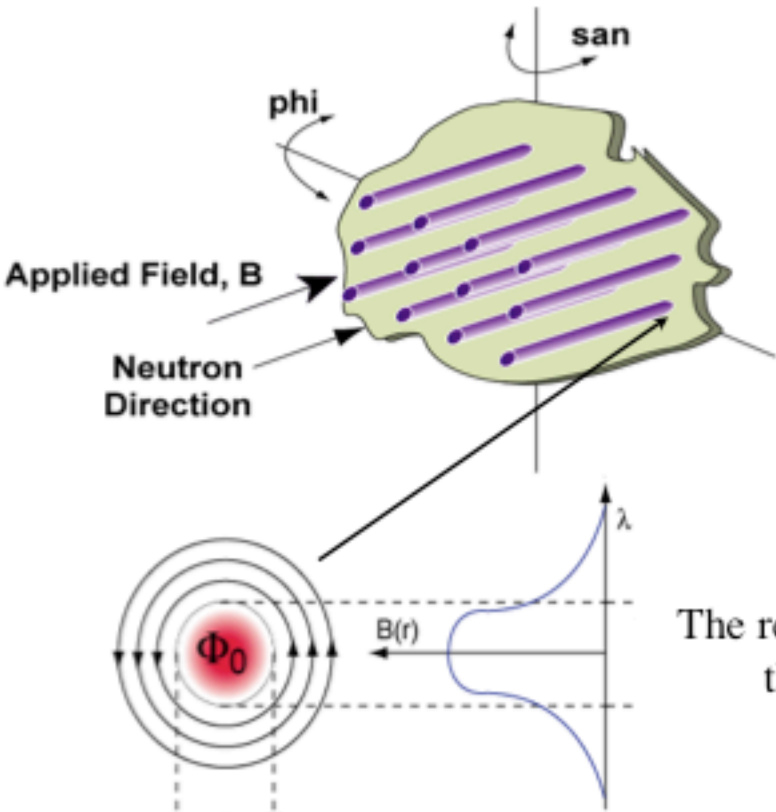


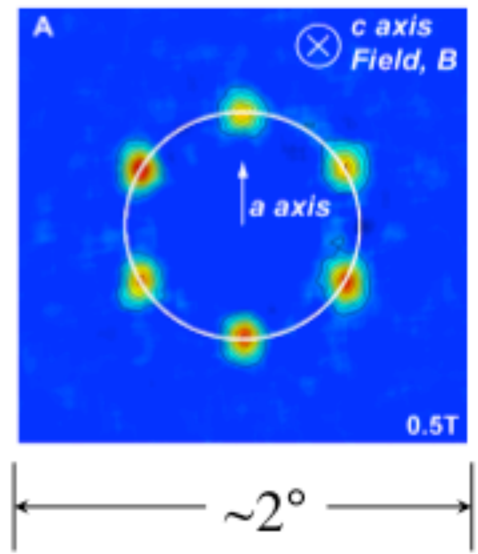
The momentum transfer, **Q**, is roughly perpendicular to the flux lines, therefore all the magnetization is seen.

(recall $\frac{d\sigma_{magnetic}}{d\Omega} = \langle \mathbf{M}_{\perp}^{*}(\mathbf{Q}) \rangle \langle \mathbf{M}_{\perp}(\mathbf{Q}) \rangle$)

A simple example of magnetic elastic scattering

MgB_2 is a superconductor below 39K, and expels all magnetic field lines (Meissner effect).
Above a critical field, flux lines penetrate the sample.





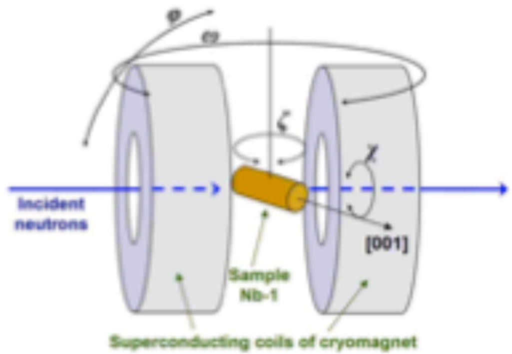
Via Bragg's Law
$2d\sin\theta=\lambda$
$\lambda = 10$ Å
$d = 425$Å

The reciprocal lattice has 60° rotational symmetry,
therefore the flux line lattice is hexagonal

Cubitt et. al. Phys. Rev. Lett. **91** 047002 (2003)
Cubitt et. al. Phys. Rev. Lett. **90** 157002 (2003)

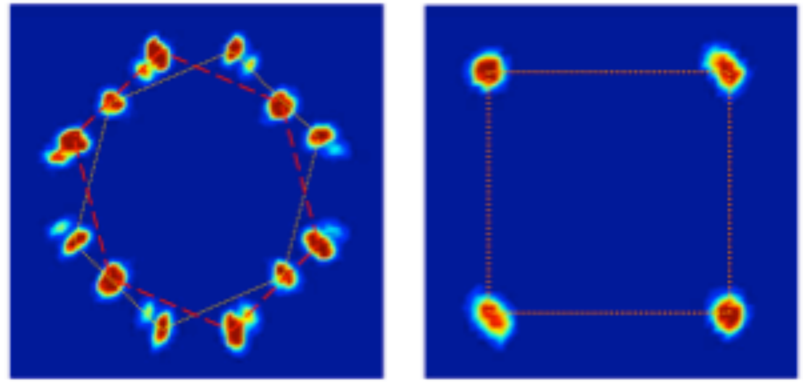
Flux line lattices

Pure Niobium



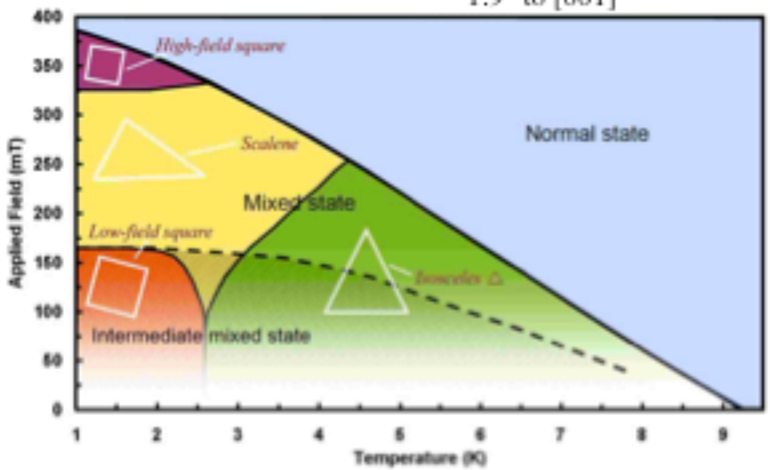
200 mT

350 mT



0.1° to (110)
1.9° to [001]

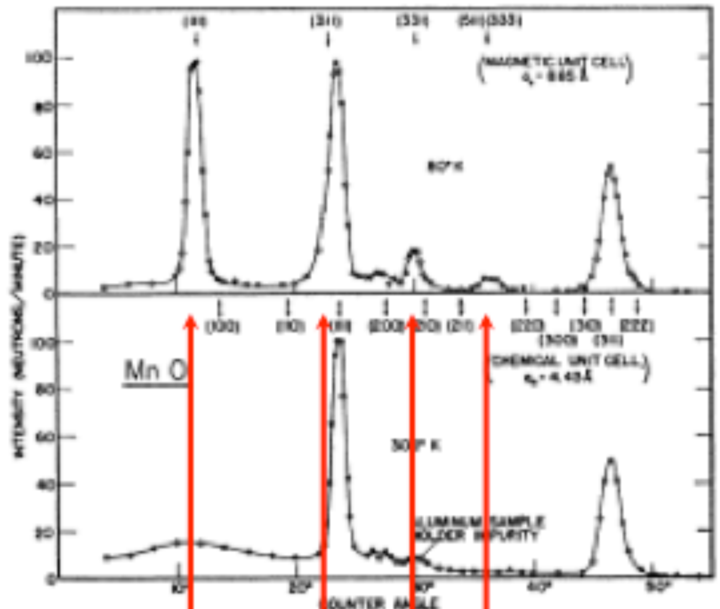
0.1° to (110)
1.9° to [001]



M. Laver et. al. Phys. Rev. B **79** 014518 (2009)

Antiferromagnetism in MnO

Bragg peaks from crystal structure



80K (antiferromagnetic)

300K (paramagnetic)

FIG. 1. Neutron diffraction patterns for MnO at room temperature and at 80°K.

New Bragg peaks

C. G. Shull & J. S. Smart, Phys. Rev. **76** (1949) 1256

Antiferromagnetism in MnO

New magnetic Bragg peaks

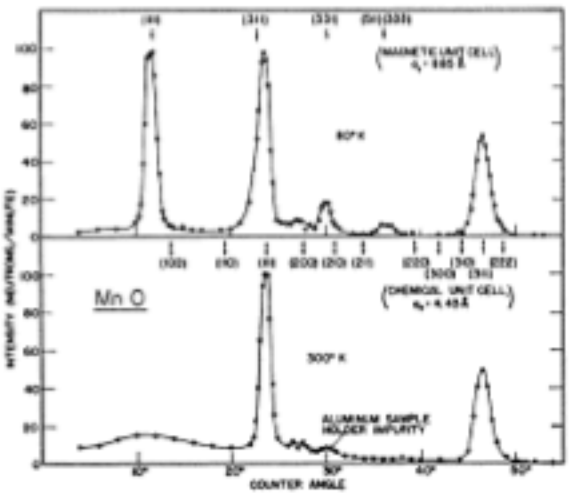
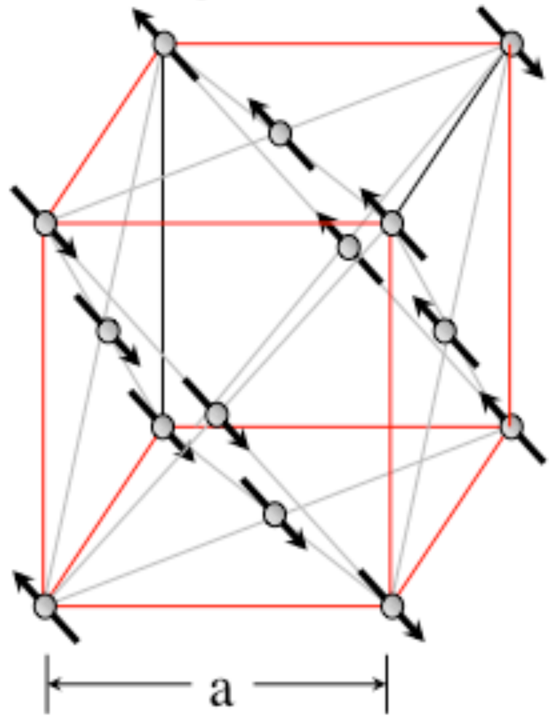


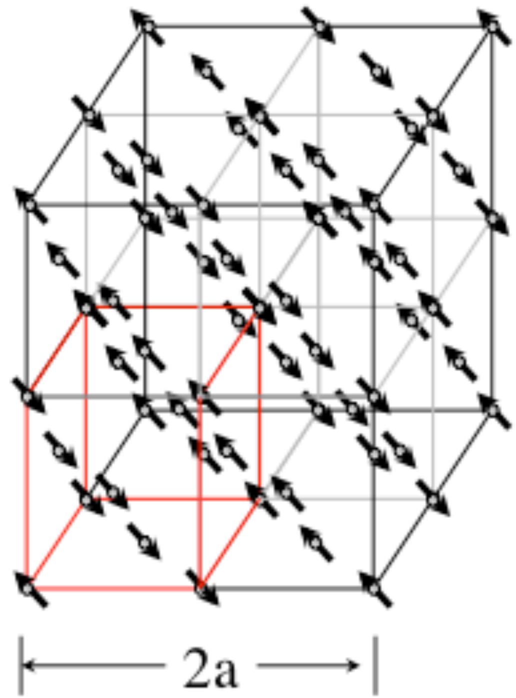
FIG. 1. Neutron diffraction patterns for MnO at room temperature and at 80°K.

C. G. Shull & J. S. Smart, Phys. Rev. **76** (1949) 1256

Magnetic structure



Magnetic unit cell



C. G. Shull *et al.*, Phys. Rev. **83** (1951) 333

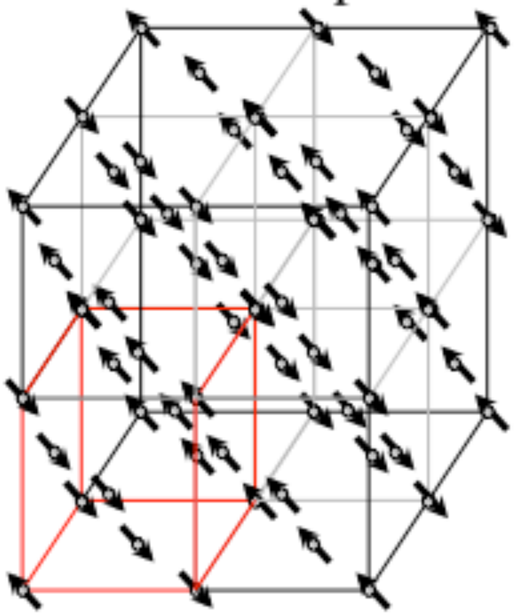
H. Shaked *et al.*, Phys. Rev. B **38** (1988) 11901

Antiferromagnetism in MnO

The moments are said to lie in the (111) plane

H. Shaked *et al*., Phys. Rev. B **38** (1988) 11901

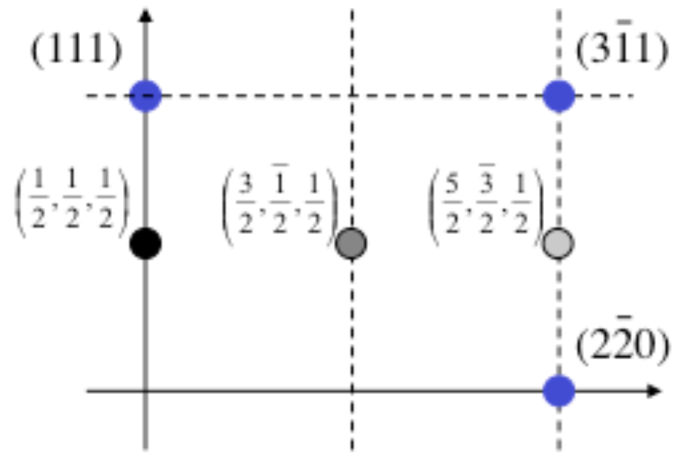
Real space



fcc lattice

(i.e. h,k,l) all even or all odd

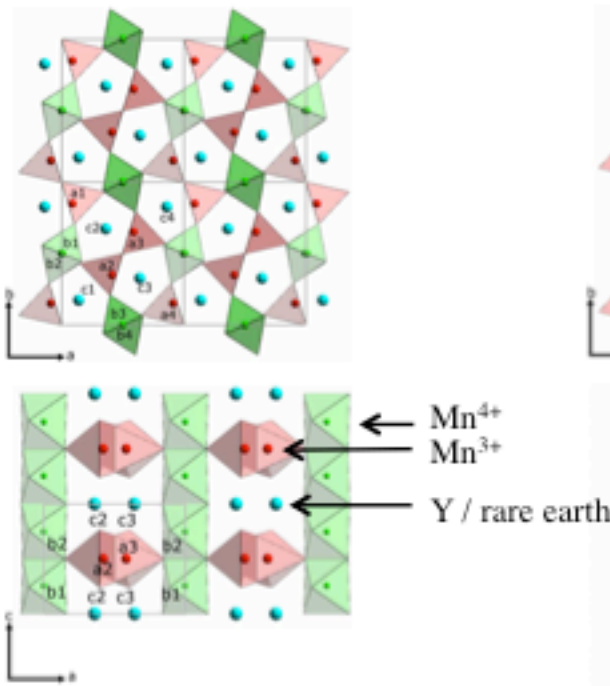
Reciprocal space



Moment direction

The magnetic structure of YMn_2O_5

Crystal structure

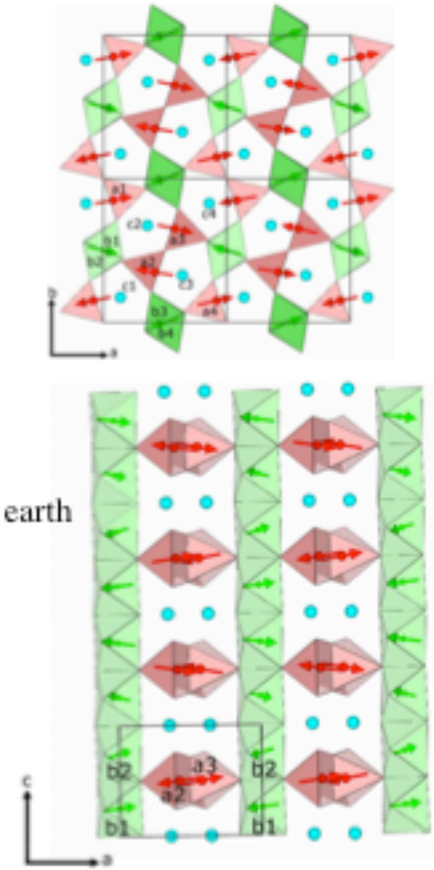


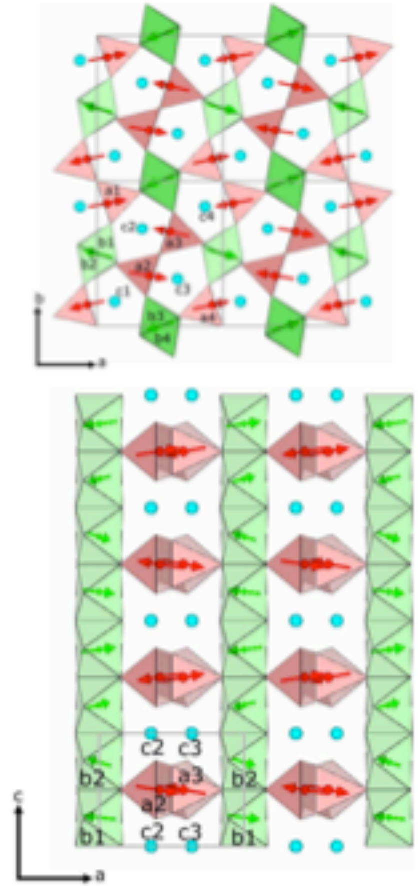
C. Vecchini *et al*., PRB **77** (2008) 134434

Domain 1

Domain 2

(domains are related by inversion symmetry)



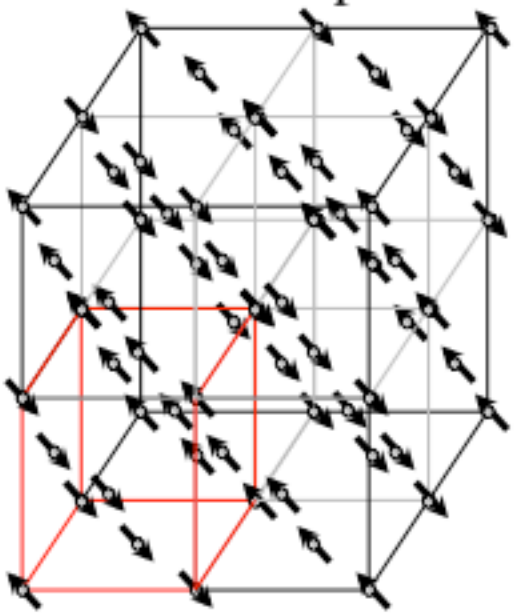


YMn_2O_5 has a *commensurate* magnetic structure

The moments are said to lie in the (111) plane

H. Shaked *et al.*, Phys. Rev. B **38** (1988) 11901

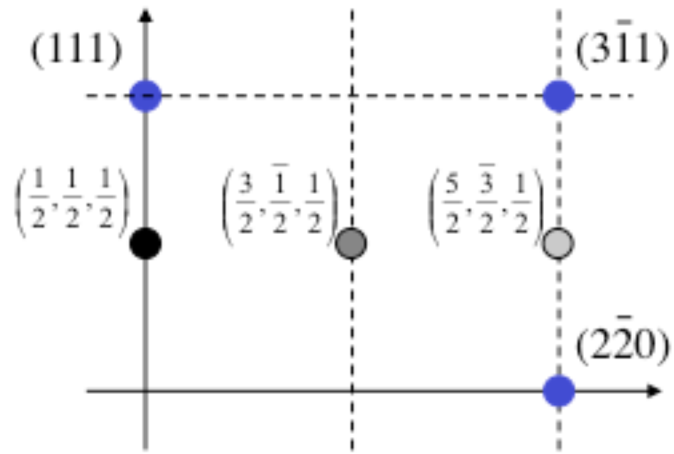
Real space



fcc lattice

(i.e. h,k,l) all even or all odd

Reciprocal space

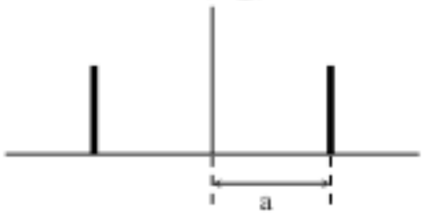


Moment direction

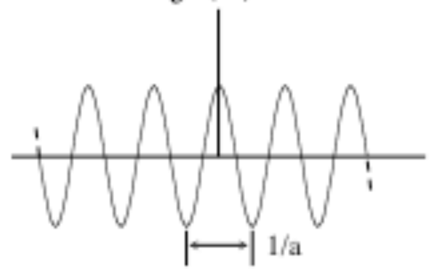
Antiferromagnetism in Chromium

The Fourier Transform for two Delta functions:

$F(\mathbf{Q})$



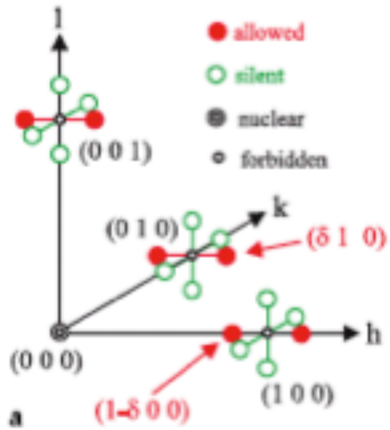
$f(\mathbf{r})$



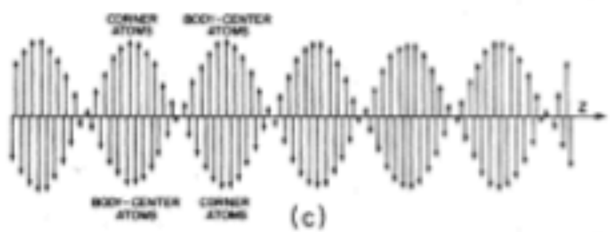
E.Fawcett, Rev. Mod. Phys. **60** (1988) 209

