

This presentation might differ from the one which really
will be presented

Outline

- Basics
- Magnetic scattering
 - Spin manipulation
 - Instruments

Basics

Reminder:

The scattering cross section:

$$\frac{d\sigma}{d\Omega} = \sum_{i,j} b_i b_j \exp \left(i\vec{q} \left(\vec{R}_i - \vec{R}_j \right) \right)$$

Suppose that at position R_i we can have different scattering length with a certain probability distribution

$$\langle b_i b_j \rangle = \langle b_i \rangle \langle b_j \rangle + \delta_{ij} \left(\langle b_i^2 \rangle - \langle b_i \rangle^2 \right)$$

$$\frac{d\sigma}{d\Omega} = \sum_{i,j} \langle b_i \rangle \langle b_j \rangle \exp \left(i\vec{q} \left(\vec{R}_i - \vec{R}_j \right) \right) + \sum_i \left(\langle b_i^2 \rangle - \langle b_i \rangle^2 \right)$$

$$I_{coherent}(\vec{q}) + I_{incoherent}(\vec{q})$$

b_i can have a distribution because:

- different isotopes exist
- the nucleus has a spin $I \Rightarrow$ with the neutron $1/2$ spin it forms two possible states

$I+1/2 \Rightarrow 2(I+1/2)+1$ states with scattering length b_+

$I-1/2 \Rightarrow 2(I-1/2)+1$ states with scattering length b_-

Coherent

$$\langle b_i \rangle = b_+ \frac{I+1}{2I+1} + b_- \frac{I}{2I+1}$$

Incoherent

$$\left(\langle b_i^2 \rangle - \langle b_i \rangle^2 \right) = (b_+ - b_-)^2 \frac{I(I+1)}{(2I+1)^2}$$

Isotope Incoherence

Spin Incoherence

the nuclear spin is usually randomly oriented EXCEPT very low T or very high B

Let's define the polarization of the beam as: $\vec{P} = 2 \langle \vec{s} \rangle = \langle \vec{\sigma} \rangle$

In terms of Pauli matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Scattering length on a nucleus with spin I:

$$b = A + \frac{1}{2} B \hat{\sigma} \hat{I}$$

$$A = b_+ \frac{I+1}{2I+1} + b_- \frac{I}{2I+1} \quad B = \frac{2(b_+ - b_-)}{2I+1}$$

σ_x and σ_y changes the spin state of the neutron => spin flip scattering

σ_z does not => non spin flip scattering

If the nuclear spin I is randomly oriented in space each one has 1/3 probability thus:

Spin Incoherent scattering 2/3 spin flip 1/3 non spin flip P => -1/3P

Magnetic scattering

The neutron has a 1/2 spin => magnetic moment $\mu_n = \gamma_n \mu_N$
 μ_N nuclear Bohr magneton and $\gamma_n = -1.913$

With a magnetic field the interaction potential Lovesey 1986:

$$V(\vec{R}) = -\vec{\mu}_n \vec{B} = \gamma_n \mu_N \left[2\mu_B \text{curl} \left(\frac{\vec{s} \times \vec{R}}{|\vec{R}^3|} \right) - \frac{e}{2m_e c} \left(\vec{p}_e \frac{\vec{\sigma} \times \vec{R}}{|\vec{R}^3|} + \frac{\vec{\sigma} \times \vec{R}}{|\vec{R}^3|} \vec{p}_e \right) \right]$$

\vec{s} = electron spin operator, \vec{p}_e = electron momentum operator

$\vec{\sigma}$ = neutron spin operator

The matrix element (scattering probability) becomes:

$$\langle k' | V_M | k \rangle = -r_0 \hat{\sigma} \hat{Q}_\perp \text{ with } r_0 = \frac{\gamma_n e^2}{m_e c^2}$$

$$\hat{Q}_\perp = \sum_i \exp(i\vec{q}\vec{r}_i) \left(\vec{q} \times (\vec{s} \times \vec{q}) - \frac{i}{\hbar |\vec{q}|} (\vec{q} \times \vec{p}_i) \right) \text{ with } \vec{q} = \frac{\vec{q}}{|\vec{q}|}$$

or in terms of the magnetization and changing to integral to account for the spatial extent of the electrons

$$\vec{Q} = -\frac{1}{2\mu_B} \int d\vec{r} \exp(i\vec{k}\vec{r}) \vec{M}(\vec{r}) \text{ and } \vec{Q}_\perp = \vec{Q} - \vec{q} (\vec{Q}\vec{q})$$

Of the sample spins (magnetization) **ONLY THE COMPONENT PERPENDICULAR** to \vec{q} contributes !! This is fundamentally different from the nuclear spin!