Chromium is an example of an *itinerant*, *incommensurate* antiferromagnet

Reciprocal space



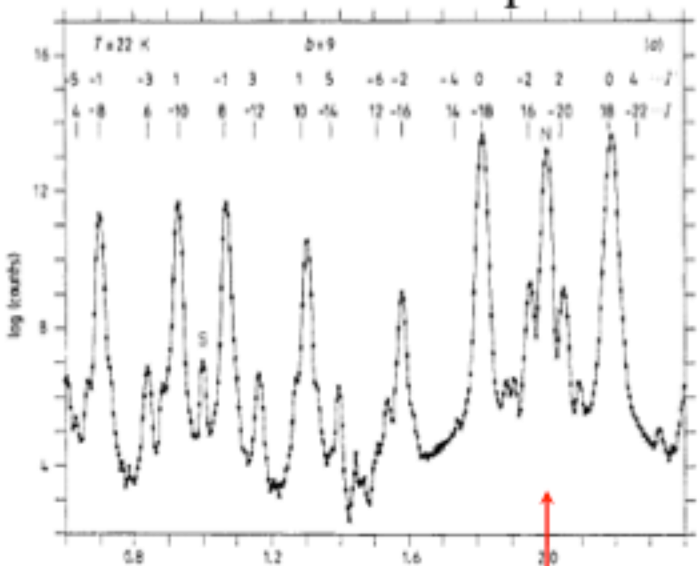
Real space



a Spin Density wave

Magnetic structure of Holmium

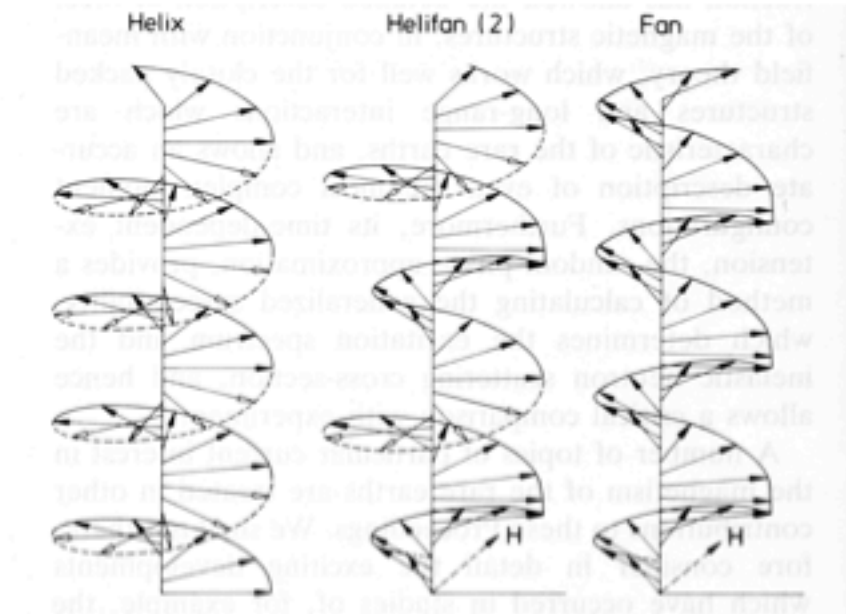
Scan along [00*l*], incommensurate peaks



l (rlu)

Nuclear peak

Real space structures

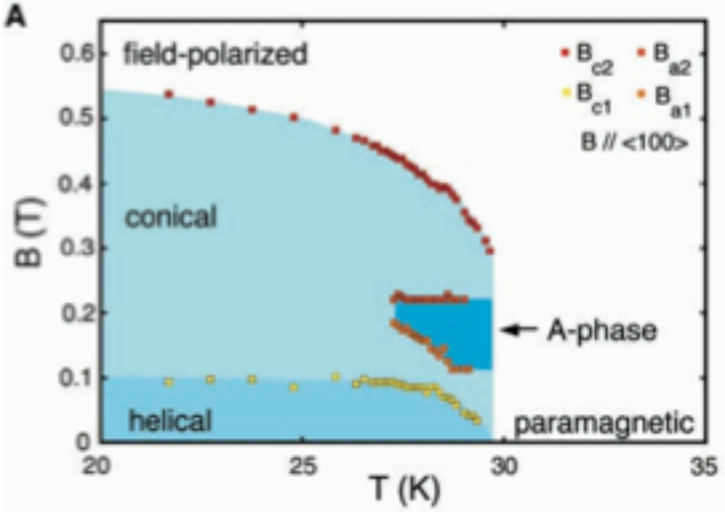


W. C. Koehler, in *Magnetic Properties of Rare Earth Metals*, ed. R. J. Elliot (Plenum Press, London, 1972) p. 81

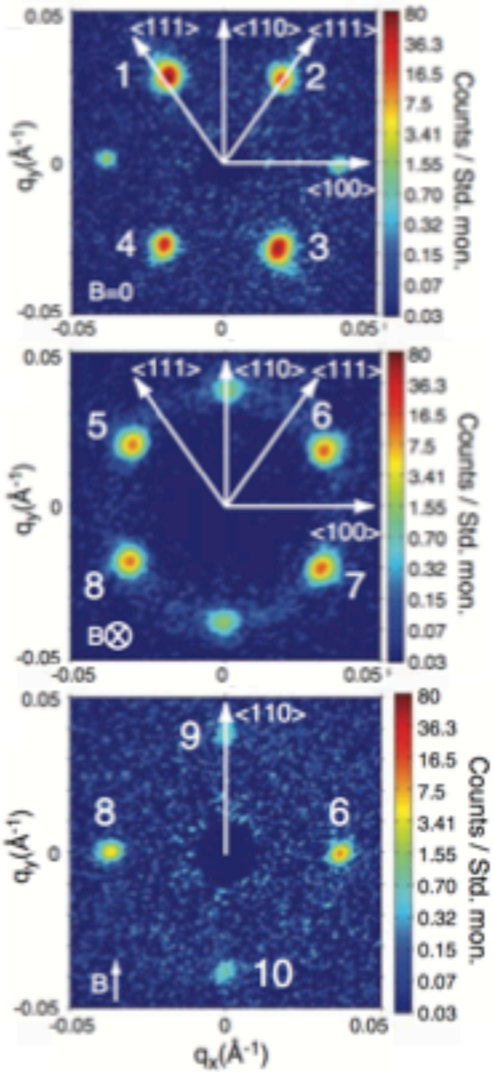
R. A. Cowley and S. Bates, J. Phys. C **21** (1988) 4113

A. R. Mackintosh and J. Jensen, Physica B **180 & 181** (1992) 1

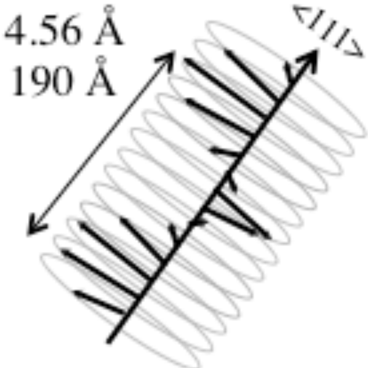
Skyrmions in MnSi

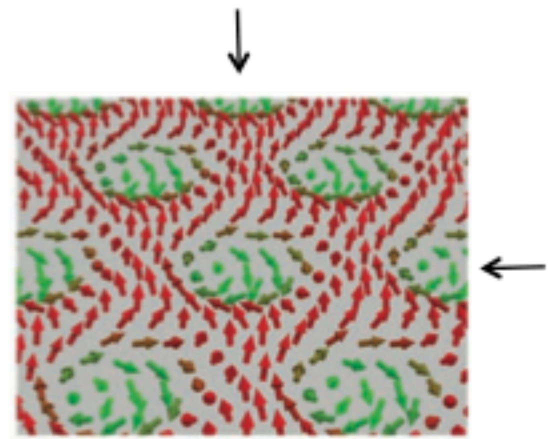


S. Mühlbauer et. al. Science **323** 915 (2009)

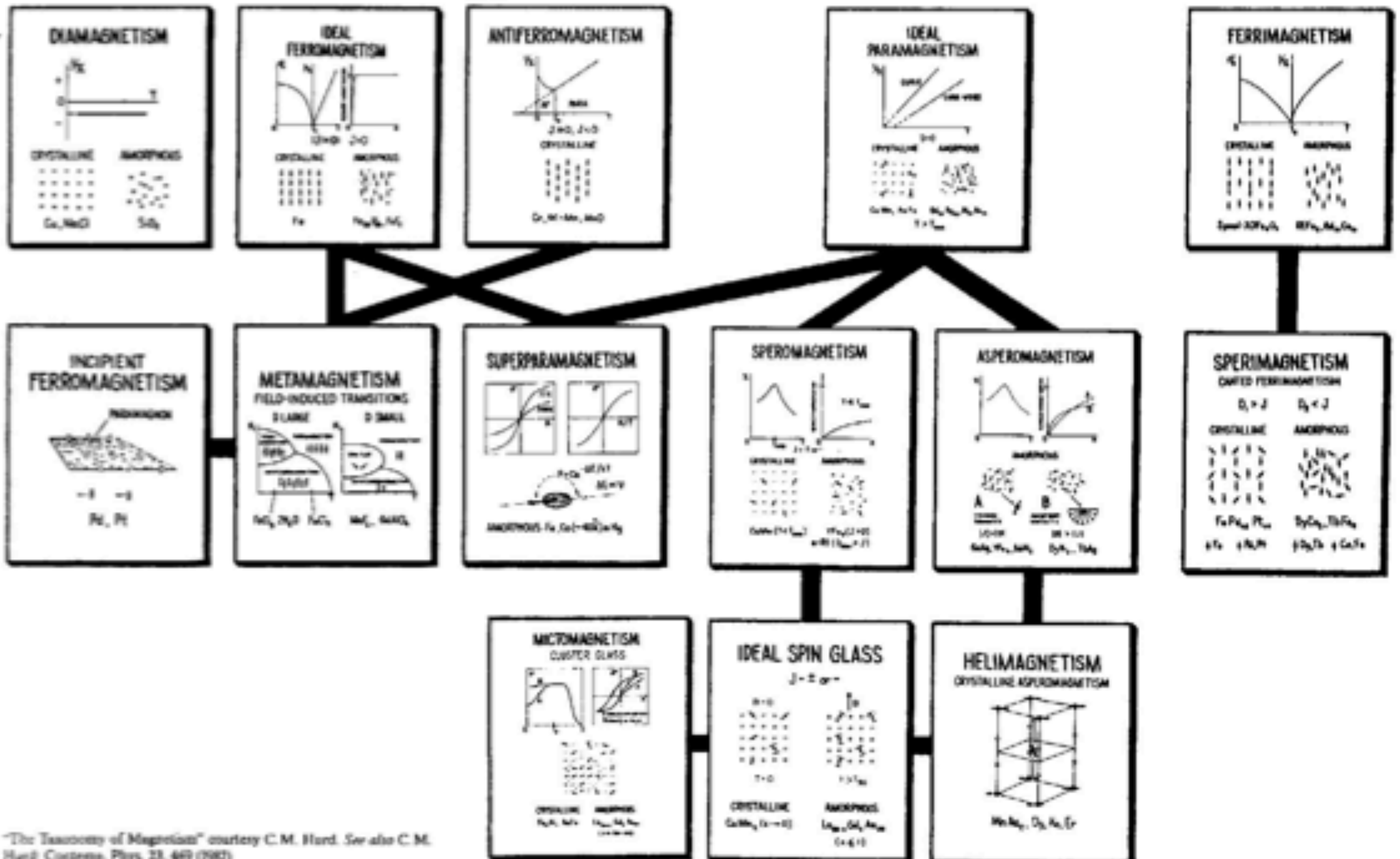


$a \approx 4.56$ Å
$\lambda = 190$ Å





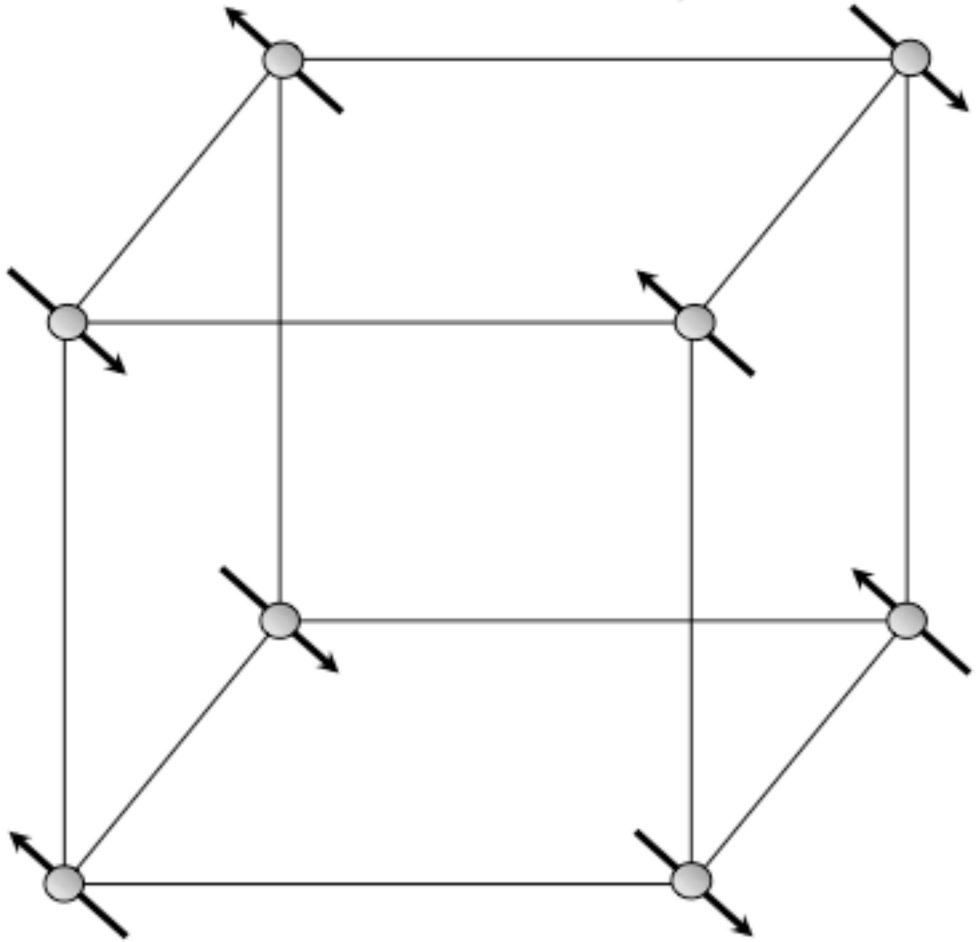
The 'Family Tree' of Magnetism



"The Taxonomy of Magnetism" courtesy C.M. Hurd. *See also* C.M. Hurd: Contemp. Phys. 23, 469 (1982)

Magnetic form factors

A magnetic moment is spread out in space
Unpaired electrons can be found in orbitals, or in band structures



Magnetic form factors

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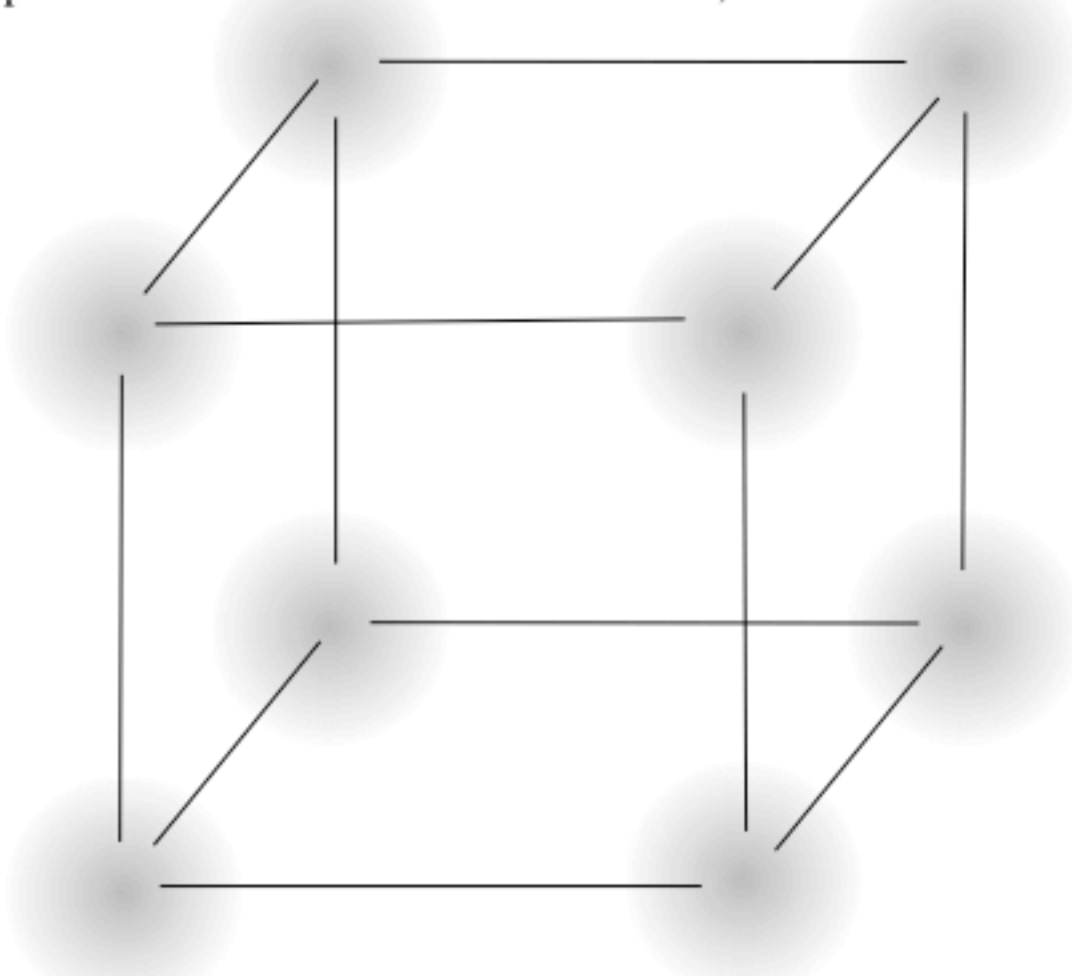
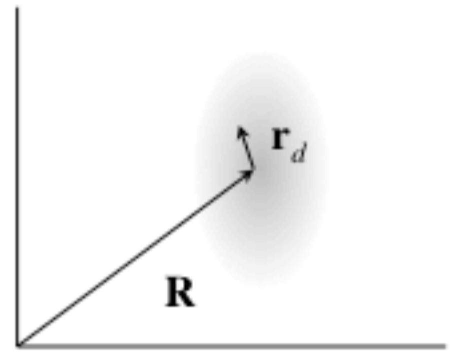


Magnetic form factors

A magnetic moment is spread out in space
Unpaired electrons can be found in orbitals, or in band structures

e.g.
Take a magnetic ion with total spin **S** at position **R**
The (normalized) density of the spin is $s_d(\mathbf{r})$
around the equilibrium position



$$\mathbf{M}(\mathbf{Q}) = \int \mathbf{M}(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r}$$

$$\propto \int s_d e^{i\mathbf{Q}\cdot\mathbf{r}_d} \cdot d\mathbf{r}_d \int S(\mathbf{R}) e^{i\mathbf{Q}\cdot\mathbf{R}} \cdot d\mathbf{R}$$

$$= f(Q) \int S(\mathbf{R}) e^{i\mathbf{Q}\cdot\mathbf{R}} \cdot d\mathbf{R}$$

$$f(Q) = \int s_d e^{i\mathbf{Q}\cdot\mathbf{r}_d} \cdot d\mathbf{r}_d$$

$f(Q)$ is the *magnetic form factor*
It arises from the *spatial distribution* of *unpaired electrons* around a magnetic atom

Magnetic form factors

$f(Q)$ is the *magnetic form factor*

It arises from the *spatial distribution* of *unpaired electrons* around a magnetic atom

$$\frac{d\sigma_{magnetic}}{d\Omega} = \left\langle \mathbf{M}^{*}_{\perp}(\mathbf{Q}) \right\rangle \left\langle \mathbf{M}_{\perp}(\mathbf{Q}) \right\rangle$$

$$\propto f^{2}(Q) \int S_{\perp}(\mathbf{R}_i) S^{*}_{\perp}\left(\mathbf{R}_j\right) e^{i\mathbf{Q}\cdot\left(\mathbf{R}_i - \mathbf{R}_j\right)} \cdot d\mathbf{R}$$

There is no form factor for nuclear scattering, as the nucleus can be considered as a point compared to the neutron wavelength

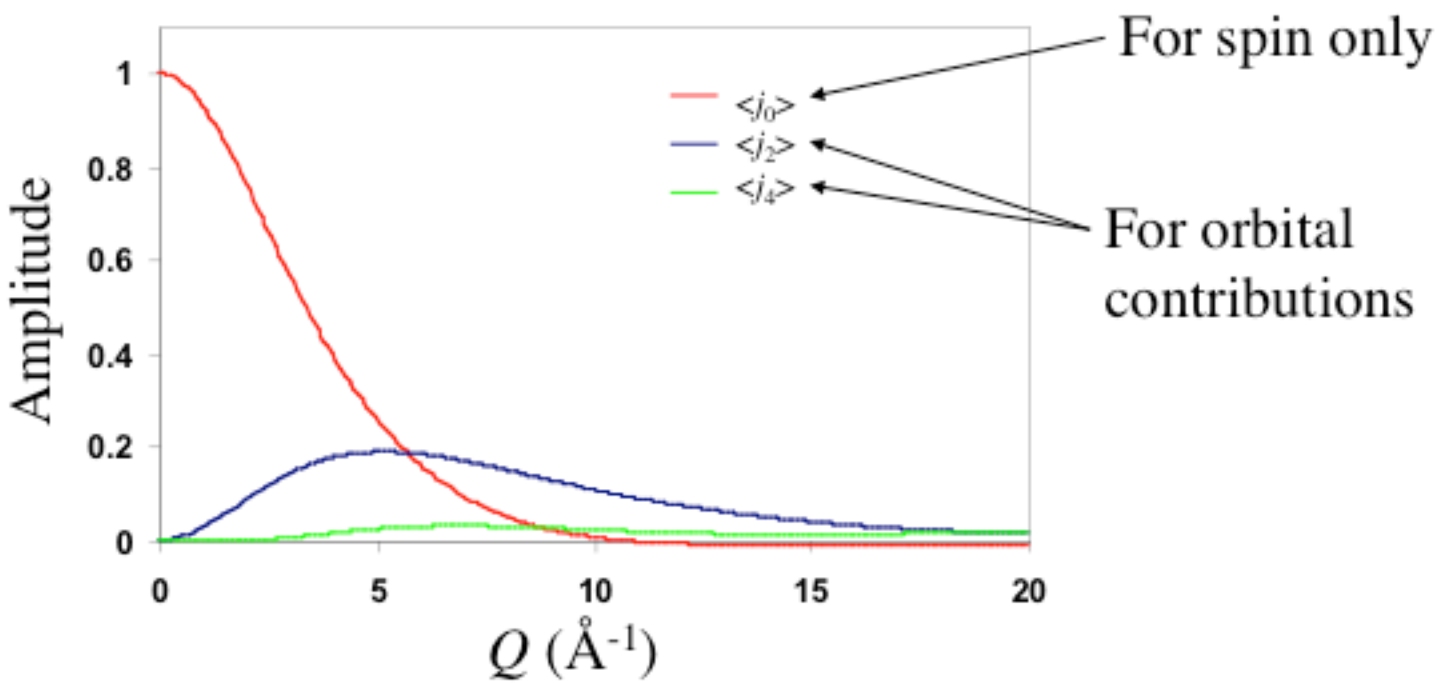
Magnetic form factors

Approximations for the form factors are tabulated

(P. J. Brown, International Tables of Crystallography, Volume C, section 4.4.5)

$$f(Q) = C_1\langle j_0(Q/4\pi)\rangle + C_2\langle j_2(Q/4\pi)\rangle + C_4\langle j_4(Q/4\pi)\rangle + \ldots$$

Form factors for iron



Magnetic electron density

Nickel

H. A. Mook., Phys. Rev. **148** (1966) 495

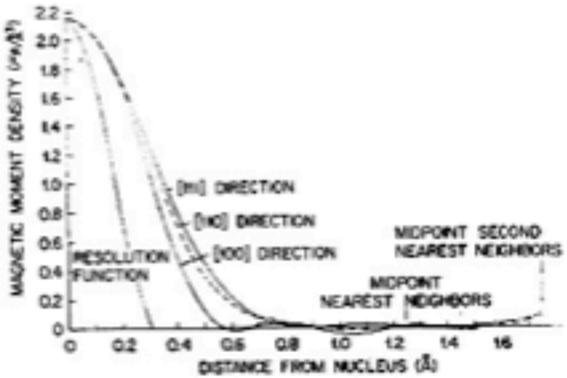


FIG. 1. Distribution of magnetic moment density along the three major crystallographic directions.