The scattered intensity is the Fourier transform of the self correlation function of the scattering length density

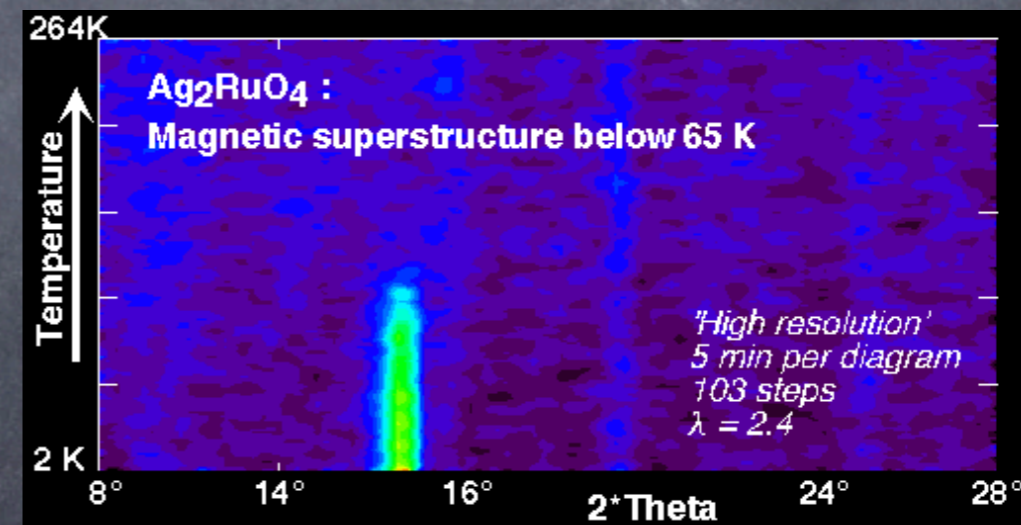
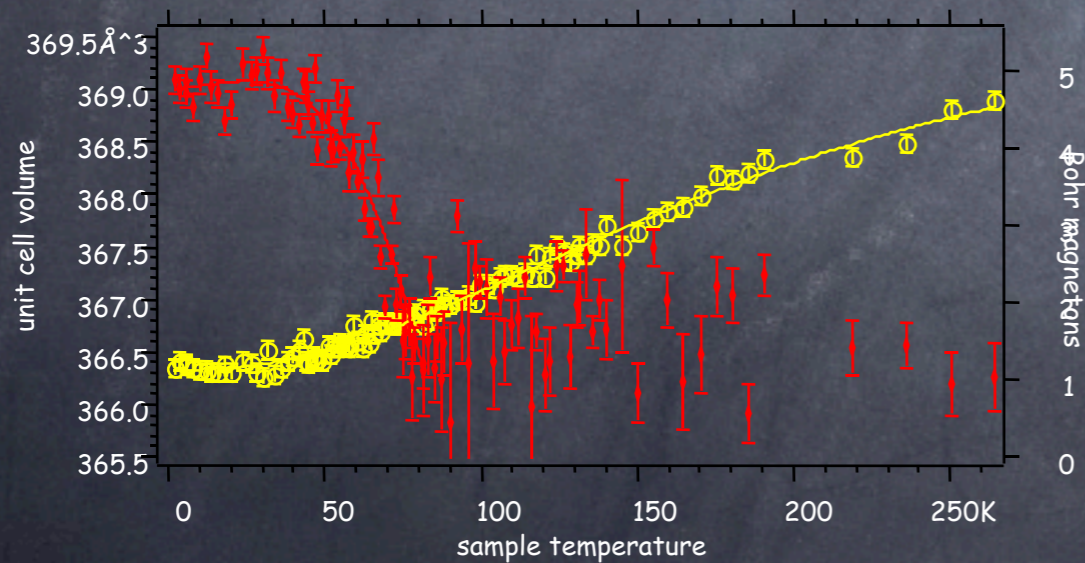
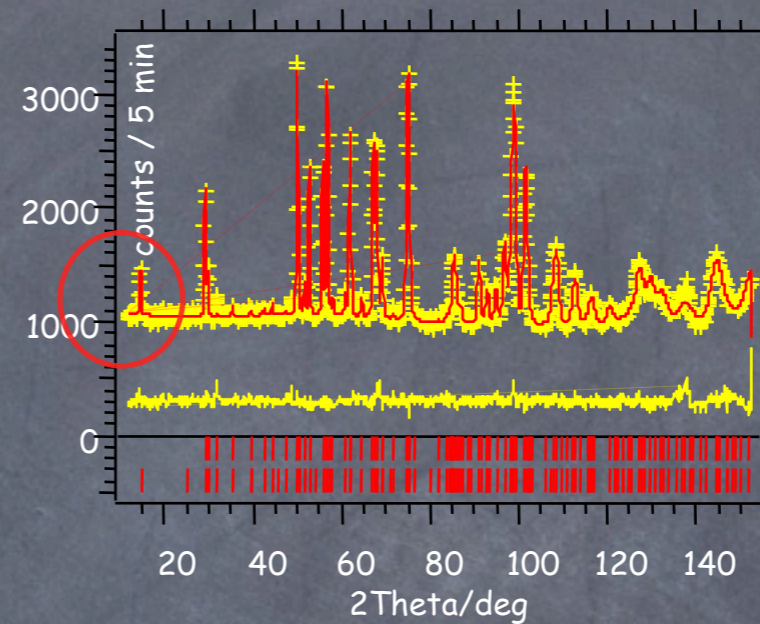
Alternatively if the Fourier transform of the scattering length density is $F(q)$

$$S(q) = F(q)F^*(q)$$

If $F(q) = F_N(q) + F_M(q)$ in the most generic case there will be four terms:

- nuclear
- magnetic
- nuclear magnetic interference
- chiral

- Unpolarized neutrons
- comparable intensity to nuclear
- identified by a priory knowledge
- temperature dependence



50% spin flip scattering

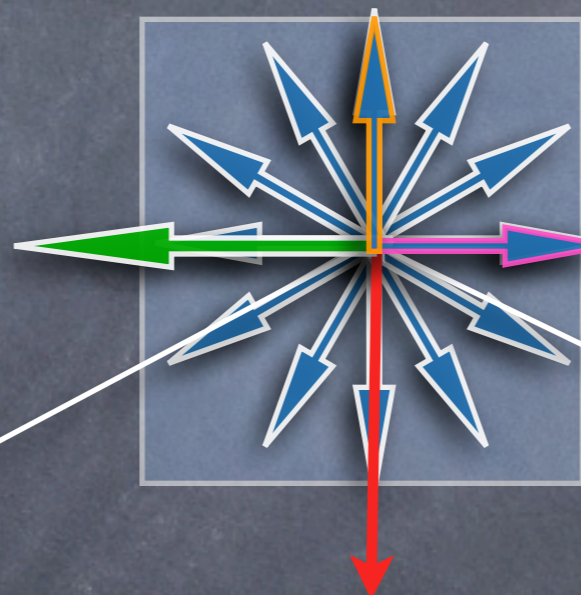
neutron beam polarization
direction

50% non spin flip scattering

neutron k_{out}

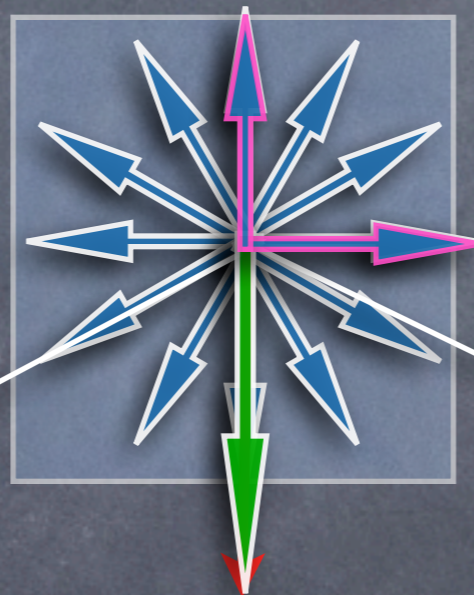
Q scattering vector

neutron k_{in}



spin flip scattering

neutron beam polarization
direction



spin flip scattering

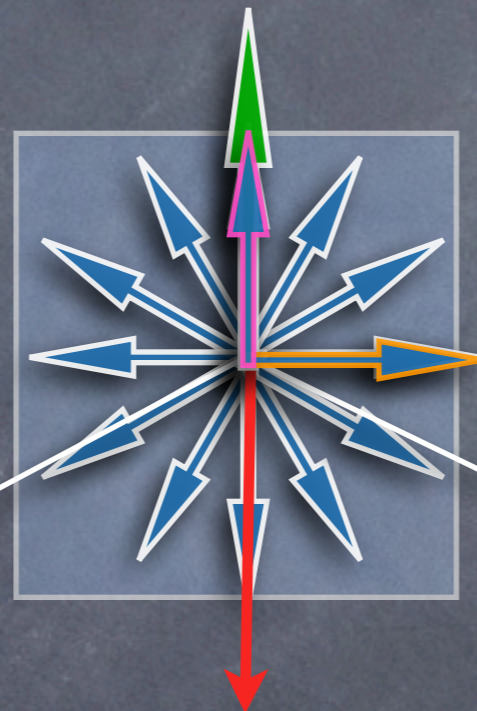
neutron k_{out}

Q scattering vector

neutron k_{in}

50% non spin flip scattering

neutron beam polarization
direction



50% spin flip scattering

neutron k_{out}

Q scattering vector

neutron k_{in}



$$UP_X = N + \frac{1}{2}M + \frac{1}{3}I$$

$$DOWN_X = \frac{1}{2}M + \frac{2}{3}I$$

$$UP_Z = N + \frac{1}{2}M + \frac{1}{3}I$$

$$DOWN_Z = \frac{1}{2}M + \frac{2}{3}I$$

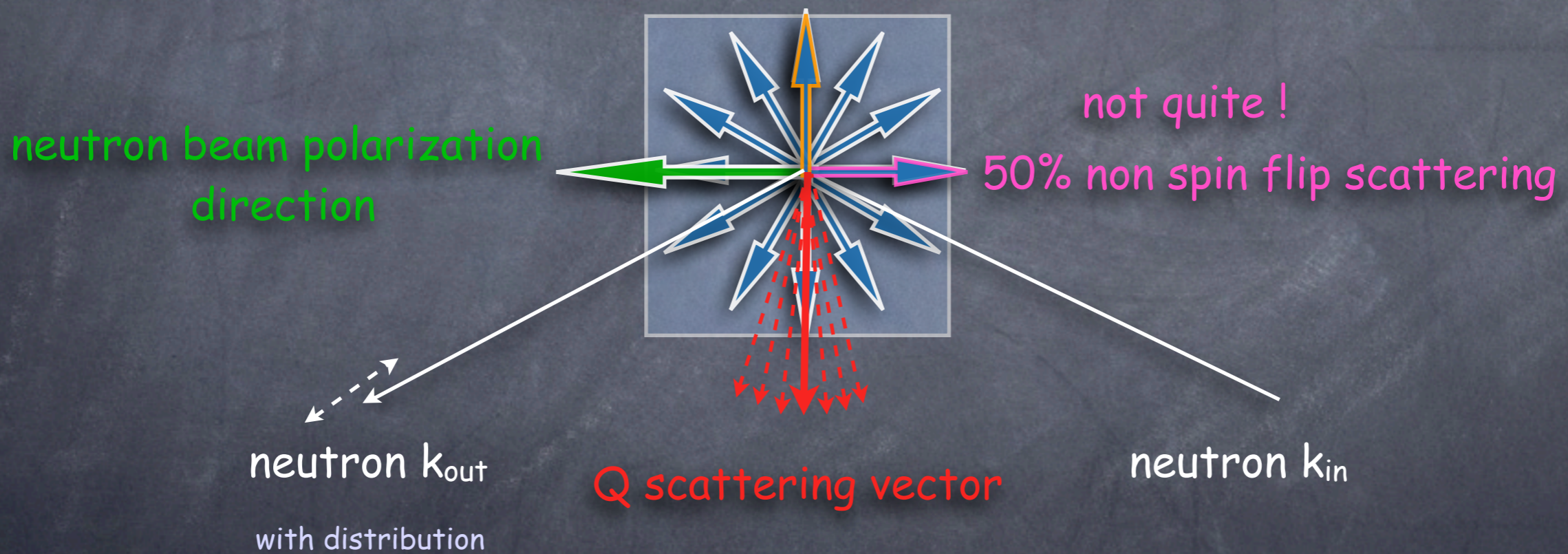
$$UP_Y = N + \frac{1}{3}I$$

$$DOWN_Y = M + \frac{2}{3}I$$

N the nuclear scattering
M the magnetic scattering
I the incoherent scattering

When the scattering is non negligibly quasielastic

50% spin flip scattering



$$UP_X = N \frac{1+fg}{2} + \frac{1}{2}M - \frac{gf}{2}M \sin^2 \epsilon + \frac{3-gf}{6}I$$

$$DOWN_X = N \frac{1-g}{2} + \frac{1}{2}M + \frac{g}{2}M \sin^2 \epsilon + \frac{3+g}{6}I$$

$$UP_Z = N \frac{1+fg}{2} + \frac{1}{2}M + \frac{3-gf}{6}I$$

$$DOWN_Z = N \frac{1-g}{2} + \frac{1}{2}M + \frac{3+g}{6}I$$

$$UP_Y = N \frac{1+fg}{2} + \frac{1-gf}{2}M + \frac{gf}{2}M \sin^2 \epsilon + \frac{3-gf}{6}I$$