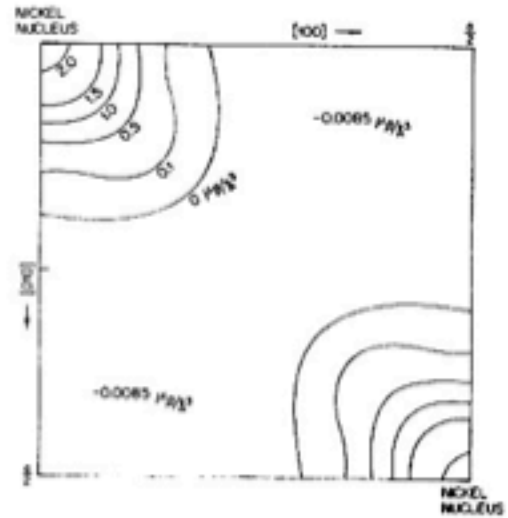


FIG. 4. The magnetic moment distribution in the [100] plane.

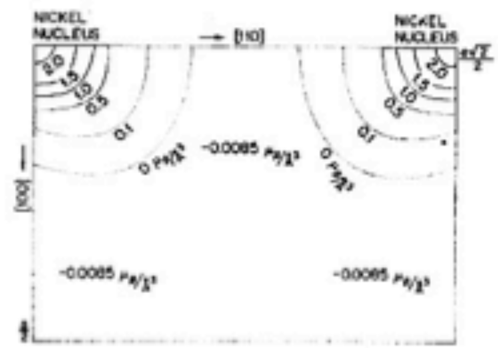
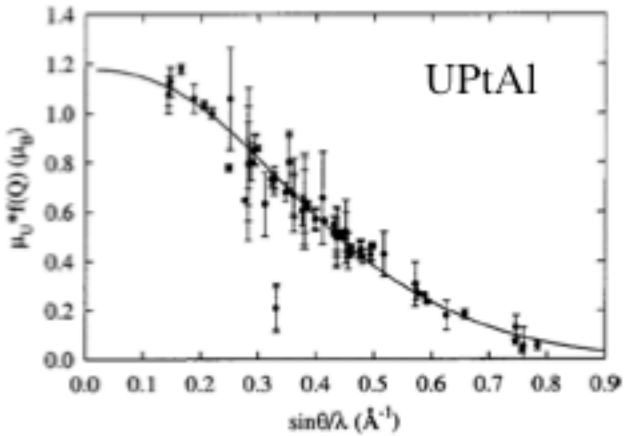


FIG. 5. The magnetic moment distribution in the [110] plane.

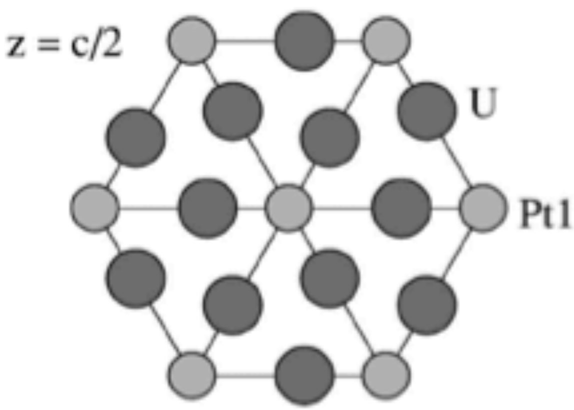
Magnetic electron density

$$f(Q) = \int s_d e^{i\mathbf{Q}\cdot\mathbf{r}_d} \cdot d\mathbf{r}_d$$

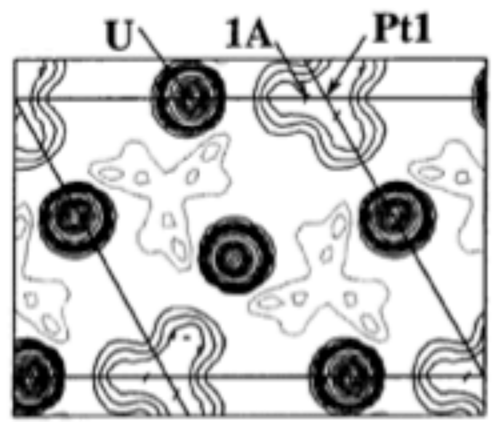


P. Javorsky *et al*., Phys. Rev. B **67** (2003) 224429

From the form factors, the magnetic moment density in the unit cell can be derived



crystal structure



magnetic moment density

Conclusions:

- Learn your Fourier transforms
- Get used to using vectors
- Neutrons only ever see the components of the magnetization, **M**, that are *perpendicular* to the scattering vector, **Q**
- Magnetization is distributed in space, therefore the magnetic scattering has a *form factor*

Lecture 2: The plan

- Inelastic scattering
  - Crystal fields and molecular magnets
  - Magnons
  - Spin wave continua
- Generalized susceptibility
- Critical scattering
- Short-ranged order

Neutron inelastic scattering

Magnetic fluctuations are governed by a wave equation:

$$H\psi = E\,\psi$$

The Hamiltonian is given by the physics of the material.

Given a Hamiltonian, *H*, the energies *E* can be calculated.
(this is sometimes very difficult)

Neutrons measure the energy, *E*, of the magnetic fluctuations, therefore probing the free parameters in the Hamiltonian.

Crystal fields

The crystal field is an electric field on an atom caused by neighbouring atoms in the sample.

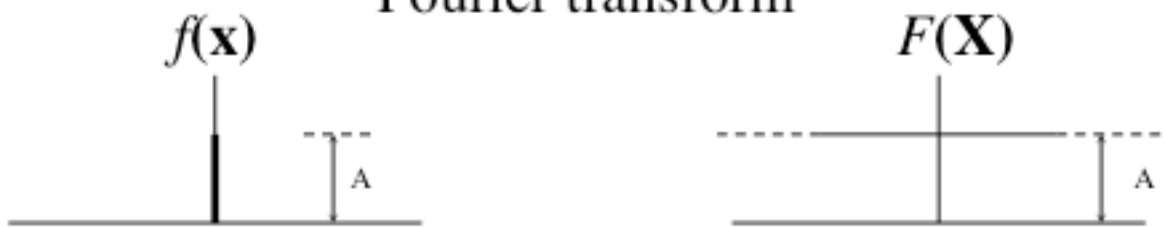
It may lift the degeneracy of the energy levels for the atomic electrons.

If the electrons are unpaired, neutrons can cause the electrons to jump between the energy levels.

Because the crystal field is at a single atomic site, the inelastic scattering is essentially independent of **Q**.

Fourier transform

A Delta function

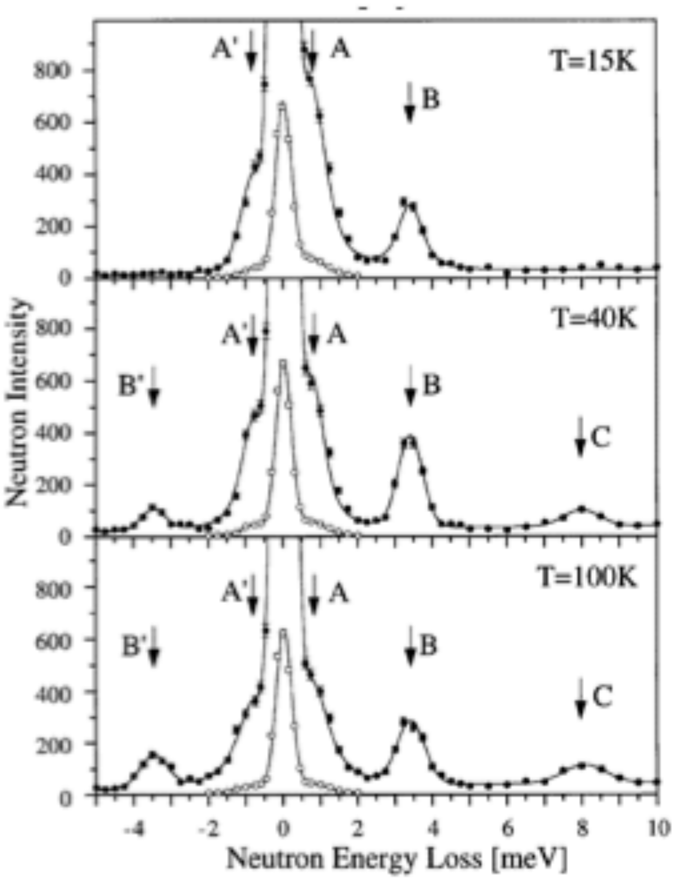


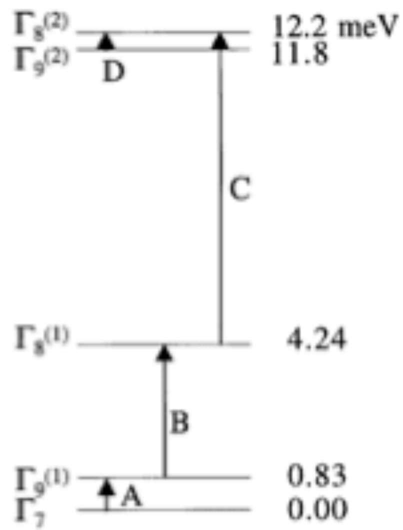
Crystal fields in $NdPd_2Al_3$

$$H = \sum_{l=2,4,6} B_l^0 O_l^0 + \sum_{l=2,4} B_l^4 O_l^4$$

O = Stevens parameters
(K. W. Stevens, Proc. Phys. Soc A**65** (1952) 209)

B = CF parameters, measured by neutrons

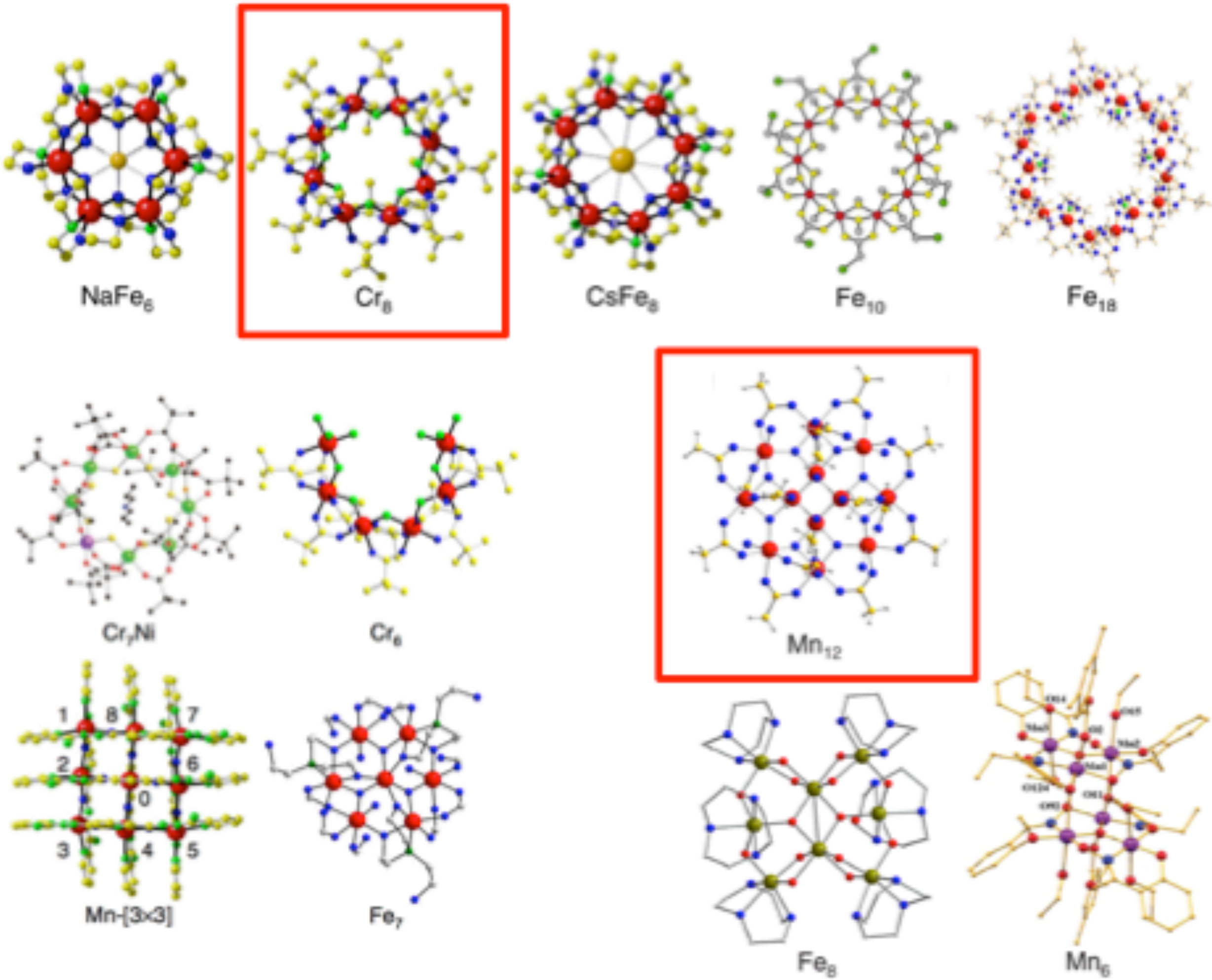




A. Dönni *et al.*, J. Phys.: Condens. Matter **9** (1997) 5921

O. Moze., *Handbook of magnetic materials* vol. 11, 1998 Elsevier, Amsterdam, p.493

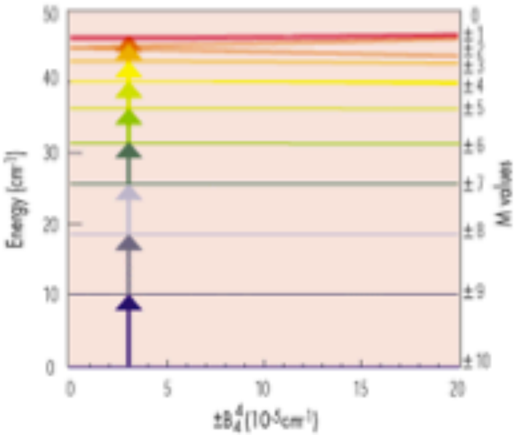
Molecular magnets



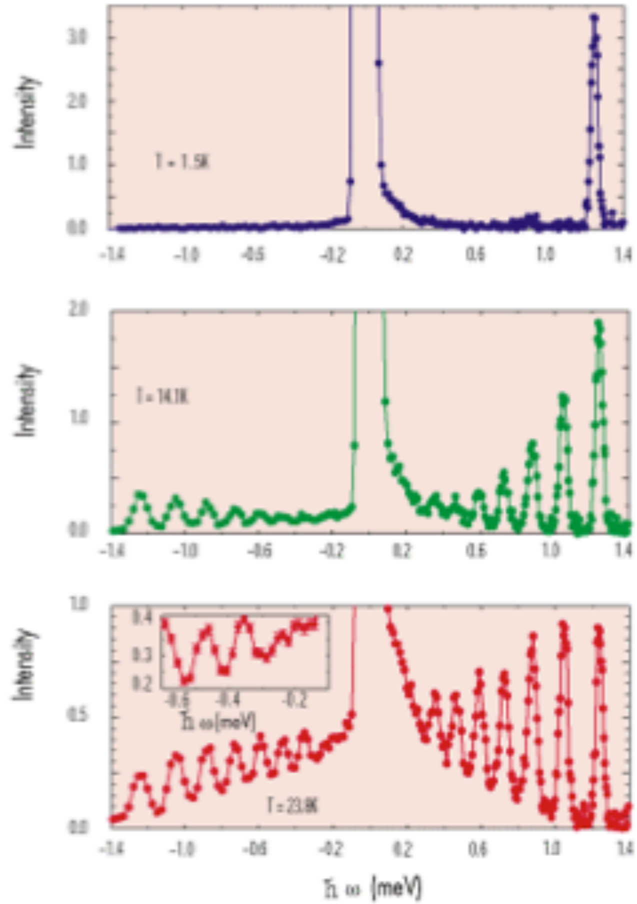
A. Furrer and O. Waldmann, Rev. Mod. Phys. **85** (2013) 367

Quantum tunneling in Mn_{12}-acetate

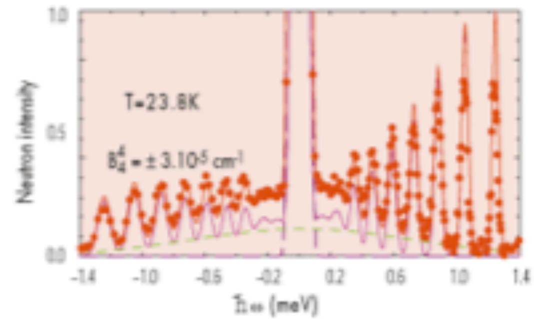
$$H = \sum_{l=2,4,6} B_l^0 O_l^0 + \sum_{l=2,4} B_l^4 O_l^4$$



Calculated energy terms



Neutron spectra



Fitted data with scattering from:

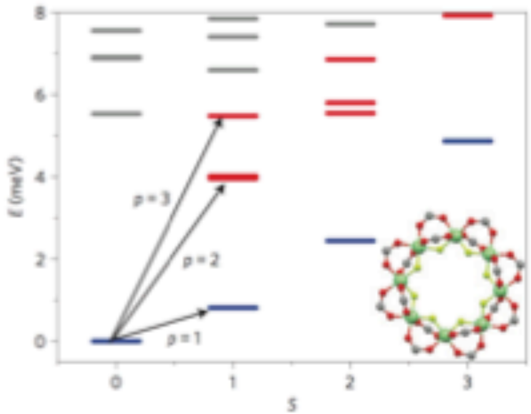
- energy levels
- elastic scattering
- incoherent background

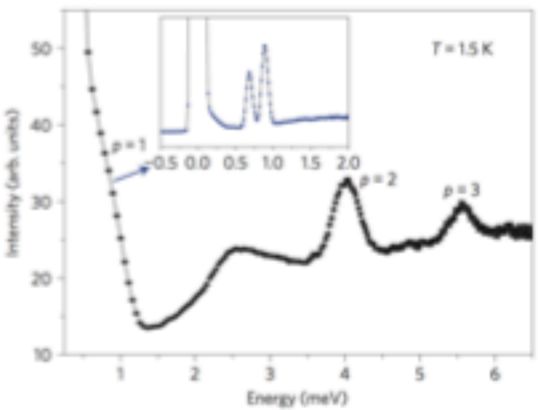
I. Mirebeau *et al*., Phys. Rev. Lett. **83** (1999) 628

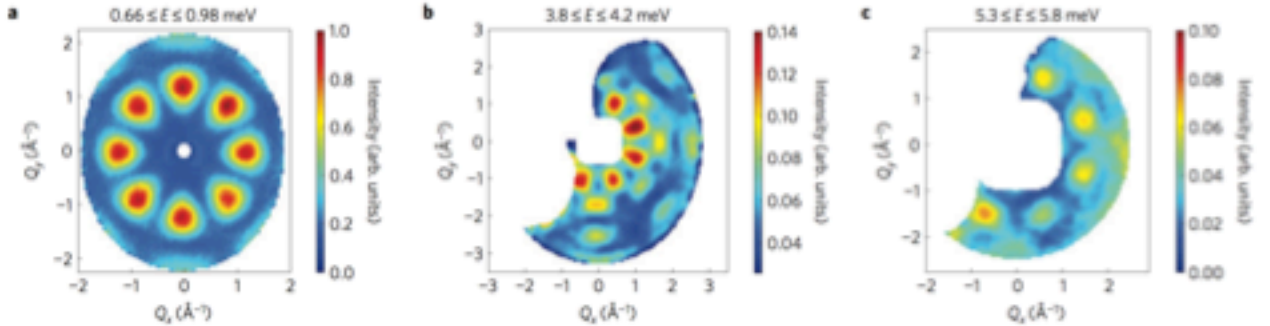
R. Bircher *et al*., Phys. Rev. B **70** (2004) 212413

Space-time correlations in Cr_8

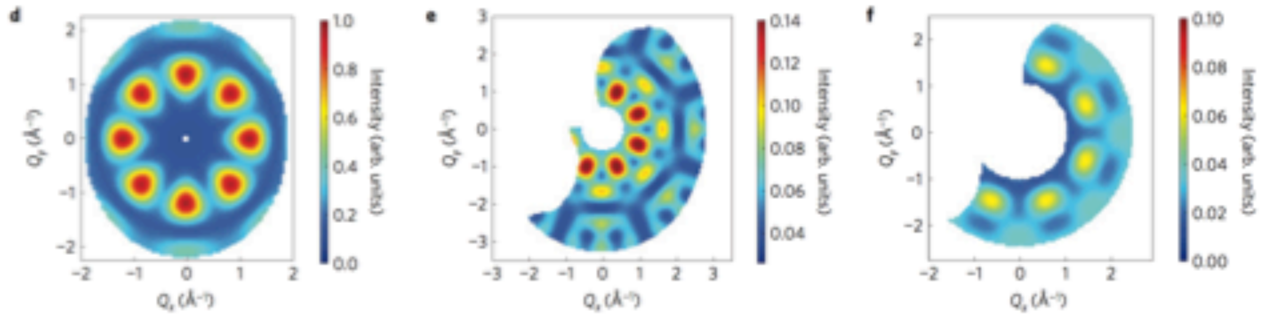
$$H = J\sum_{i=1}^{8} \mathbf{s}_i \cdot \mathbf{s}_{i+1}$$



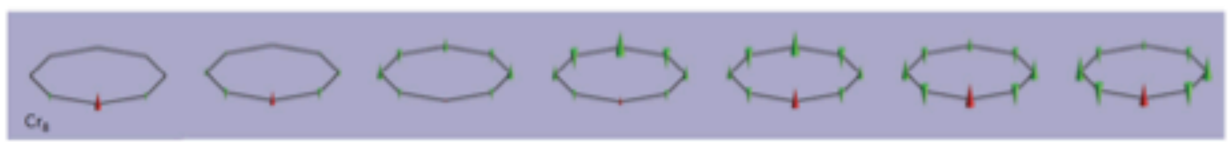




Data



Fit



M. Baker *et al.*, Nature Phys. **8** (2012) 906

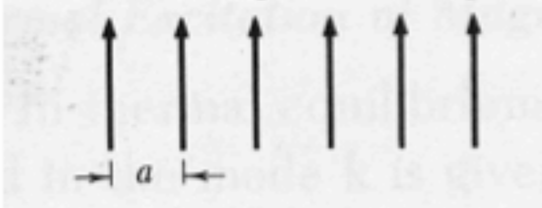
Spin waves and magnons

A simple Hamiltonian for spin waves is:

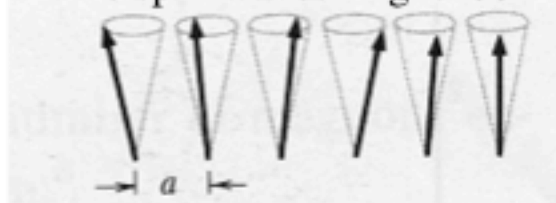
$$H = -J\sum_{i,j}\mathbf{s}_i\mathbf{s}_j$$

J is the magnetic exchange integral, which can be measured with neutrons.

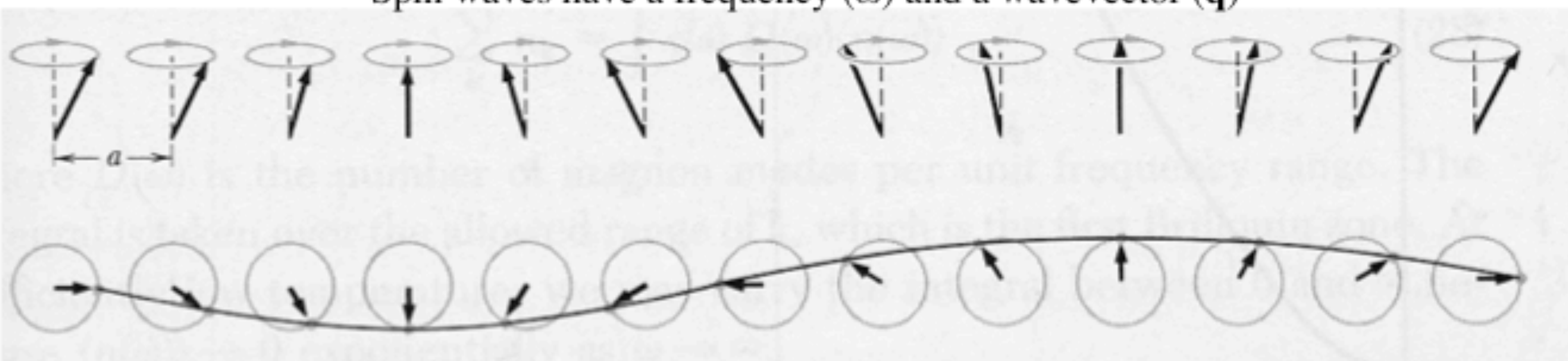
Take a simple ferromagnet:



The spin waves might look like this:

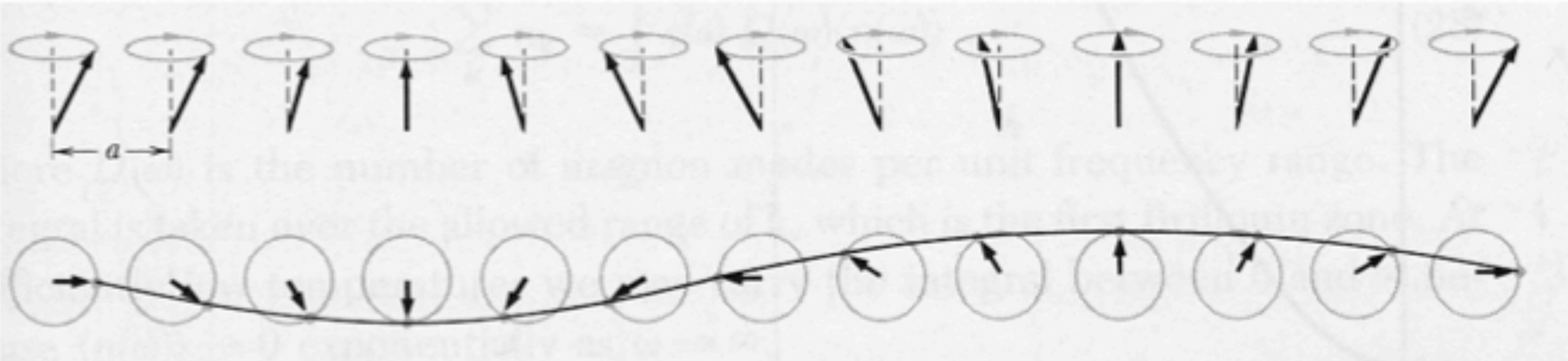


Spin waves have a frequency (ω) and a wavevector (**q**)



The frequency and wavevector of the waves are *directly measurable* with neutrons

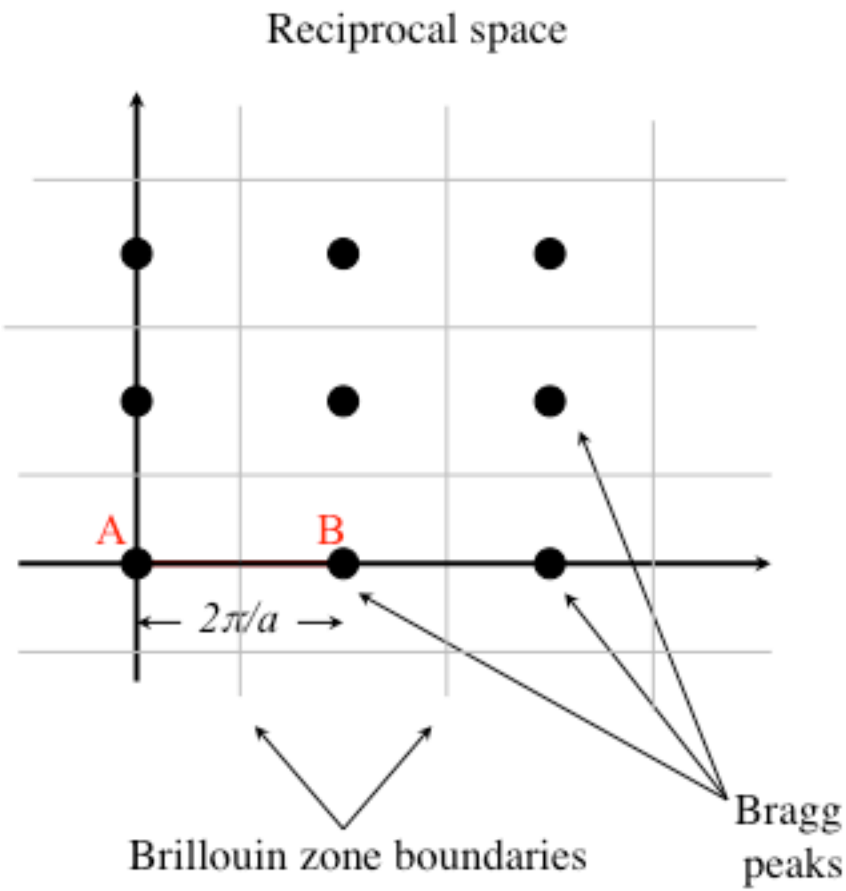
Magnons

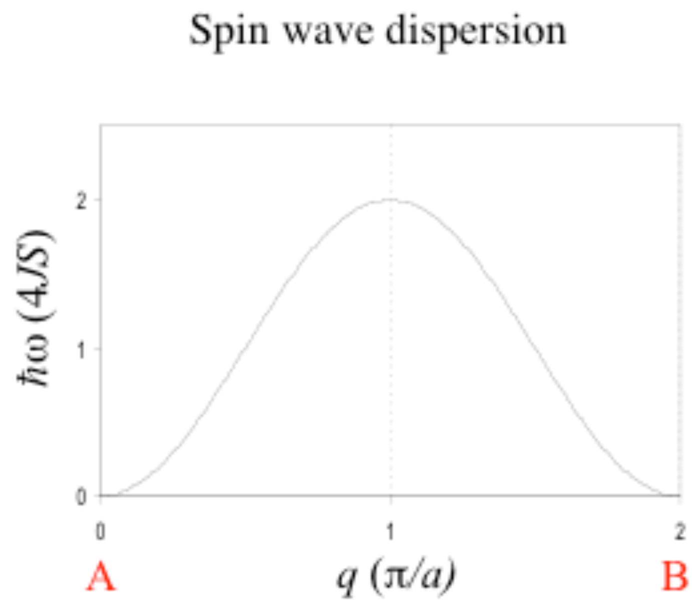


The Fourier Transform for a periodic function:

Each wavelength for the magnon has its own periodicity.
Each wavevector for the magnon has its own frequency (energy)

Magnons and reciprocal space



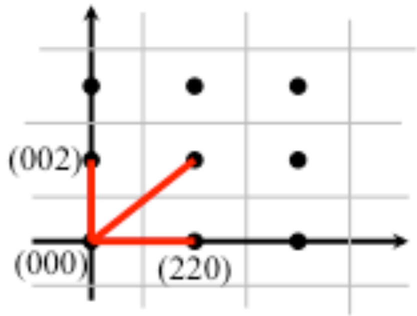


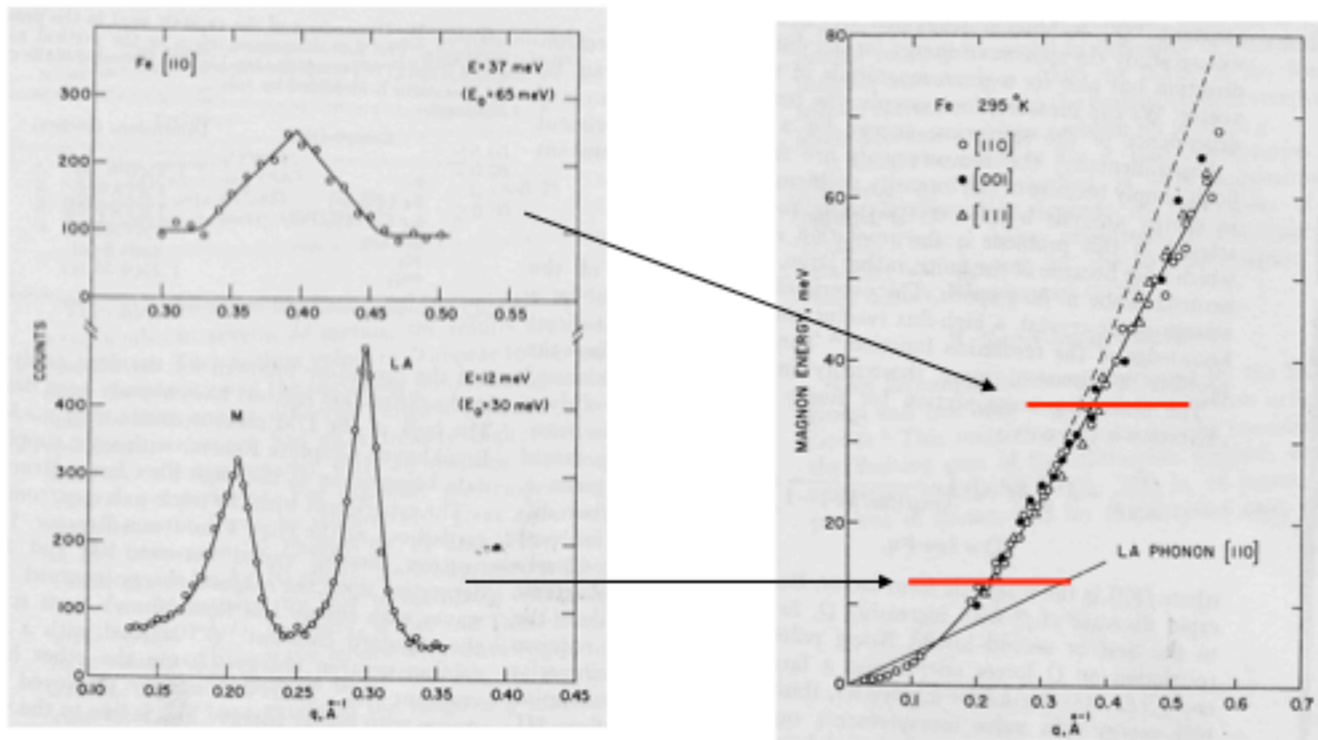
$$\hbar\omega = 4JS(1 - \cos qa)$$
$$= Dq^2 \quad (\text{for } qa << 1)$$
$$D = 2JSa^2$$

C. Kittel, *Introduction to Solid State Physics*, 1996, Wiley, New York

F. Keffer, *Handbuch der Physik* vol 18II, 1966 Springer-Verlag, Berlin

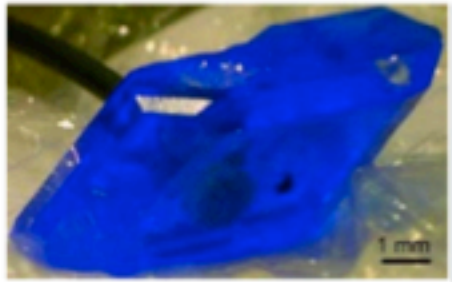
Magnons in crystalline iron

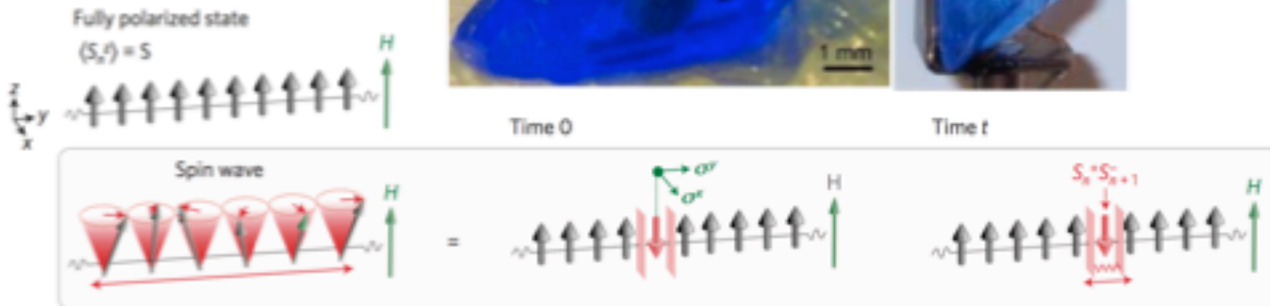




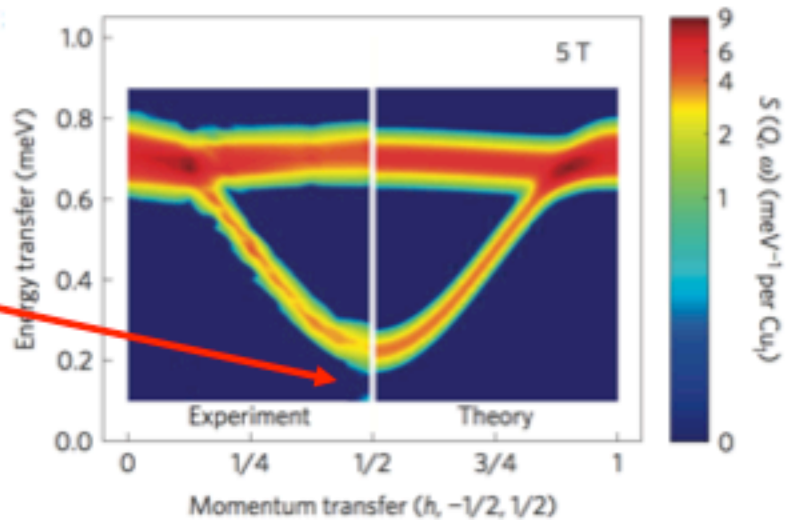
G. Shirane *et al.*, J. Appl. Phys. **39** (1968) 383

Magnetic excitations in $CuSO_4$



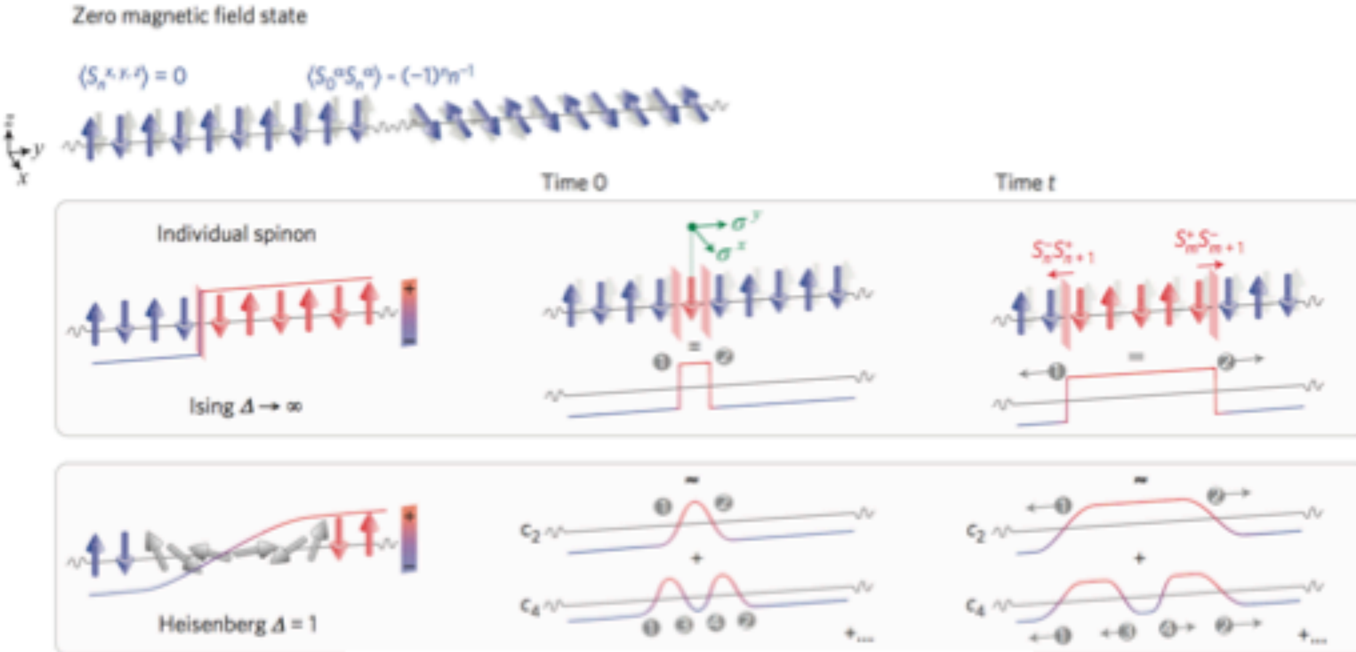


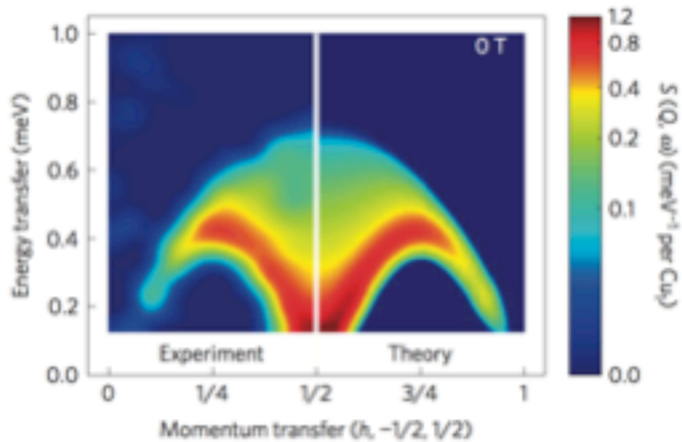
Gap in dispersion due to magnetic anisotropy



M. Mourigal *et al.*,
Nature Phys. **9** (2013) 435

Magnetic excitations in $CuSO_4$





M. Mourigal *et al.*,

Nature Phys. **9** (2013) 435

Neutron scattering and magnetic susceptibility

Magnetic susceptibility is a fundamental property of a material. It is defined as:

$$\chi = \frac{M}{H}$$

In a magnetic system, **M** is a vector which varies as a function of space, **r**, and (due to fluctuations) as a function of time, *t*.