$$DOWN_Y = N \frac{1-g}{2} + \frac{1+g}{2}M - \frac{g}{2}M \sin^2 \epsilon + \frac{3+g}{6}I$$

g is the polarizing efficiency
 f the flipper efficiency
 N the nuclear scattering
 M the magnetic scattering
 I the incoherent scattering
 ε is the angle of Q to Q_{elastic}

Spin manipulation

Quantum mechanical description of the neutron spin state:

$$|\chi\rangle = a|+\rangle + b|-\rangle \quad \text{where } a \text{ and } b \text{ can be complex and } \sqrt{a^2 + b^2} = 1$$

and conveniently

$$|\chi\rangle = e^{-i\varphi/2} \cos \frac{\theta}{2} |+\rangle + e^{i\varphi/2} \sin \frac{\theta}{2} |-\rangle$$

this leads to

$$\langle \hat{\sigma}_x \rangle = \sin \theta \cos \varphi, \quad \langle \hat{\sigma}_y \rangle = \sin \theta \sin \varphi, \quad \langle \hat{\sigma}_z \rangle = \cos \theta$$

Time evolution of the neutron spin in magnetic field

$$\frac{\partial \hat{\sigma}}{\partial t} = \frac{1}{\hbar} [\hat{\sigma}, H_s] = -\frac{\gamma}{2} [\hat{\sigma}, (\hat{\sigma} \vec{H})] = \dots = -\gamma (\vec{H} \times \hat{\sigma})$$

The Polarization of the beam $P = \langle \sigma \rangle$ behaves as a "classical" Larmor precession

$$\gamma_L = 2957 \text{ Hz/Gauss} \quad \frac{\Delta\phi}{\Delta x} [\text{deg/cm}] = 2.65 \lambda [\text{\AA}] H [\text{Gauss}]$$

What happens if the direction of the magnetic field changes in space?

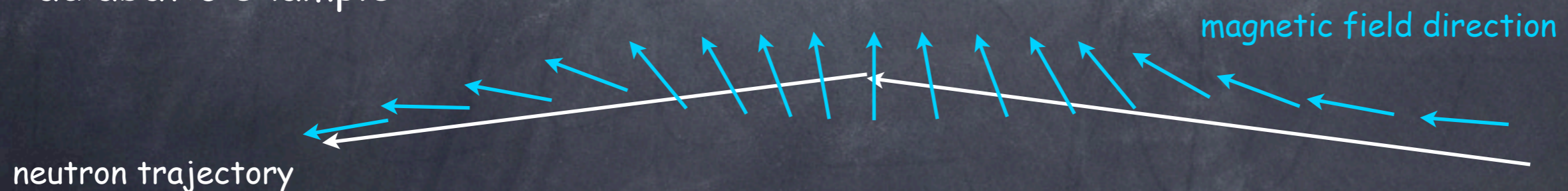
The moving neutron will see a B field which changes its direction in time

Two limiting cases

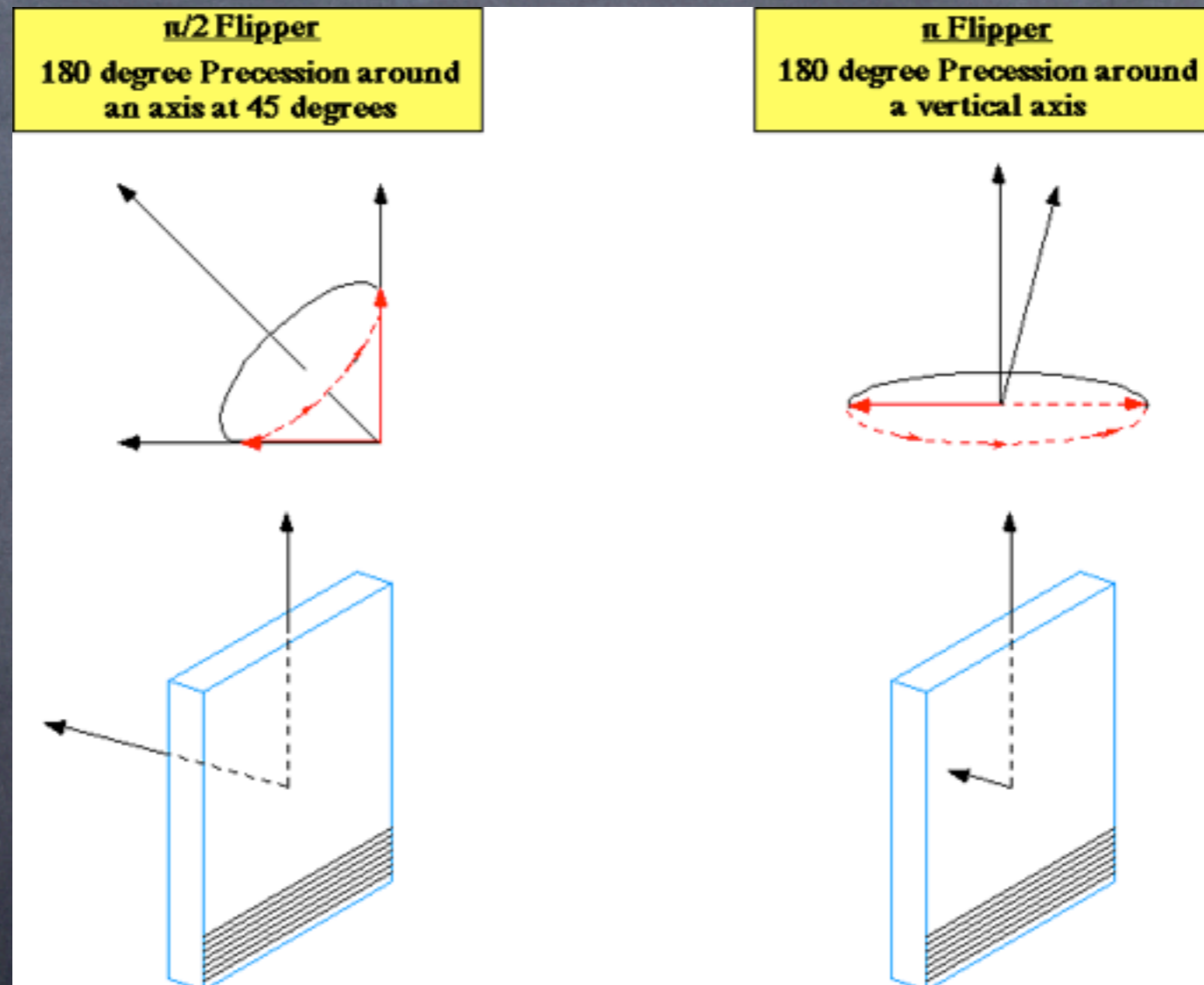
if $\omega_B \ll \omega_L$ it will follow adiabatically