The time is related to the susceptibility by:

$$M_\alpha(t) \propto \chi_{\alpha\alpha}(\omega) H_{0\alpha} e^{-i\omega t} + \chi^*_{\alpha\alpha}(\omega) H^*_{0\alpha} e^{i\omega t}$$

The susceptibility is a complex tensor:

$$\chi_{\alpha\alpha}(\omega) = \chi'_{\alpha\alpha}(\omega) + i\chi''_{\alpha\alpha}(\omega)$$

The rate of energy gain is given by:

$$\frac{d\bar{E}}{dt} = -M_\alpha \frac{dH}{dt} \propto \chi''_{\alpha\alpha}(\omega)$$

and the inelastic cross-section is then related to a *generalized* susceptibility:

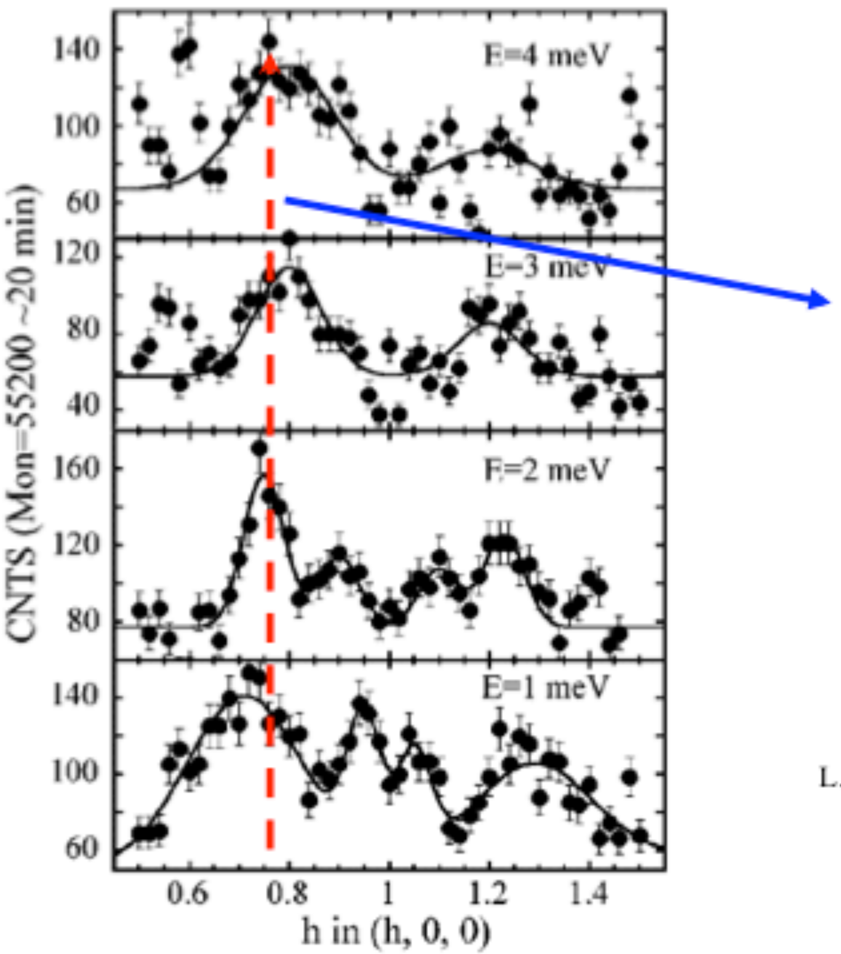
$$\frac{d^2\sigma}{d\Omega dE} \propto \frac{k'}{k} \sum_\alpha \frac{\chi''_{\alpha\alpha}(\mathbf{Q},\omega)}{\left(1 - e^{-\beta\hbar\omega}\right)}$$

T. J. Hicks, *Magnetism in disorder*, Oxford Unversity Press, Oxford, 1995

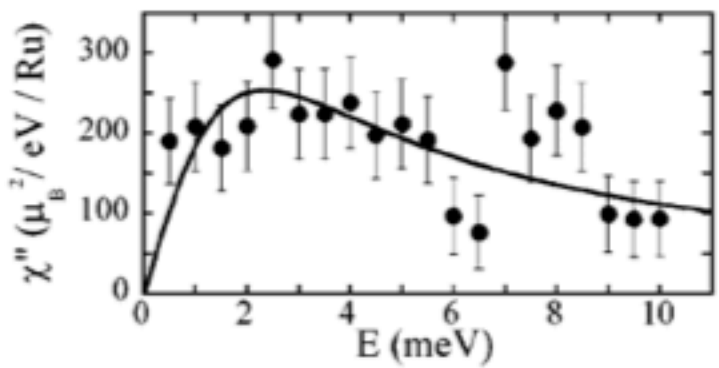
S. W. Lovesey, *Theory of neutron scattering from condensed matter*, Oxford University Press, Oxford, 1986

Spin excitations in $Sr_3Ru_2O_7$

$Sr_3Ru_2O_7$ is from a family of materials that are low-dimensional magnetic, and superconductors



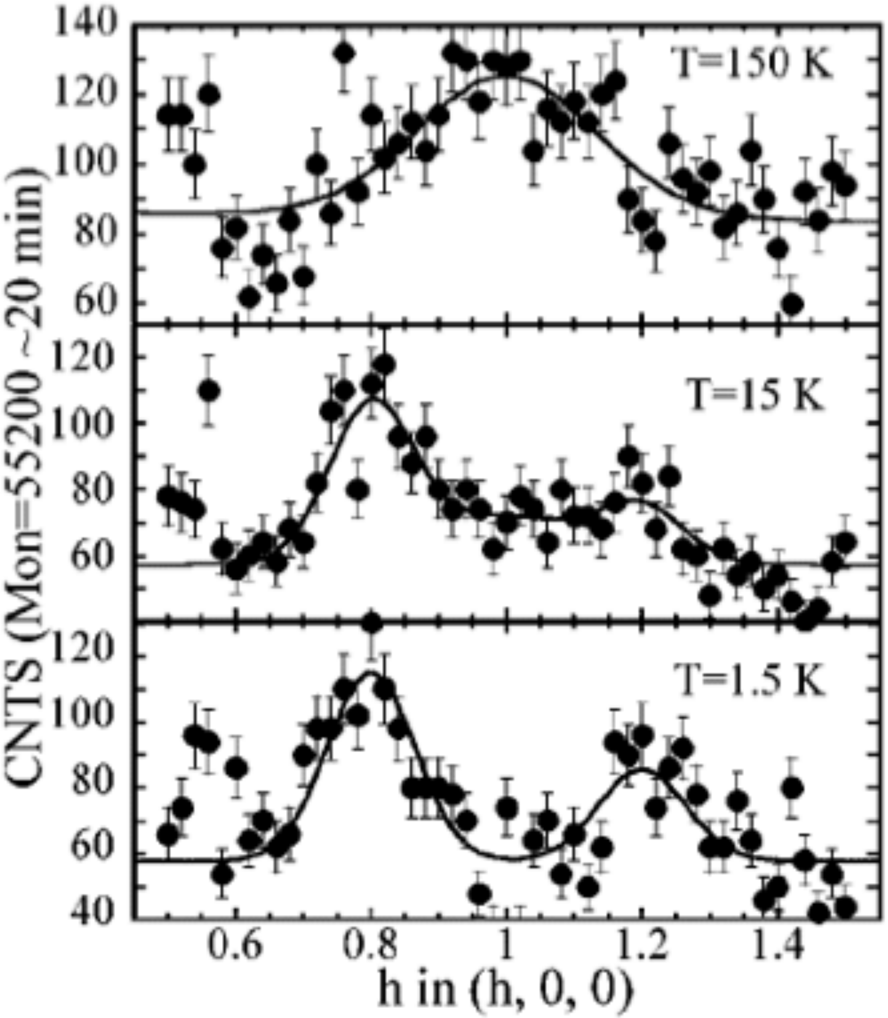
Inelastic neutron scattering at 1.5K

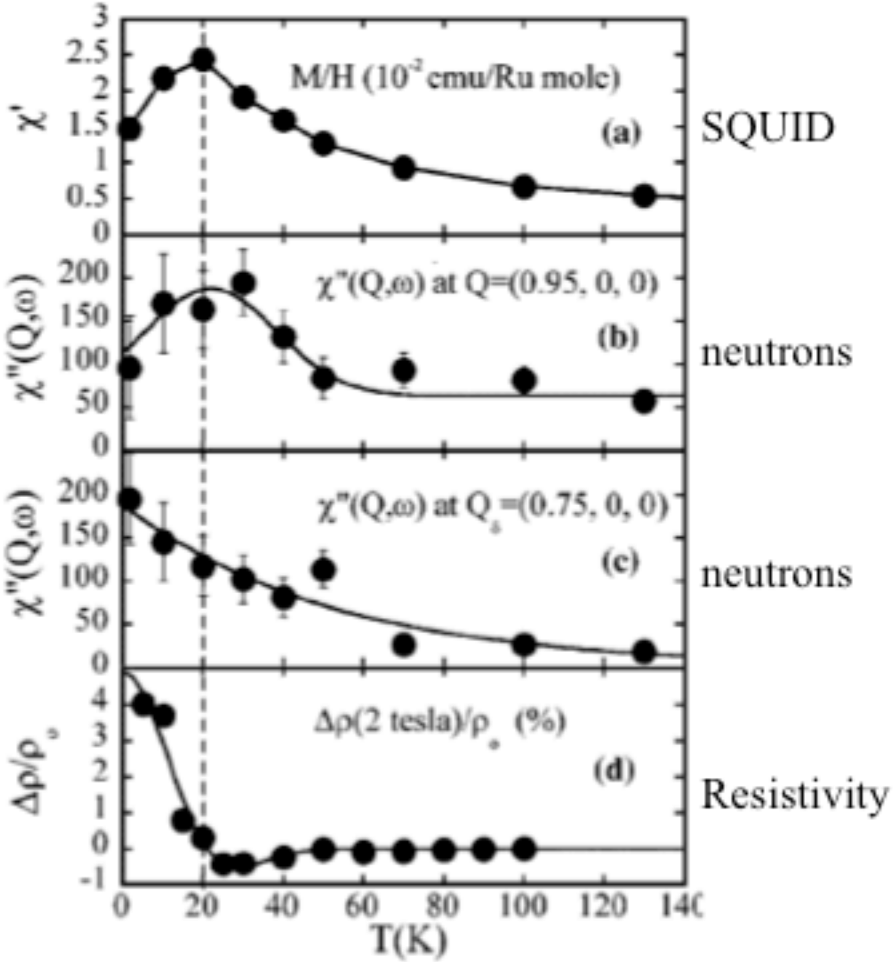


Susceptibility

L. Capogna *et al*., Phys. Rev. B. **87** (1998) 143

Temperature dependence of the spin excitations in $Sr_3Ru_2O_7$

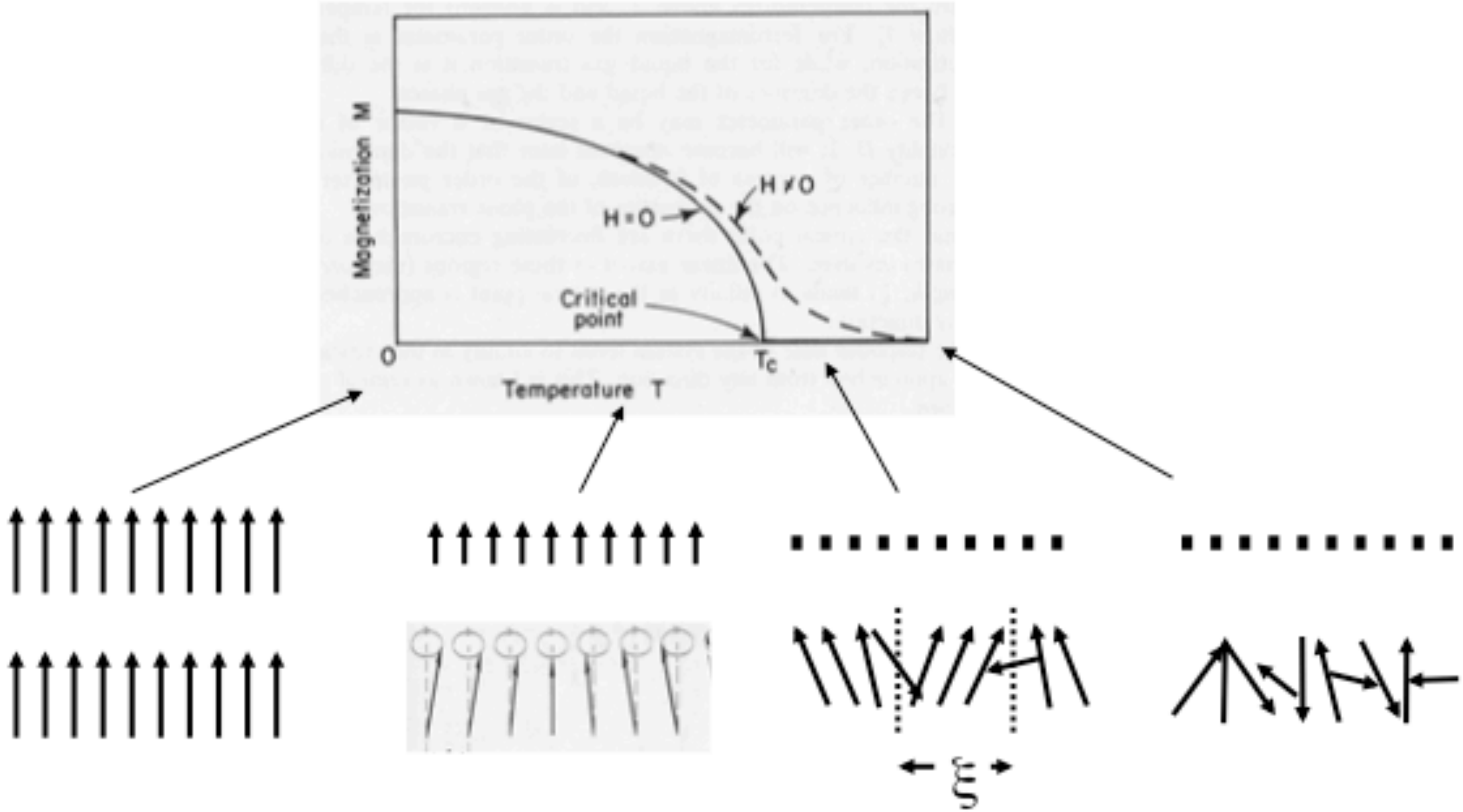




L. Capogna *et al*., Phys. Rev. B **87** (1998) 143

Magnetic phase transitions

The magnetic structure of a simple ferromagnet as a function of temperature



Time-average ($\hbar\omega = 0$)

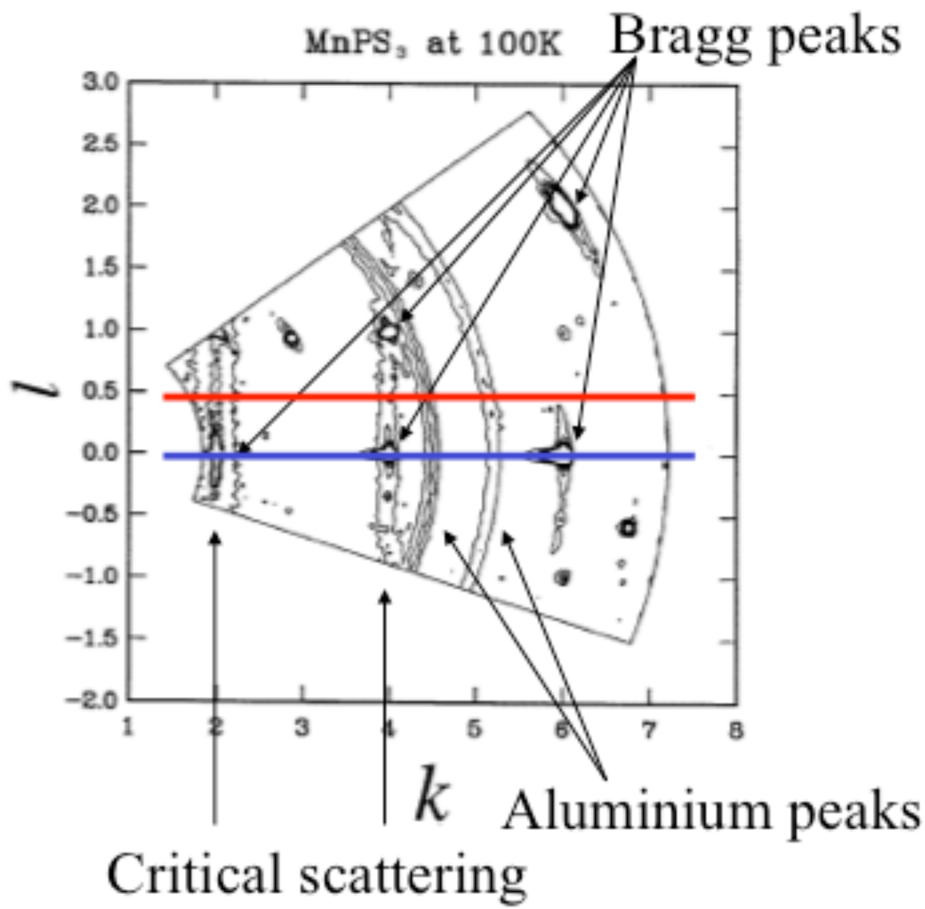
Snapshot ($\int_{-\infty}^{\infty} \hbar\omega$)

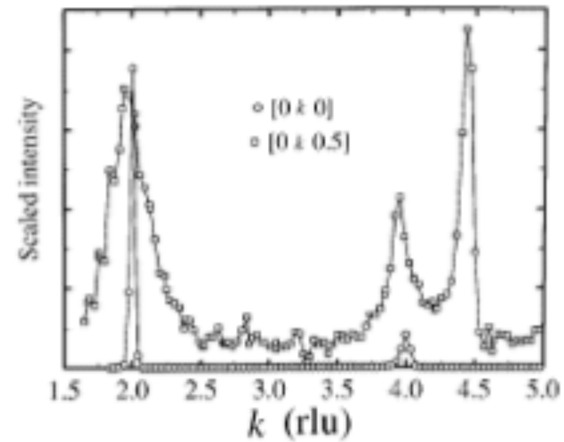
Renormalization theory and universality:

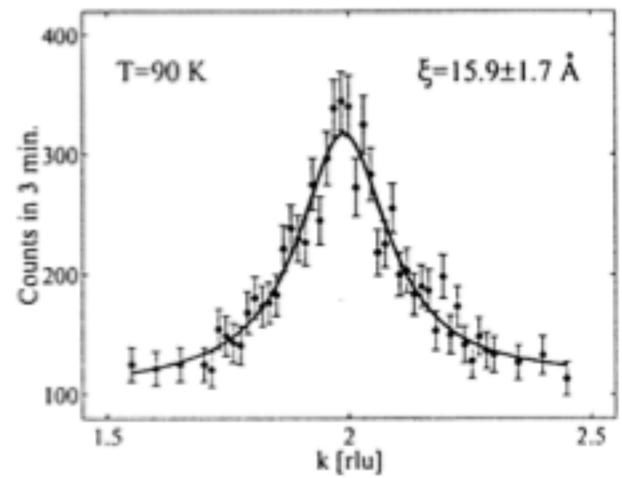
$$M(T) \propto (T-T_N)^{-\beta} \quad \xi(T) \propto (T-T_N)^{-\nu}$$

M. F. Collins, *Magnetic critical scattering*, 1989, Oxford University, Oxford

Magnetic critical scattering in $MnPS_3$





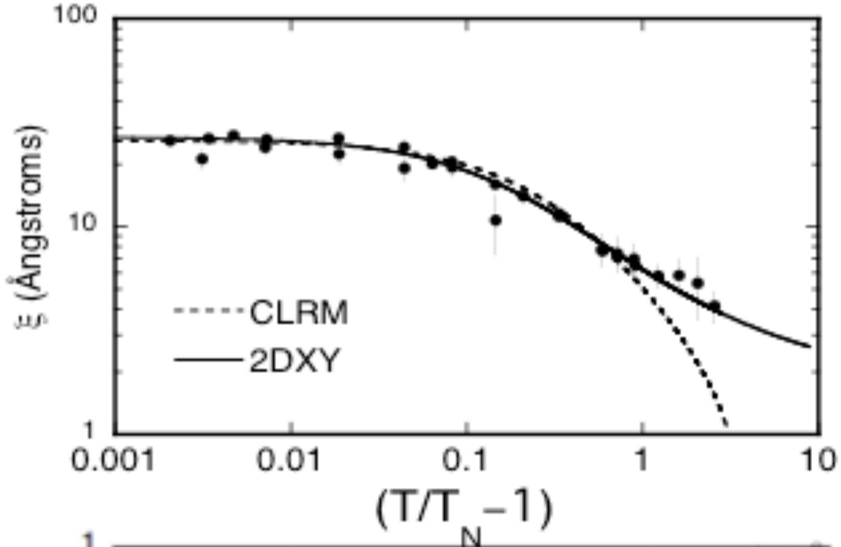


A.R. Wildes *et al*., J. Magn. Magn. Mater. **87** (1998) 143

Magnetic critical scattering in $MnPS_3$

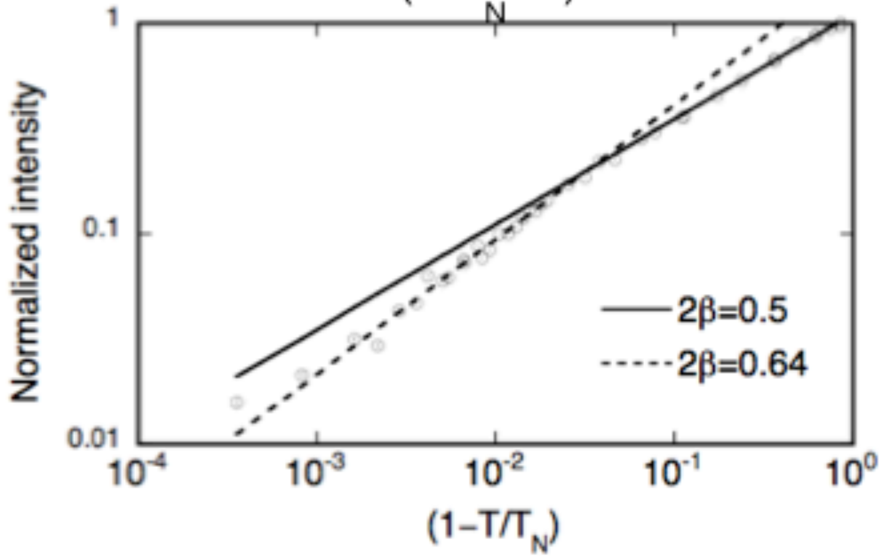
Correlation length
(from the widths of the rods, *energy integrated*)

H.M. Rønnow *et al.*, Physica B **276-278** (2000) 676



Sublattice magnetization
(from the intensities of the Bragg peaks, *zero energy transfer*)

A. R. Wildes *et al.*, PRB **74** (2006) 094422



Elastic scattering

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{m_n}{2\pi\hbar^2}\right)^2 \sum_{\zeta',s'} p_s \left|\langle \boldsymbol{k}', s' | \hat{V}(\mathbf{r}) | \boldsymbol{k}, s\rangle\right|^2$$

$$\propto \int \left|\langle \hat{V} \rangle\right|^2 e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r} + \left(\left|\langle \hat{V}^2 \rangle\right| - \left|\langle \hat{V} \rangle\right|^2\right) \int z(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r}$$

The contribution from deviations from the average structure: *Short-range* order

The contribution from the average structure of the sample: *Long-range* order

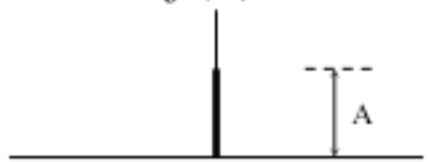
Diffuse scattering from a paramagnet

An ideal paramagnet has no correlation in space *or* time

Fourier transforms

$f(\mathbf{x})$ $F(\mathbf{X})$

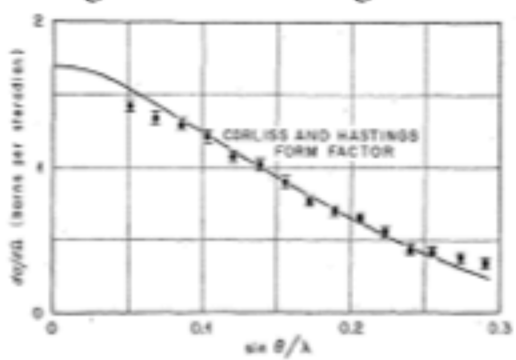
A Delta function



For a paramagnet, $z(\mathbf{r}) = \delta(r)$, and must *integrate* over energy

$$\frac{d\sigma^{\pm\mp}}{d\Omega} \propto \frac{2}{3} f^2(Q) S(S+1)$$

Paramagnetic scattering from MnF_2



R. M. Moon, T. Riste and W. K. Koehler, Phys. Rev. **181** (1969) 920

Neutrons and bulk susceptibility

If neutrons measure the generalized susceptibility,
it must be possible to convert between bulk susceptibility measurements and neutron cross-sections.
This can be done using the Kramers-Krönig relation:

$$\int d\omega \frac{\chi''(\mathbf{Q},\omega)}{\omega} = \pi\chi'(\mathbf{Q},\omega)$$

Integrate the neutron scattering over all energies:

$$\frac{d\sigma}{d\Omega} = \int \hbar\, d\omega \left(\frac{d^2\sigma}{d\Omega d\omega}\right)$$

$$\propto k_B T \sum_{\alpha} \chi'_{\alpha\alpha}(\mathbf{Q},0)$$

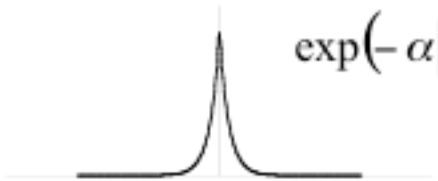
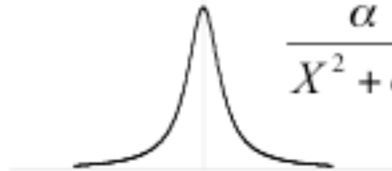
Bulk susceptibility measures the real part of the susceptibility.
Bulk susceptibility averages over all the sample, which is equivalent to $\mathbf{Q} = 0$.

$$i.e.\ \frac{d\sigma}{d\Omega}(\mathbf{Q}=0) \propto k_B T \chi'$$

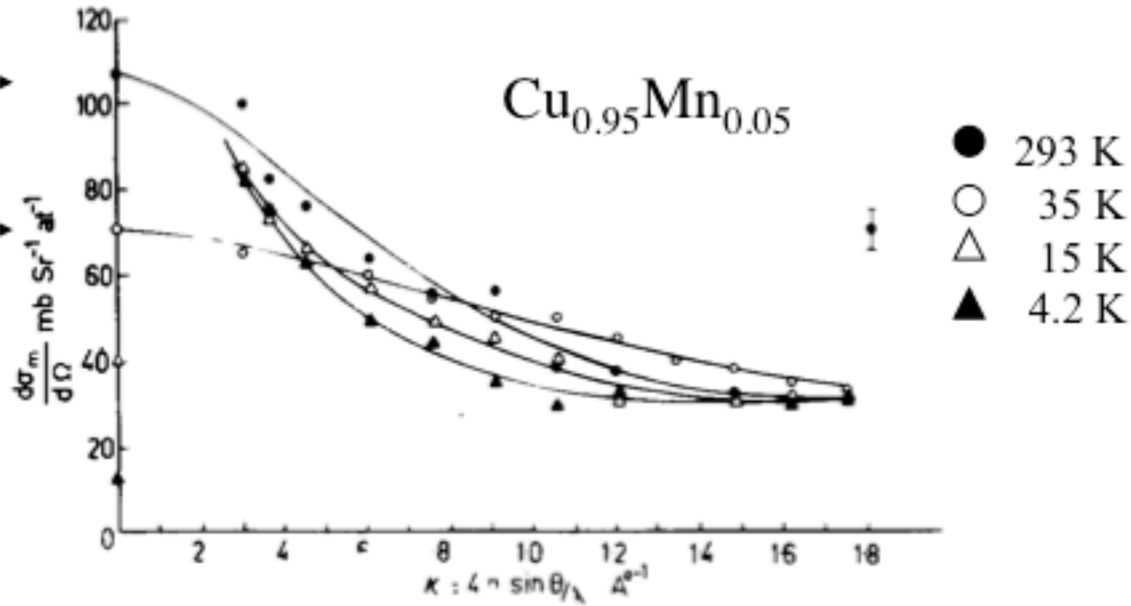
Bulk susceptibility can be put as a point on a neutron scattering plot!

Diffuse scattering from a paramagnet

Paramagnets/spin glasses always have some correlations, particularly in time

	$f(\mathbf{x})$		$F(\mathbf{X})$	
An exponential		$\exp(-\alpha\lvert x\rvert)$		$\dfrac{\alpha}{X^2+\alpha^2}$

Points taken from bulk susceptibility



N. Ahmed and T. J. Hicks, Solid State Comm. **15** (1974) 415

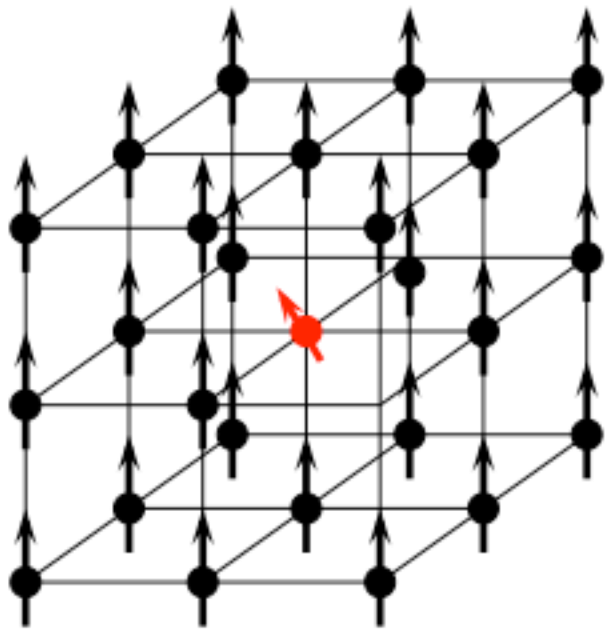
T. J. Hicks, *Magnetism in disorder*, 1995, Clarendon, Oxford

Diffuse scattering and short-ranged order

$$\left(\left|\left\langle \hat{V}^2 \right\rangle\right| - \left|\left\langle \hat{V} \right\rangle\right|^2\right) \int z(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r}$$

Take a magnetic moment at the origin.

$z(\mathbf{r})$ describes the probability of finding the same moment at position $\mathbf{r}$.



"Short-ranged" order can still extend over many nanometres

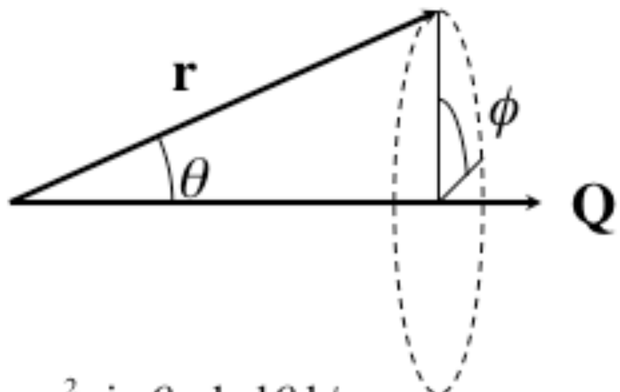
Calculation of the magnetic structure of $Ni_{0.65}Fe_{0.35}$

256 atoms, Yang *et al*. J. Appl. Phys. **81** (1997) 3973

Isotropic Fourier Transforms

Most magnetic diffuse scattering experiments are done on powders

In the case of scattering that is isotropic in three dimensions:



$$\mathrm{d}\mathbf{r} = r^2 \sin\theta \cdot \mathrm{d}r\mathrm{d}\theta\mathrm{d}\phi$$

$$\mathbf{Q} \cdot \mathbf{r} = Qr\cos\theta$$

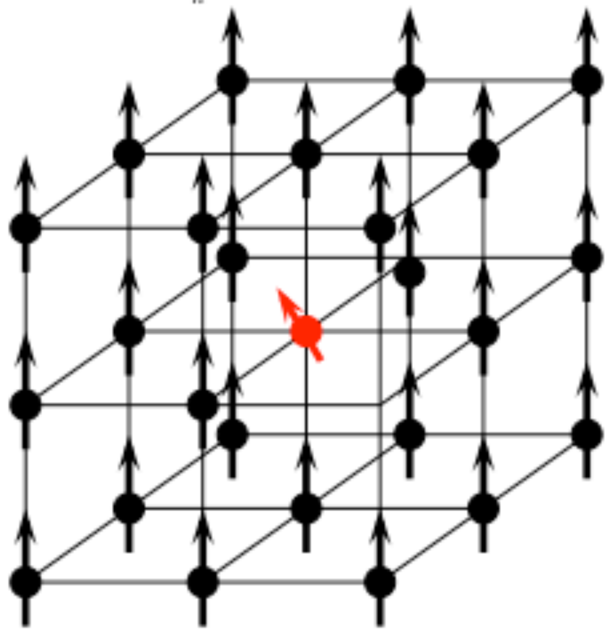
$$\int_{-\infty}^{\infty} f(\mathbf{Q}) \exp(i\mathbf{Q} \cdot \mathbf{r}) \cdot \mathrm{d}\mathbf{r} = 4\pi \int_{0}^{\infty} r^2 f(r) \frac{\sin Qr}{Qr} \cdot \mathrm{d}r$$

Diffuse scattering and short-ranged order

$$\left(\left| \left\langle \hat{V}^2 \right\rangle \right| - \left| \left\langle \hat{V} \right\rangle \right|^2 \right) \int z(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r}$$

In a crystalline material the integral becomes a sum over neighbours

$$\propto S(S+1) + \sum_{R_n} \left\langle \mathbf{S}_0 \cdot \mathbf{S}_{R_n} \right\rangle \frac{\sin(QR_n)}{QR_n}$$

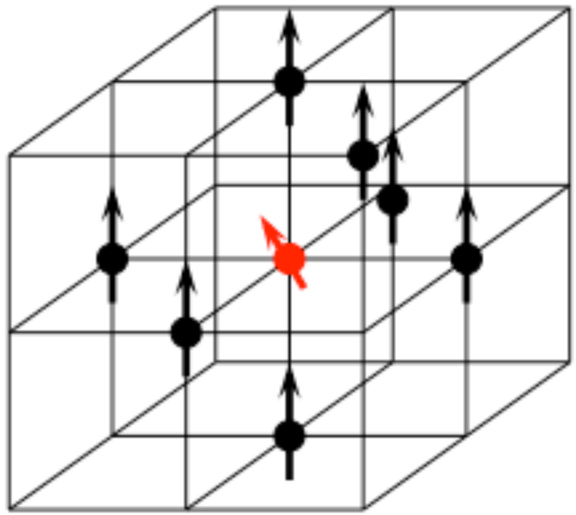


Diffuse scattering and short-ranged order

$$\left(\left|\left\langle \hat{V}^2 \right\rangle\right| - \left|\left\langle \hat{V} \right\rangle\right|^2\right) \int z(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r}$$

In a crystalline material the integral becomes a sum over neighbours

$$\propto S(S+1) + \sum_{R_n} \left\langle \mathbf{S}_0 \cdot \mathbf{S}_{R_n} \right\rangle \frac{\sin(QR_n)}{QR_n}$$



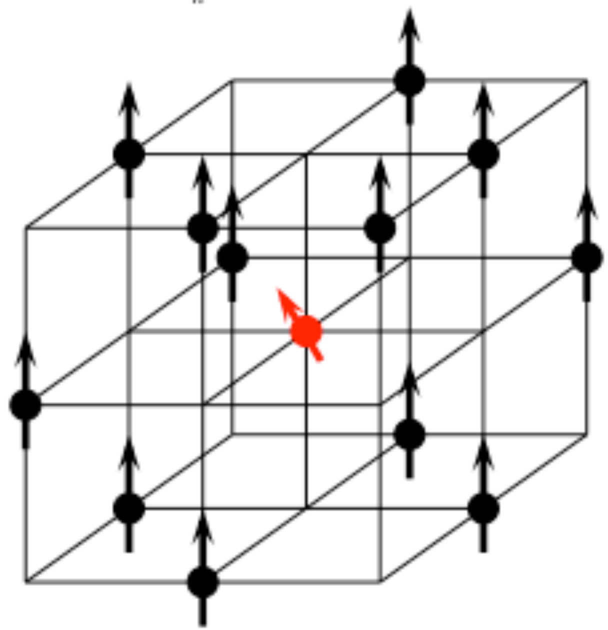
First nearest neighbour shell

Diffuse scattering and short-ranged order

$$\left(\left|\left\langle \hat{V}^2 \right\rangle\right| - \left|\left\langle \hat{V} \right\rangle\right|^2\right) \int z(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r}$$

In a crystalline material the integral becomes a sum over neighbours

$$\propto S(S+1) + \sum_{R_n} \left\langle \mathbf{S}_0 \cdot \mathbf{S}_{R_n} \right\rangle \frac{\sin(QR_n)}{QR_n}$$



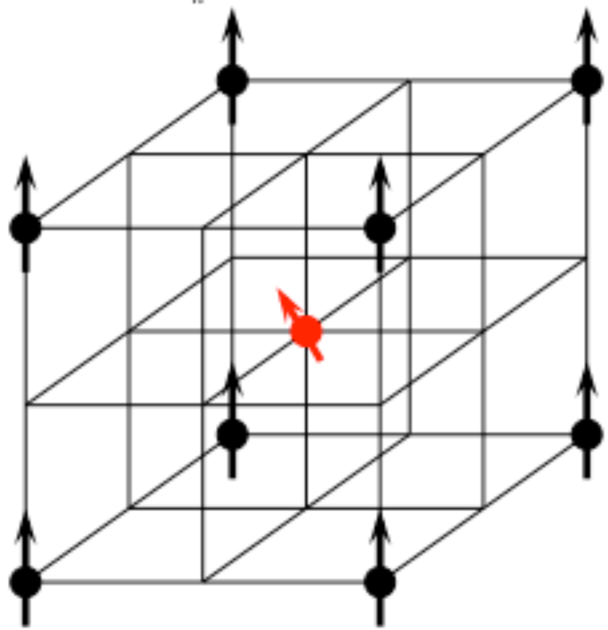
Second nearest neighbour shell
(note: already out of the box!)

Diffuse scattering and short-ranged order

$$\left(\left|\left\langle \hat{V}^2 \right\rangle\right| - \left|\left\langle \hat{V} \right\rangle\right|^2\right) \int z(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r}$$

In a crystalline material the integral becomes a sum over neighbours

$$\propto S(S+1) + \sum_{R_n} \left\langle \mathbf{S}_0 \cdot \mathbf{S}_{R_n} \right\rangle \frac{\sin(QR_n)}{QR_n}$$

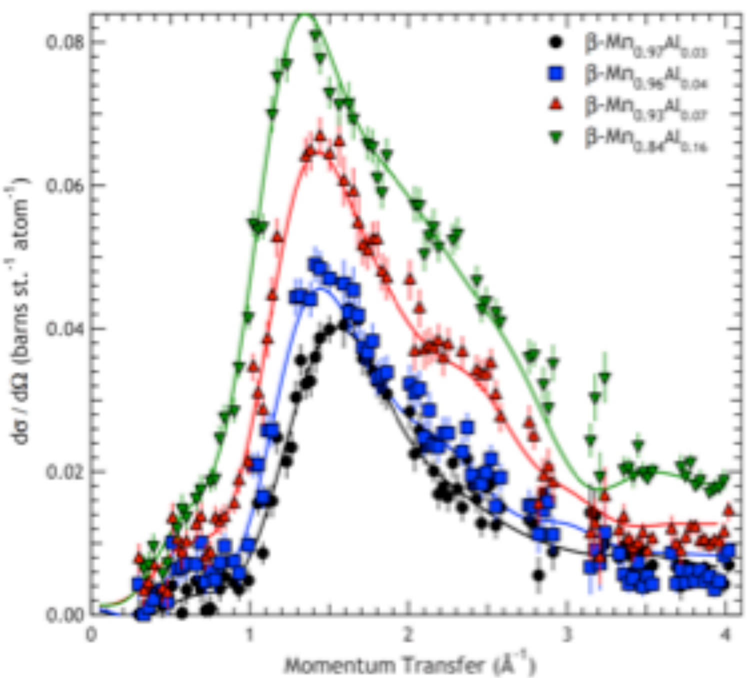


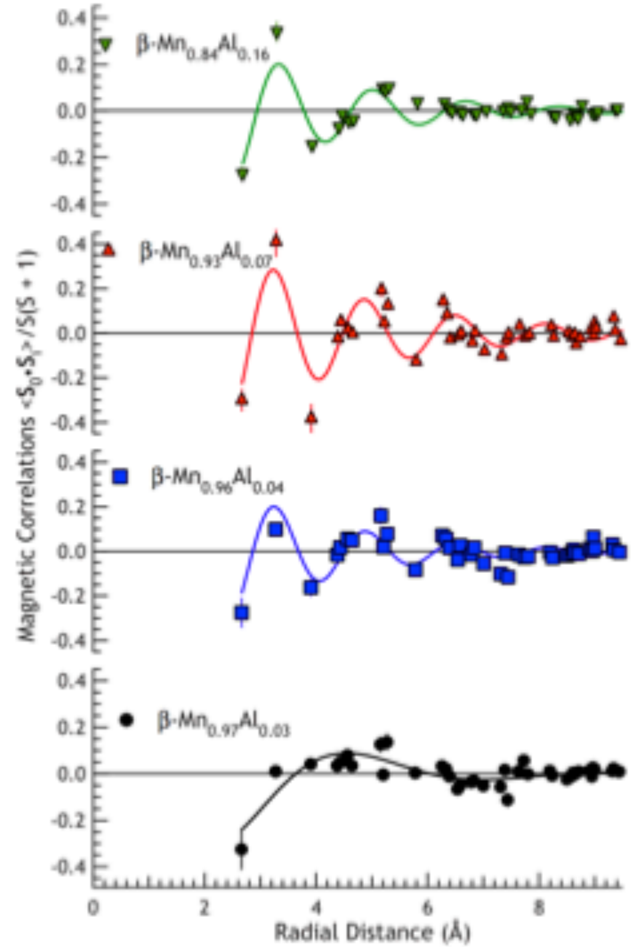
Third nearest neighbour shell
(note: already out of the box!)

Short-range order in β-MnAl

$$\left(\left|\left\langle \hat{V}^2 \right\rangle\right| - \left|\left\langle \hat{V} \right\rangle\right|^2\right) \int z(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r}$$

$$\propto S(S+1) + \sum_{R_n} \left\langle \mathbf{S}_0 \cdot \mathbf{S}_{R_n} \right\rangle \frac{\sin(QR_n)}{QR_n}$$

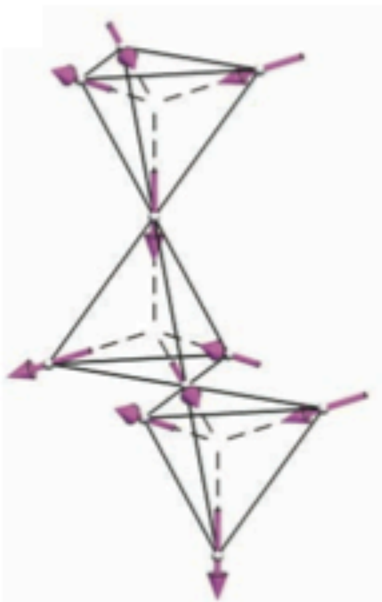




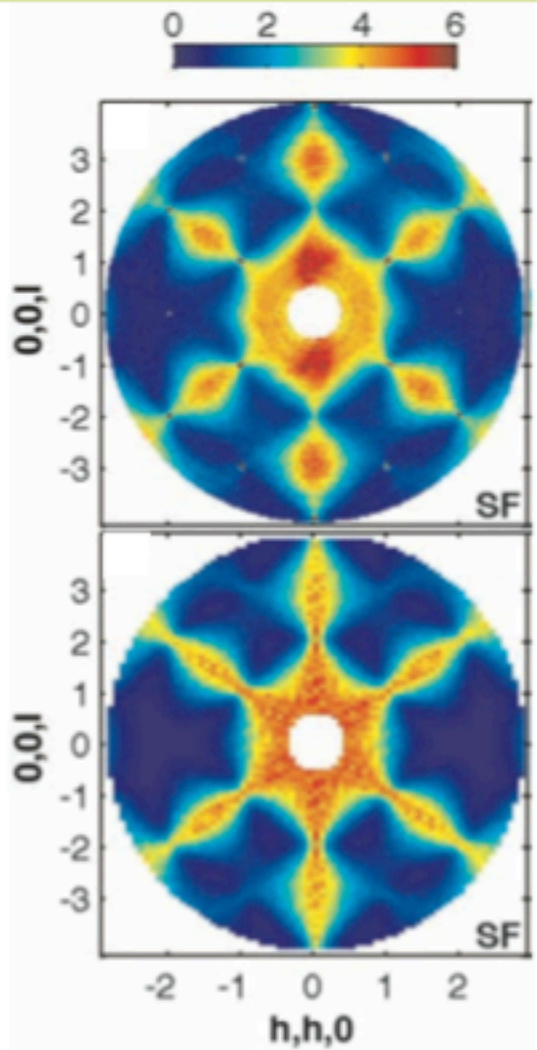
J. R. Stewart *et al*., Phys. Rev. B **78** (2008) 014428

Diffuse scattering from $Ho_2Ti_2O_7$

$Ho_2Ti_2O_7$ is a 'spin-ice' compound



T. Fennell *et al.*, Science **326** (2009) 1177582

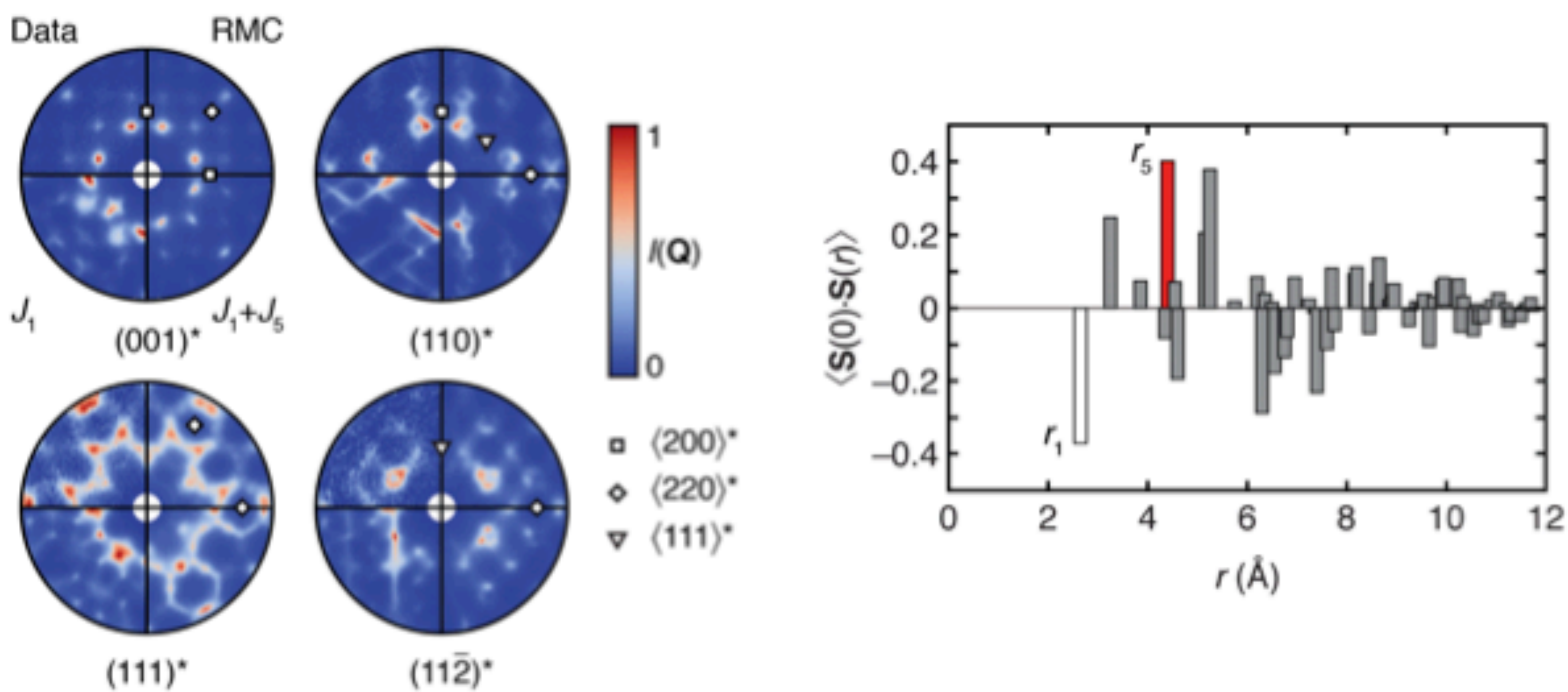


Data

Calculation

Diffuse scattering in $Mn_{0.8}Co_{0.2}$

Use of "Reverse Monte-Carlo" to fit data



J. A. M. Paddison *et al.*, PRL **110** (2013) 267207

Conclusions

- Learn your Fourier transforms (for space *and* time)
- Neutron scattering can measure a susceptibility
- Be conscious of the relationship between time and energy, particularly for diffuse scattering
- Analysing diffuse scattering is non-trivial.