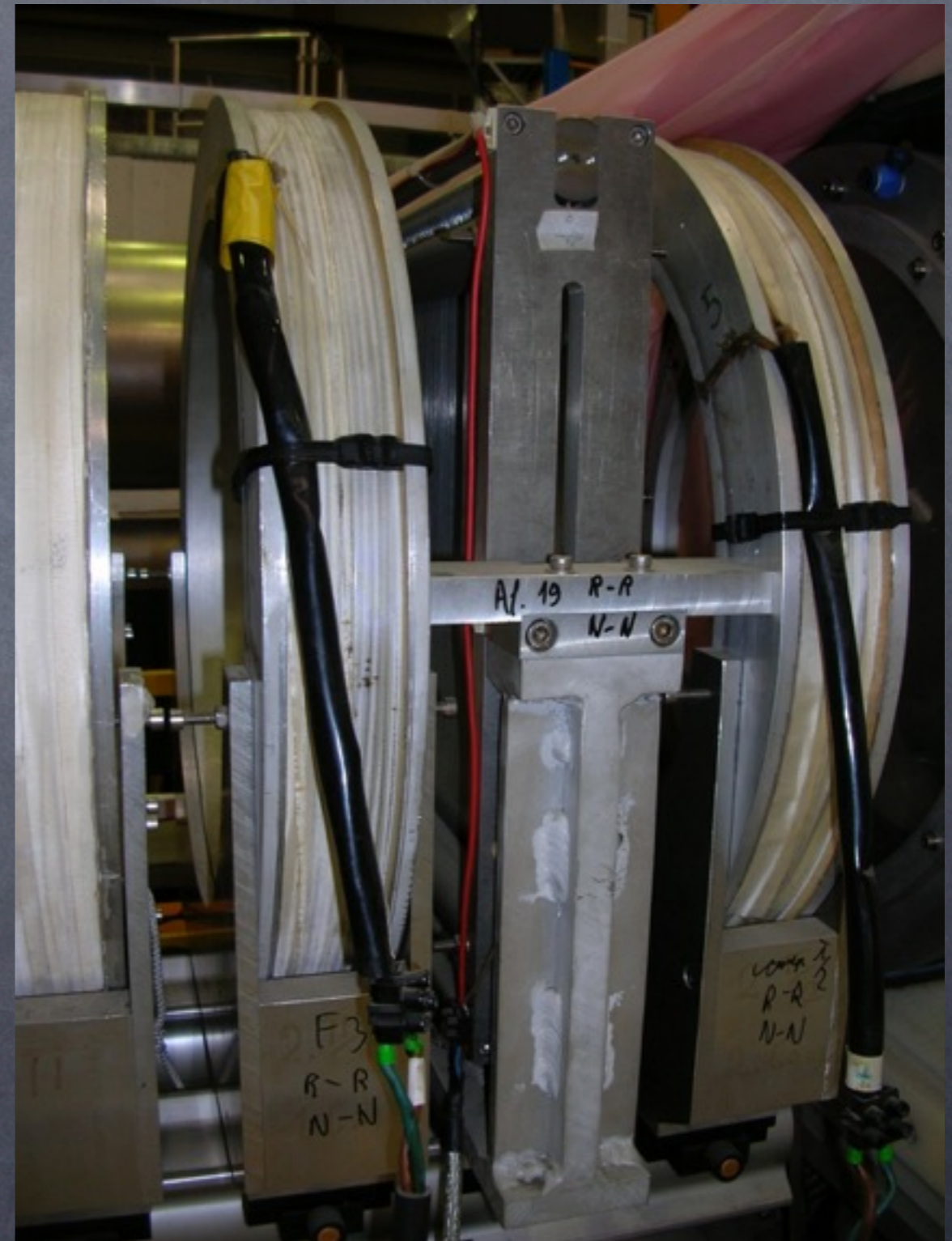
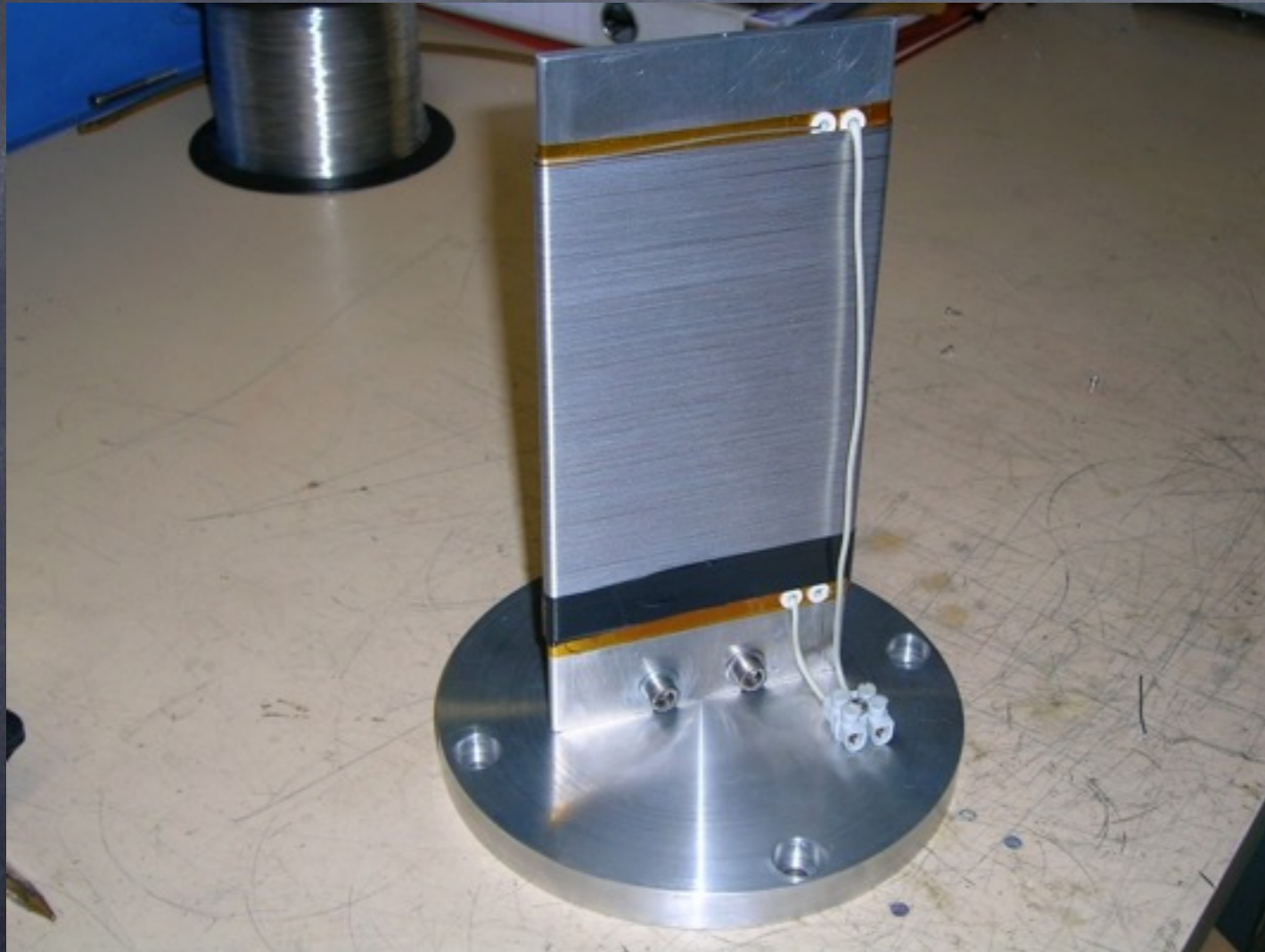
if $\omega_B \gg \omega_L$ it will start to precess around the new field direction

adiabatic example:



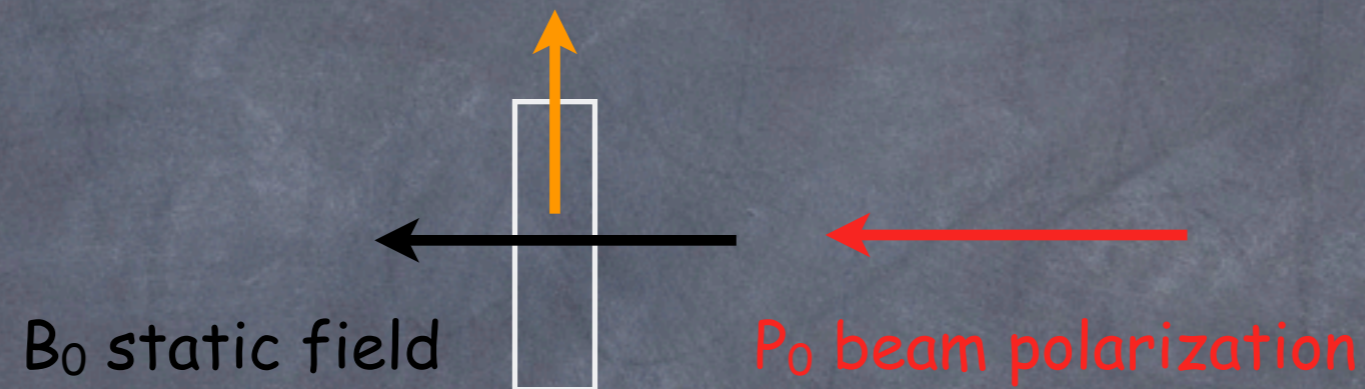
Non adiabatic example : Mezei flipper





RF flipper

B_{rf} oscillating ω_{RF} radiofrequency field

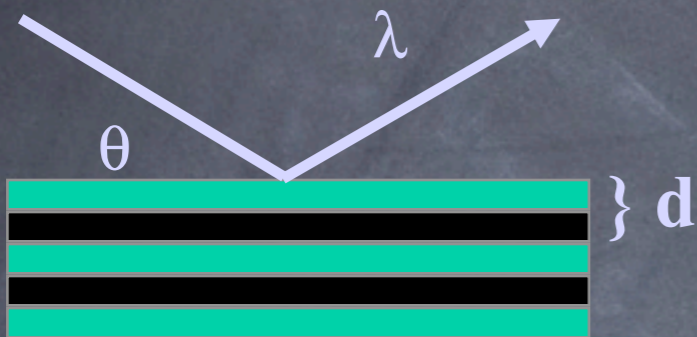


Makes up a π flipper

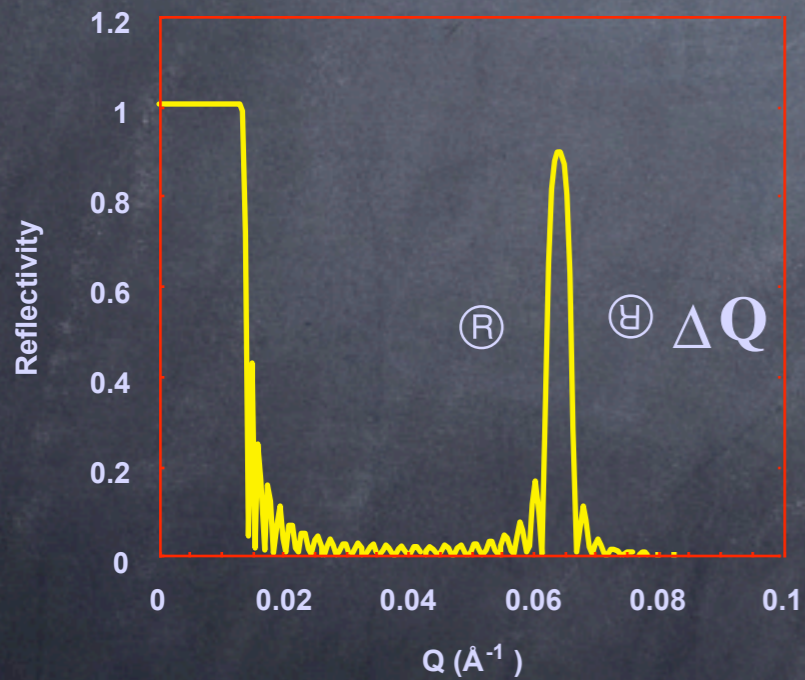
- if $B_0 \gamma_L = \omega_{RF}$
- and B_{RF} is just enough for a π turn during the flight time

Haussler Xtal Cu_2MnAl (111):

- $F_M(q) = F_N(q)$ for one of the Bragg reflections
- easy to saturate
- grow single crystal
- controlled mosaicity
- low $\lambda/2$ contamination (or filter)

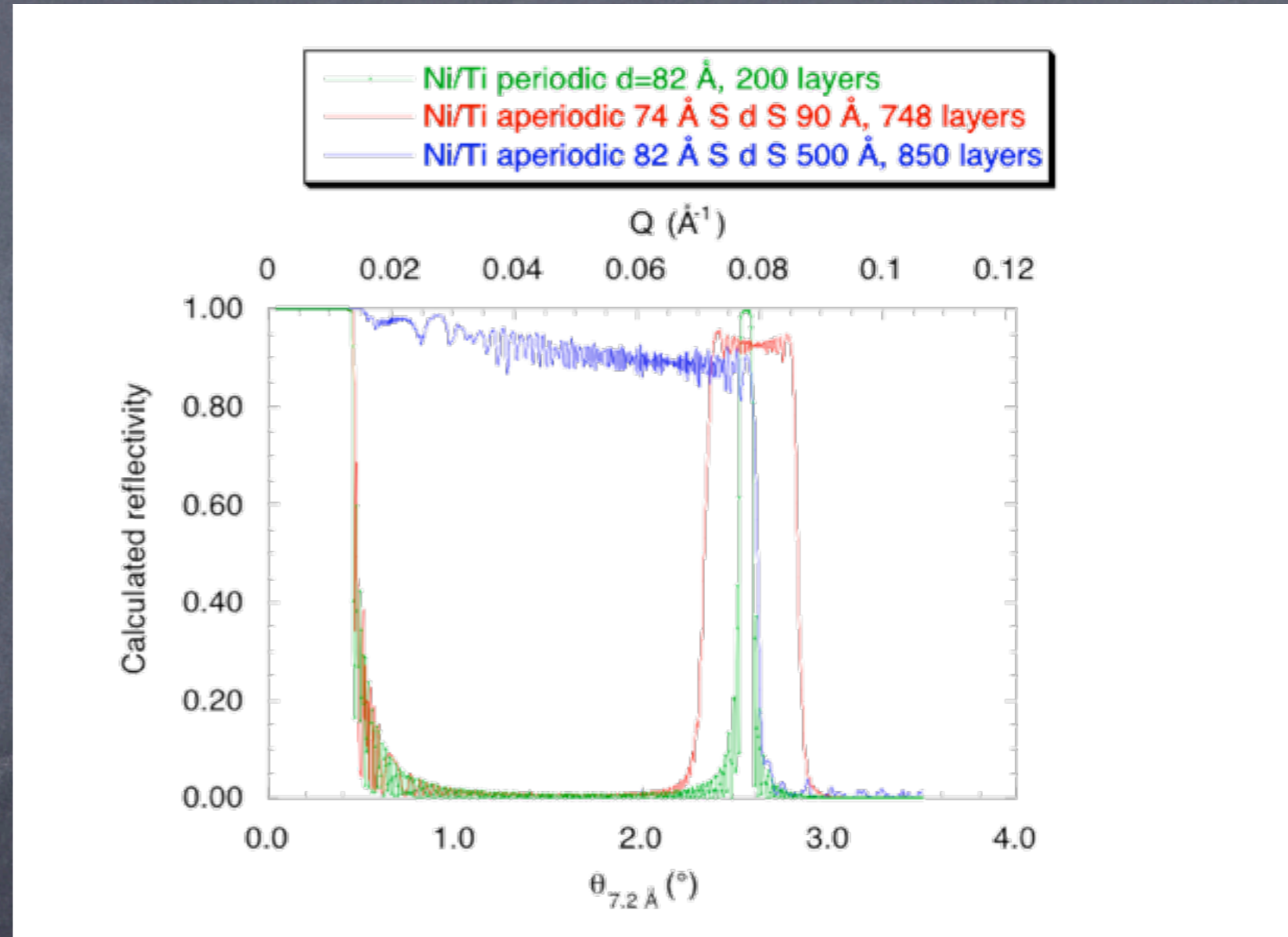
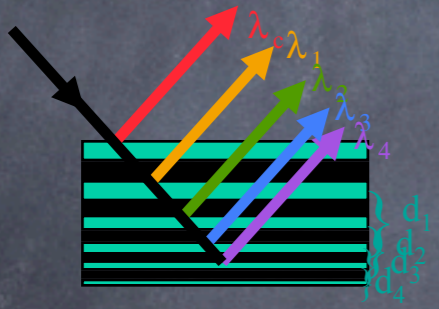


$$R = \frac{4N^2 d^4 (f_1 - f_2)^2}{\pi^2 n^4}$$