Bonus lecture: The plan

- Neutron polarization analysis
- The *spin dependent* neutron potential
- Contributions to the scattering
  - Nuclear spin incoherent
  - Nuclear coherent and isotopic incoherent
  - Magnetic
- **P** and **Q**

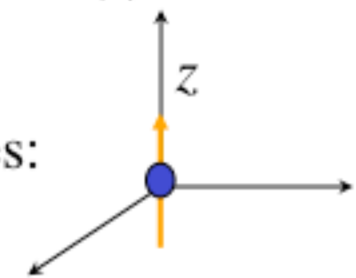
Polarized neutrons

REMEMBER: the only visible components of the magnetization are PERPENDICULAR to **Q**

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{m_n}{2\pi\hbar^2}\right)^2 \sum_{\xi',s'} p_s \left| \langle \boldsymbol{k}', s' | \hat{V}(\mathbf{r}) | \boldsymbol{k}, s \rangle \right|^2$$

$$U^{ss'} = \langle s' | \hat{V} | s \rangle = \langle s' | b + B\hat{\mathbf{I}} \cdot \hat{\boldsymbol{\sigma}} - \mathbf{M}_\perp \cdot \hat{\boldsymbol{\sigma}} | s \rangle$$

Neutron polarization coordinates:
z is parallel to **P**



$$U^{++} = b - M_{\perp z} + BI_z$$

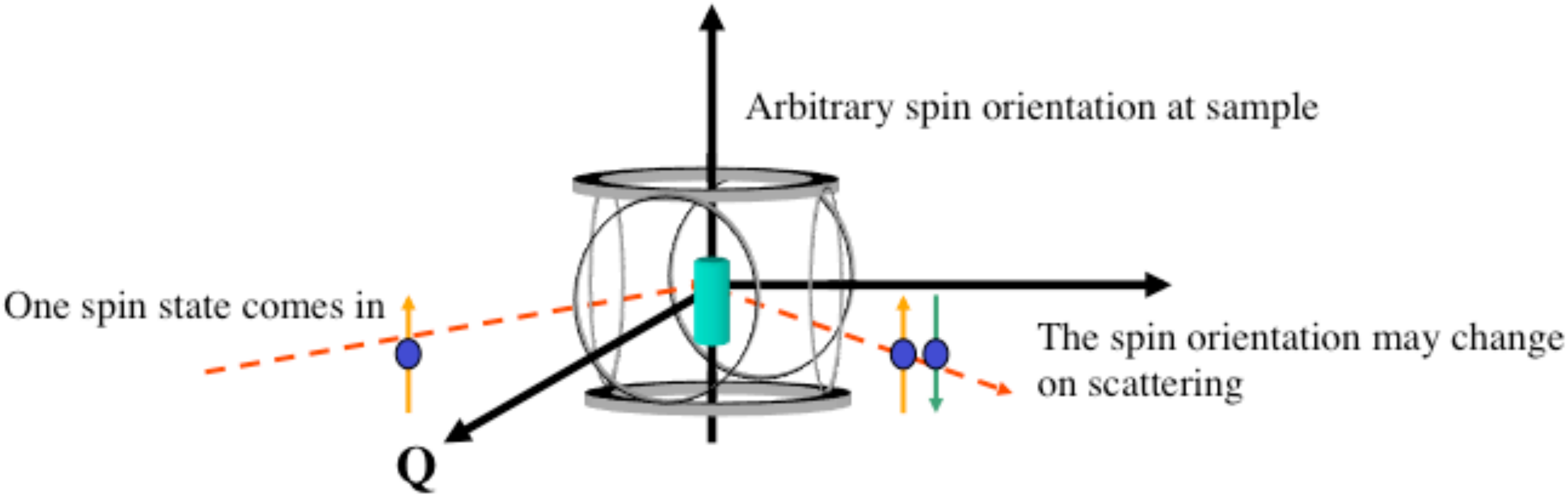
$$U^{--} = b + M_{\perp z} - BI_z$$

$$U^{+-} = -\left(M_{\perp x} + iM_{\perp y}\right) + B\left(I_x + iI_y\right)$$

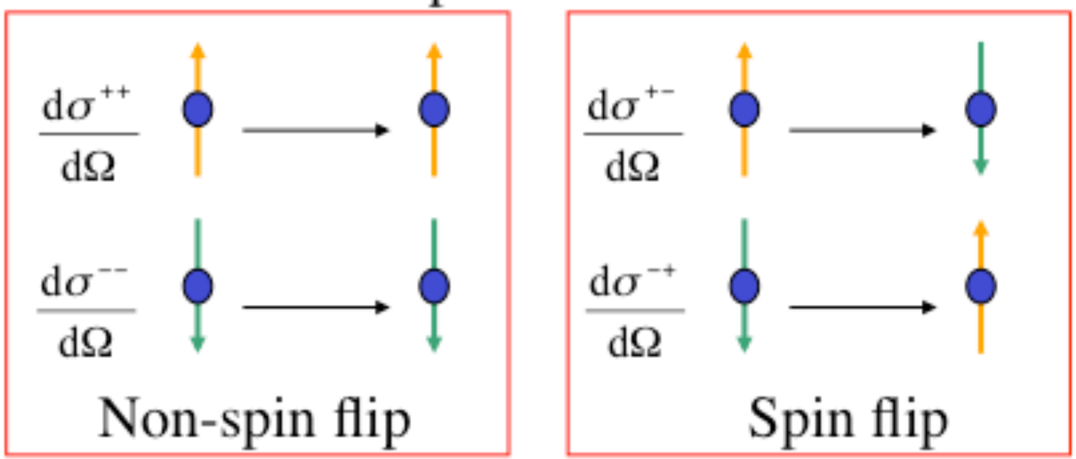
$$U^{-+} = -\left(M_{\perp x} - iM_{\perp y}\right) + B\left(I_x - iI_y\right)$$

R. M. Moon, T. Riste and W. K. Koehler, Phys. Rev. **181** (1969) 920
J. R. Stewart *et al*., J. Appl. Cryst. **42** (2009) 69

Polarized neutrons



Four possibilities:



Nuclear coherent scattering

$$\frac{d\sigma}{d\Omega} = \left(\frac{m_n}{2\pi\hbar^2}\right)^2 \sum_{\zeta',s'} p_s \left| \langle \boldsymbol{k}', s' | \hat{V}(\mathbf{r}) | \boldsymbol{k}, s \rangle \right|^2$$

$$U^{ss'} = \langle s' | \hat{V} | s \rangle = \langle s' | b + B\hat{\mathbf{I}} \cdot \hat{\boldsymbol{\sigma}} - \mathbf{M}_\perp \cdot \hat{\boldsymbol{\sigma}} | s \rangle$$

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$$U^{-+} = -\left(M_{\perp x} - iM_{\perp y}\right) + B\left(I_x - iI_y\right)$$

b only appears in the non-spin flip potentials

Nuclear coherent scattering is always non-spin flip

Nuclear spin incoherent scattering

$$U^{++} = BI_z$$

$$U^{--} = -BI_z$$

$$U^{+-} = B\left(I_x + iI_y\right)$$

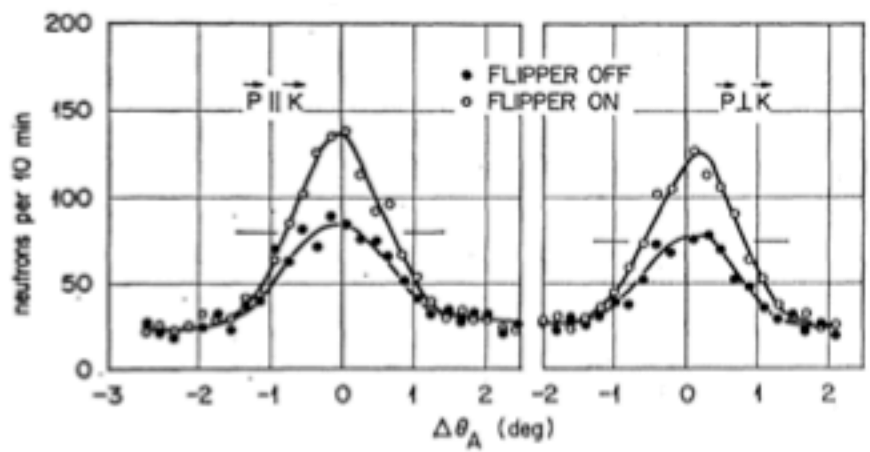
$$U^{-+} = B\left(I_x - iI_y\right)$$

$$\frac{d\sigma^{++}}{d\Omega} = \frac{d\sigma^{--}}{d\Omega} = \frac{d\sigma^{nsf}}{d\Omega}$$

$$\frac{d\sigma^{+-}}{d\Omega} = \frac{d\sigma^{-+}}{d\Omega} = \frac{d\sigma^{sf}}{d\Omega}$$

$$\frac{d\sigma^{sf}}{d\Omega} = 2\frac{d\sigma^{nsf}}{d\Omega}$$

Scattering from Vanadium



R. M. Moon, T. Riste and W. K. Koehler, Phys. Rev. **181** (1969) 920

R. M. Moon, T. Riste and W. K. Koehler, Phys. Rev. **181** (1969) 920
J. R. Stewart *et al.*, J. Appl. Cryst. **42** (2009) 69

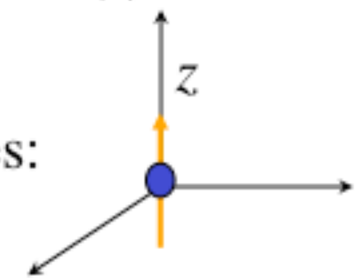
Polarized neutrons

REMEMBER: the only visible components of the magnetization are PERPENDICULAR to **Q**

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{m_n}{2\pi\hbar^2}\right)^2 \sum_{\xi',s'} p_s \left|\langle \boldsymbol{k}', s' | \hat{V}(\mathbf{r}) | \boldsymbol{k}, s \rangle\right|^2$$

$$U^{ss'} = \langle s' | \hat{V} | s \rangle = \langle s' | b + B\hat{\mathbf{I}} \cdot \hat{\boldsymbol{\sigma}} - \mathbf{M}_\perp \cdot \hat{\boldsymbol{\sigma}} | s \rangle$$

Neutron polarization coordinates:
z is parallel to **P**



Neglect nuclear spin for the moment

$$U^{++} = b - M_{\perp z}$$

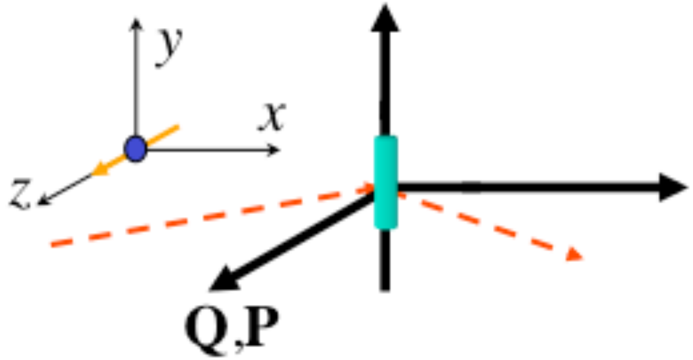
$$U^{--} = b + M_{\perp z}$$

$$U^{+-} = -\left(M_{\perp x} + iM_{\perp y}\right)$$

$$U^{-+} = -\left(M_{\perp x} - iM_{\perp y}\right)$$

R. M. Moon, T. Riste and W. K. Koehler, Phys. Rev. **181** (1969) 920

Elastic scattering, **P** ∥ **Q**



$$\frac{d\sigma^{\pm\pm}}{d\Omega} = \left|\int b(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r}\right|^2$$

$$\frac{d\sigma^{\pm\mp}}{d\Omega} = \left|\int \left(-M_{\perp x}(\mathbf{r}) \pm iM_{\perp y}(\mathbf{r})\right) e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r}\right|^2$$

$$= \int \left[M_\perp(\mathbf{r}_i) M_\perp^*(\mathbf{r}_j) \mp i\hat{z}\left(M_\perp(\mathbf{r}_i) \times M_\perp^*(\mathbf{r}_j)\right)\right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \cdot d\mathbf{r}$$

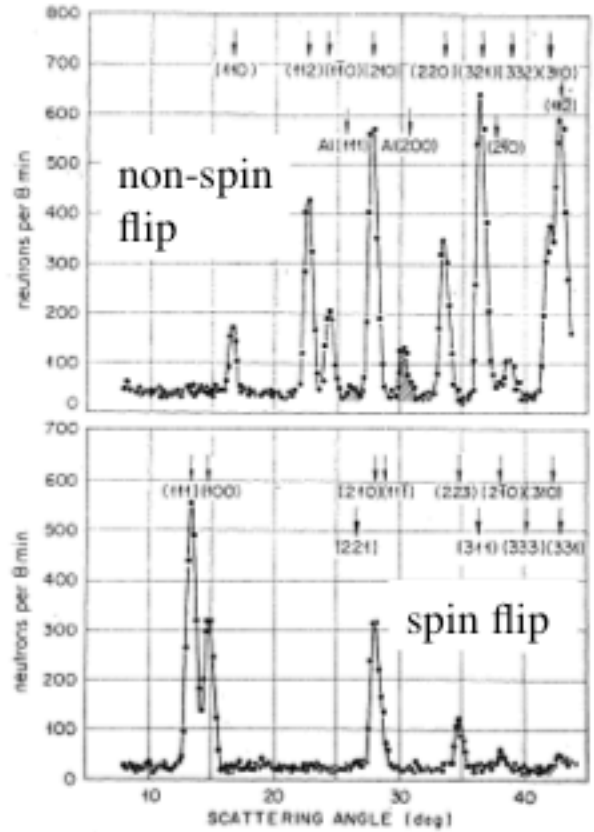
- The non spin-flip cross-sections have only nuclear components
- The spin-flip cross-sections have only the magnetic components
- There is a *complete separation* of nuclear from magnetic scattering.
- The cross term in the spin flip cross-sections usually cancels,
  i.e. $M_\perp(\mathbf{r}_i) \times M_\perp^*(\mathbf{r}_j) = -\left(M_\perp(\mathbf{r}_i) \times M_\perp^*(\mathbf{r}_j)\right)$

(isotopic incoherent is purely non spin-flip
nuclear spin incoherent is 2/3 spin-flip and 1/3 non spin-flip)

A polarization analysis measurement of α-Fe_2O_3

α-Fe_2O_3 is antiferromagnetic.

Powder diffraction gives Bragg peaks with mixed nuclear and magnetic intensities



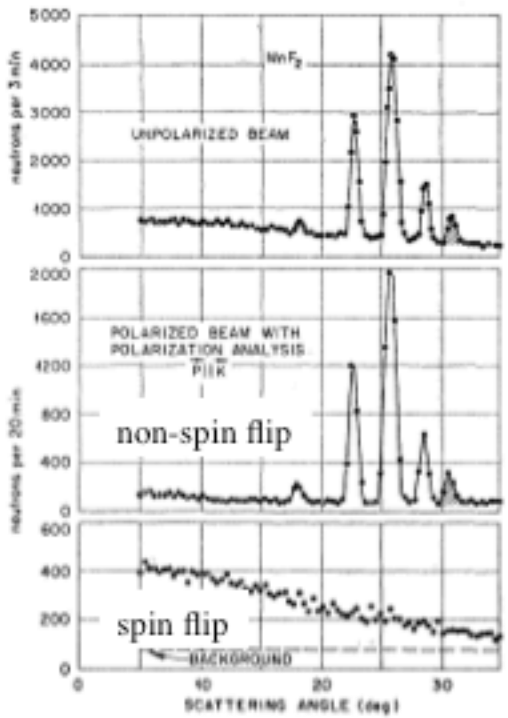
Nuclear Bragg peaks

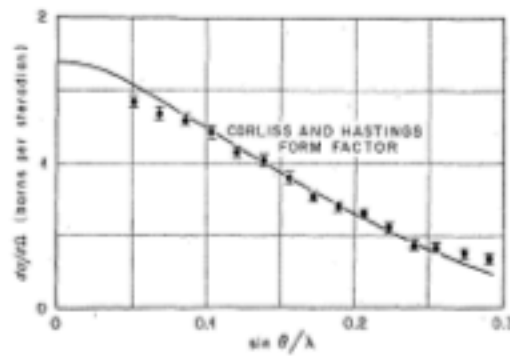
Magnetic Bragg peaks

R. M. Moon, T. Riste and W. K. Koehler, Phys. Rev. **181** (1969) 920

Diffuse scattering from a paramagnet, MnF_2

For a paramagnet, $z(\mathbf{r}) = \delta(r)$, $\quad \dfrac{d\sigma^{\pm\mp}}{d\Omega} \propto \dfrac{2}{3} f^2(Q) S(S+1)$

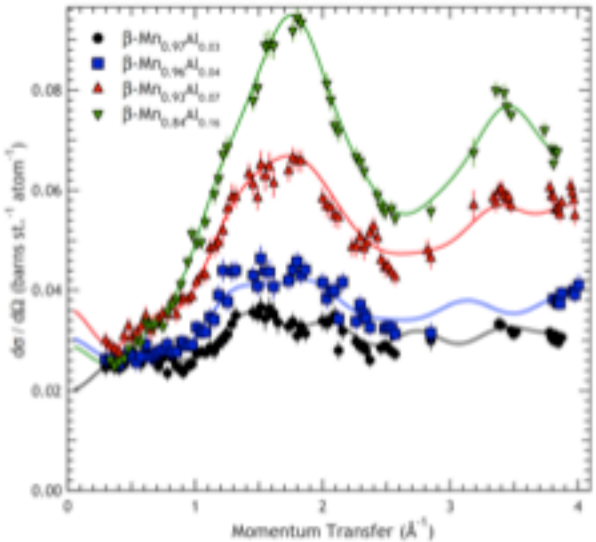




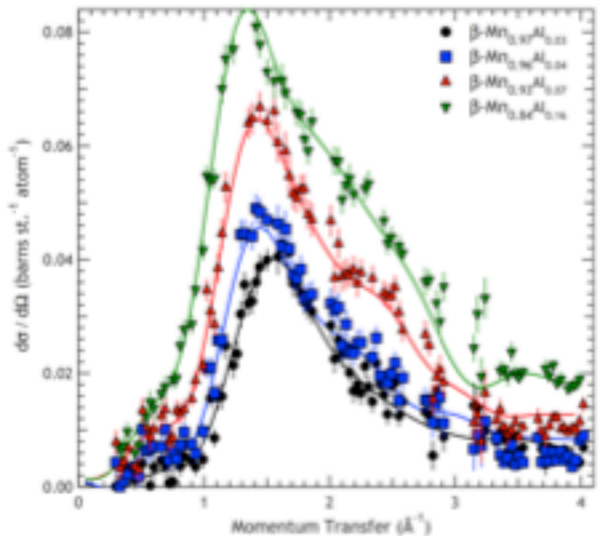
R. M. Moon, T. Riste and W. K. Koehler, Phys. Rev. **181** (1969) 920

Short-range order in β-MnAl

Nuclear diffuse

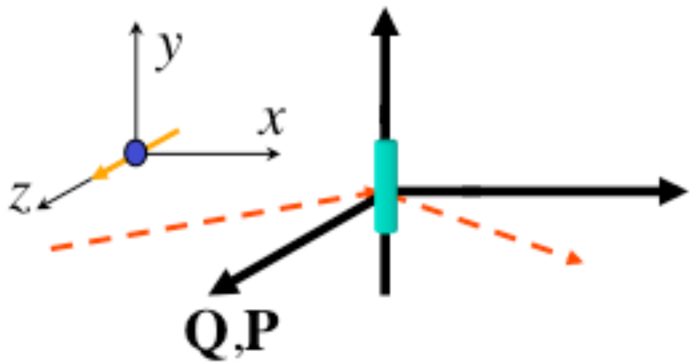


Magnetic diffuse



J. R. Stewart *et al*., Phys. Rev. B **78** (2008) 014428

Elastic scattering, **P** || **Q**



$$\frac{d\sigma^{\pm\pm}}{d\Omega} = \left| \int b(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r} \right|^2$$

$$\frac{d\sigma^{\pm\mp}}{d\Omega} = \left| \int \left(-M_{\perp x}(\mathbf{r}) \pm i M_{\perp y}(\mathbf{r}) \right) e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r} \right|^2$$

$$= \int \left[M_{\perp}(\mathbf{r}_i) M^*_{\perp}(\mathbf{r}_j) \mp i\hat{z} \left(M_{\perp}(\mathbf{r}_i) \times M^*_{\perp}(\mathbf{r}_j) \right) \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \cdot d\mathbf{r}$$

- The non spin-flip cross-sections have only nuclear components
- The spin-flip cross-sections have only the magnetic components
- There is a *complete separation* of nuclear from magnetic scattering.
- The cross term in the spin flip cross-sections usually cancels,
  i.e. $M_{\perp}(\mathbf{r}_i) \times M^*_{\perp}(\mathbf{r}_j) = -\left(M_{\perp}(\mathbf{r}_i) \times M^*_{\perp}(\mathbf{r}_j) \right)$

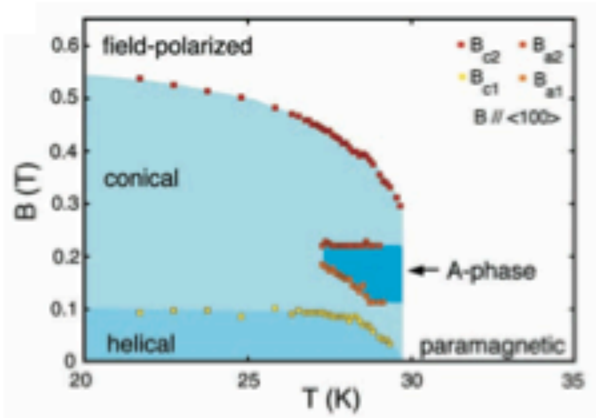
(but sometimes it doesn't)

(isotopic incoherent is purely non spin-flip
nuclear spin incoherent is 2/3 spin-flip and 1/3 non spin-flip)

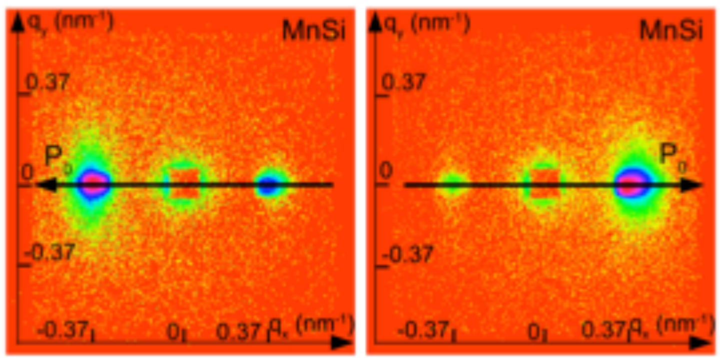
Elastic scattering, **P** ∥ **Q**

$$M_{\perp}(\mathbf{r}_i) \times M^*_{\perp}(\mathbf{r}_j) \neq -\left(M_{\perp}(\mathbf{r}_i) \times M^*_{\perp}(\mathbf{r}_j)\right)$$

MnSi

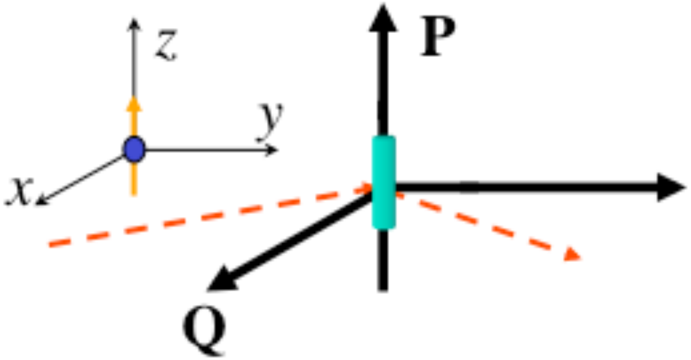


S. Mühlbauer *et al*., Science **323** (2009) 915



S. V. Grigoriev *et al*., Phys. Rev. Lett. **102** (2009) 037204

Elastic scattering, $\mathbf{P} \perp \mathbf{Q}$



$$\frac{d\sigma^{\pm\pm}}{d\Omega} = \left| \int \left(b(\mathbf{r}) \mp M_{\perp z}(\mathbf{r}) \right) e^{i\mathbf{Q}\cdot\mathbf{r}} d\mathbf{r} \right|^2$$

$$\frac{d\sigma^{\pm\mp}}{d\Omega} = \left| \int M_{\perp y}(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} d\mathbf{r} \right|^2$$

- The non-spin flip cross-sections probe the components of the magnetization parallel to the neutron polarization
- The spin-flip cross-sections probe the components of the magnetization that are perpendicular to *both* the polarization and to **Q**
- There can be a difference between the two non-spin flip cross-sections, the difference is called nuclear-magnetic interference.
- The two spin-flip cross-sections are equivalent

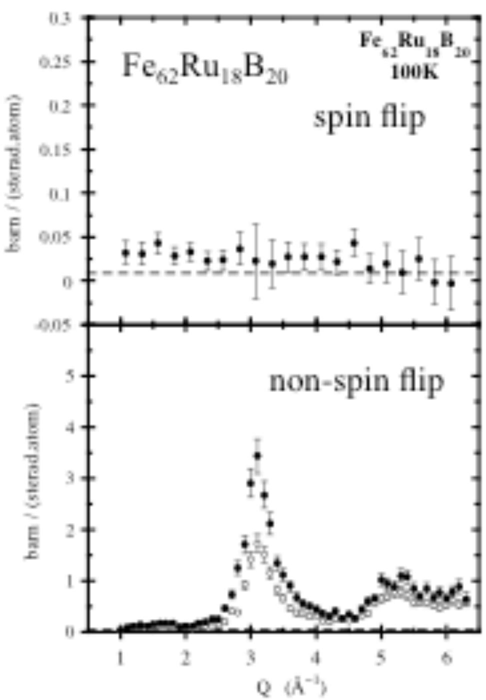
(isotopic incoherent is purely non spin-flip
nuclear spin incoherent is 2/3 spin-flip and 1/3 non spin-flip)

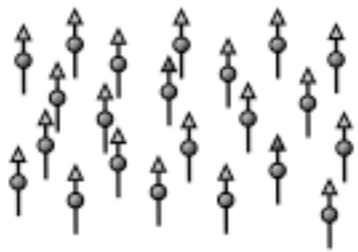
Diffuse scattering in metallic glasses, $Fe_{80-x}Ru_xB_{20}$

Metallic glasses have no long-ranged order, but each magnetic moment has a common axis. They are therefore *ferromagnetic*.

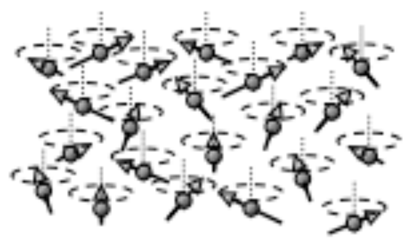
$$\frac{d\sigma^{\pm\pm}}{d\Omega} = \left| \int \left(b(\mathbf{r}) \mp M_{\perp z}(\mathbf{r}) \right) e^{i\mathbf{Q}\cdot\mathbf{r}} d\mathbf{r} \right|^2$$

$$\frac{d\sigma^{\pm\mp}}{d\Omega} = \left| \int M_{\perp y}(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} d\mathbf{r} \right|^2$$

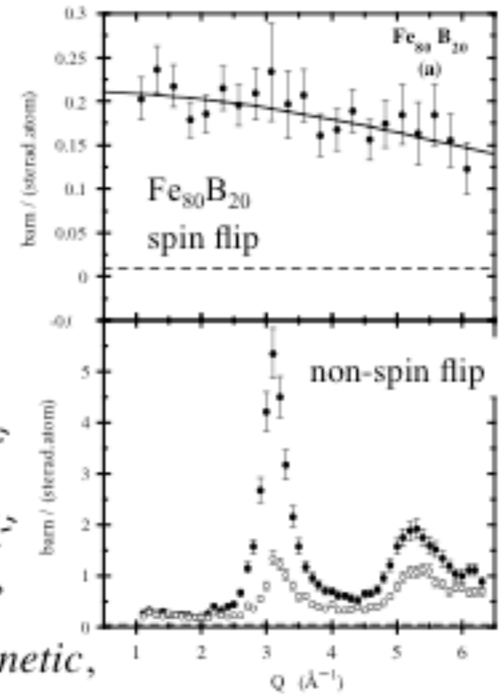




Collinear ferromagnetic.

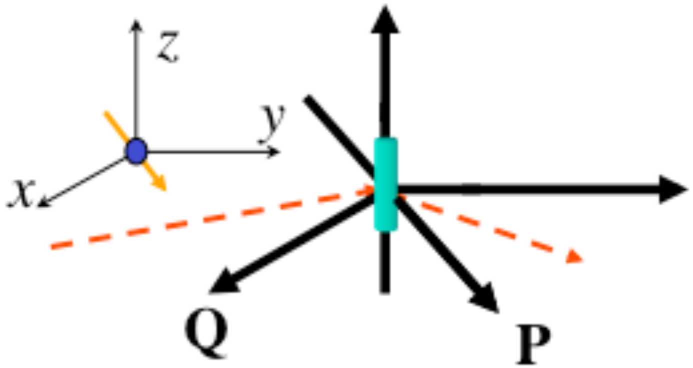


Non-collinear ferromagnetic, asperromagnetic

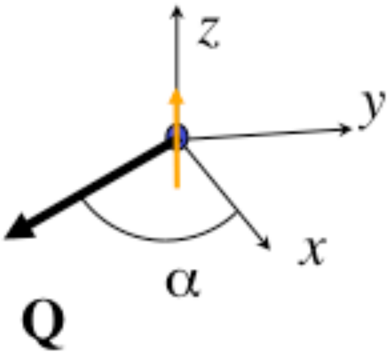


A. R. Wildes and N. Cowlam, *Recent research developments in magnetism and magnetic materials*, 2003, Transworld Research Network, India

Elastic scattering



XYZ Polarization Analysis



Condition: the sample has **no net magnetization**

$$\left(\frac{d\sigma}{d\Omega}\right)_X^{NSF} = \frac{1}{2}\sin^2\alpha\left(\frac{d\sigma}{d\Omega}\right)_{mag} + \frac{1}{3}\left(\frac{d\sigma}{d\Omega}\right)_{SI} + \left(\frac{d\sigma}{d\Omega}\right)_{nuc+II}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_X^{SF} = \frac{1}{2}\left(\cos^2\alpha + 1\right)\left(\frac{d\sigma}{d\Omega}\right)_{mag} + \frac{2}{3}\left(\frac{d\sigma}{d\Omega}\right)_{SI}$$

x-direction

$$\left(\frac{d\sigma}{d\Omega}\right)_Y^{NSF} = \frac{1}{2}\cos^2\alpha\left(\frac{d\sigma}{d\Omega}\right)_{mag} + \frac{1}{3}\left(\frac{d\sigma}{d\Omega}\right)_{SI} + \left(\frac{d\sigma}{d\Omega}\right)_{nuc+II}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_Y^{SF} = \frac{1}{2}\left(\sin^2\alpha + 1\right)\left(\frac{d\sigma}{d\Omega}\right)_{mag} + \frac{2}{3}\left(\frac{d\sigma}{d\Omega}\right)_{SI}$$

y-direction

$$\left(\frac{d\sigma}{d\Omega}\right)_Z^{NSF} = \frac{1}{2}\left(\frac{d\sigma}{d\Omega}\right)_{mag} + \frac{1}{3}\left(\frac{d\sigma}{d\Omega}\right)_{SI} + \left(\frac{d\sigma}{d\Omega}\right)_{nuc+II}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_Z^{SF} = \frac{1}{2}\left(\frac{d\sigma}{d\Omega}\right)_{mag} + \frac{2}{3}\left(\frac{d\sigma}{d\Omega}\right)_{SI}$$

z-direction

O. Schärpf and H. Capellmann, Phys. Stat. Sol a **135** (1993) 359
J. R. Stewart *et al*., J. Appl. Cryst. **42** (2009) 69

XYZ Polarization Analysis

$$\left(\frac{d\sigma}{d\Omega}\right)_{mag} = 2\left(\frac{d\sigma}{d\Omega}\right)_x^{sf} + 2\left(\frac{d\sigma}{d\Omega}\right)_y^{sf} - 4\left(\frac{d\sigma}{d\Omega}\right)_z^{sf}$$

$$= 4\left(\frac{d\sigma}{d\Omega}\right)_z^{sf} - 2\left(\frac{d\sigma}{d\Omega}\right)_x^{sf} - 2\left(\frac{d\sigma}{d\Omega}\right)_y^{sf}$$

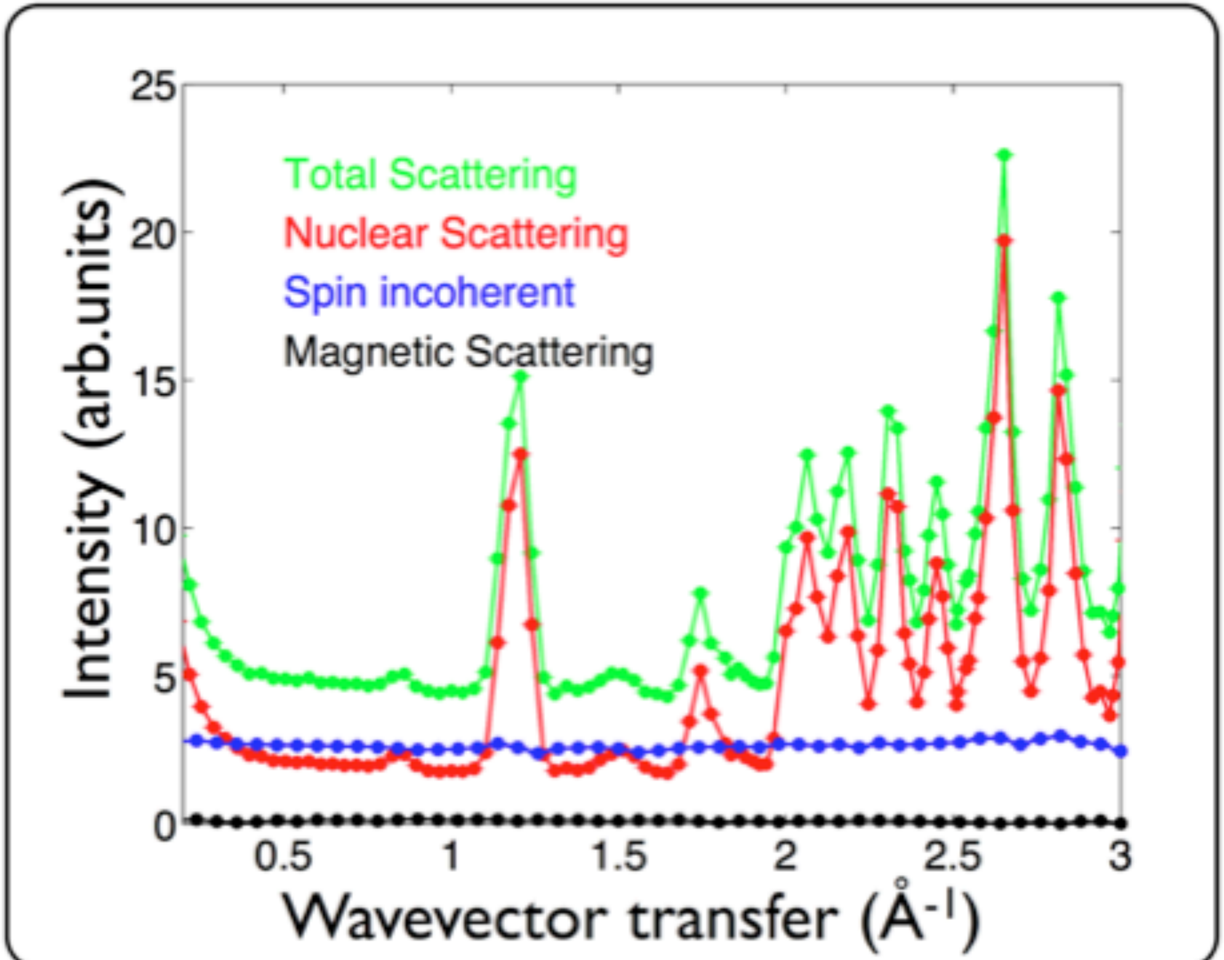
$$\left(\frac{d\sigma}{d\Omega}\right)_{nuc+II} = \frac{1}{6}\left[2\sum_{\alpha=x,y,z}\left(\frac{d\sigma}{d\Omega}\right)_\alpha^{nsf} - \sum_{\alpha=x,y,z}\left(\frac{d\sigma}{d\Omega}\right)_\alpha^{sf}\right]$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{SI} = \frac{1}{2}\sum_{\alpha=x,y,z}\left(\frac{d\sigma}{d\Omega}\right)_\alpha^{sf} - \left(\frac{d\sigma}{d\Omega}\right)_{mag}$$

O. Schärpf and H. Capellmann, Phys. Stat. Sol a **135** (1993) 359
J. R. Stewart *et al*., J. Appl. Cryst. **42** (2009) 69

Volborthite ($Cu_3V_2O_7(OH)_2 \cdot 2H_2O$)

S = 1/2 Kagome Heisenberg antiferromagnet (True quantum spin liquid?)



J. R. Stewart *et al.*, J. Appl. Cryst. **42** (2009) 69

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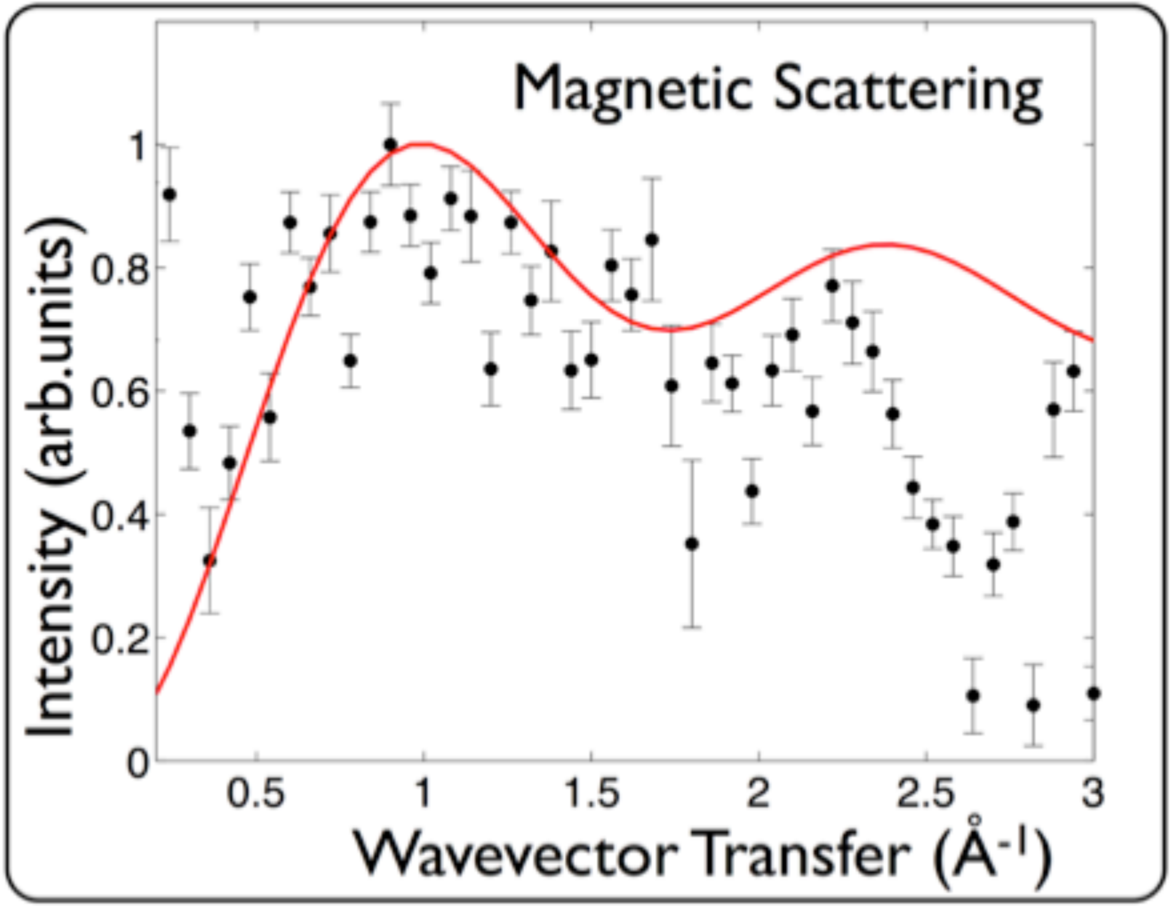

J. R. Stewart *et al.*, J. Appl. Cryst. **42** (2009) 69

Polarimetry

The previous discussion of neutron polarization analysis was over-simplified. The neutron spin is not really ‘flipped’ by a magnetic field, it *precesses*

The previous discussion dealt with the *projection* of the neutron spin along the initial polarization axis

Bragg Scattering with Polarization Analysis

$$\vec{P}_f \sigma = \begin{bmatrix} \vec{P}_i NN^* & \text{nucl} \\ -\vec{P}_i(\vec{M}_\perp . \vec{M}_\perp^*) + \vec{M}_\perp(\vec{P}_i . \vec{M}_\perp^*) + \vec{M}_\perp^*(\vec{P}_i . \vec{M}_\perp) - i\left(\vec{M}_\perp^* \times \vec{M}_\perp\right) & \text{mag} \\ +N\vec{M}_\perp^* + N^*\vec{M}_\perp - i\left(N\vec{M}_\perp^* - N^*\vec{M}_\perp\right) \times \vec{P}_i & \text{nucl-mag int} \end{bmatrix}$$

σ	total scattering cross-section of Bragg peak
N	nuclear structure factor
$\vec{M}_\perp$	magnetic interaction vector $\propto \vec{Q} \times \vec{M} \times \vec{Q}$ where $\vec{M}$ is the magnetic structure factor
$\vec{P}_i, \vec{P}_f$	incident, final polarization vector

M. Blume, Phys. Rev. **130** (1963) 1670
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T. J. Hicks, Adv. Phys. **46** (1996) 243

The polarization tensor

Measuring the rotation of polarization due to the scattering at the sample gives a unique solution for complex magnetic structures

This is a measurement of the polarization tensor:

$$\vec{P}_f = \mathbf{P}\vec{P}_i + \vec{T}, \text{ where } \mathbf{P} = \begin{bmatrix} P_{xx} & P_{xy} & P_{xz} \\ P_{yx} & P_{yy} & P_{yz} \\ P_{zx} & P_{zy} & P_{zz} \end{bmatrix}$$

It is often shown as a stereographic projection: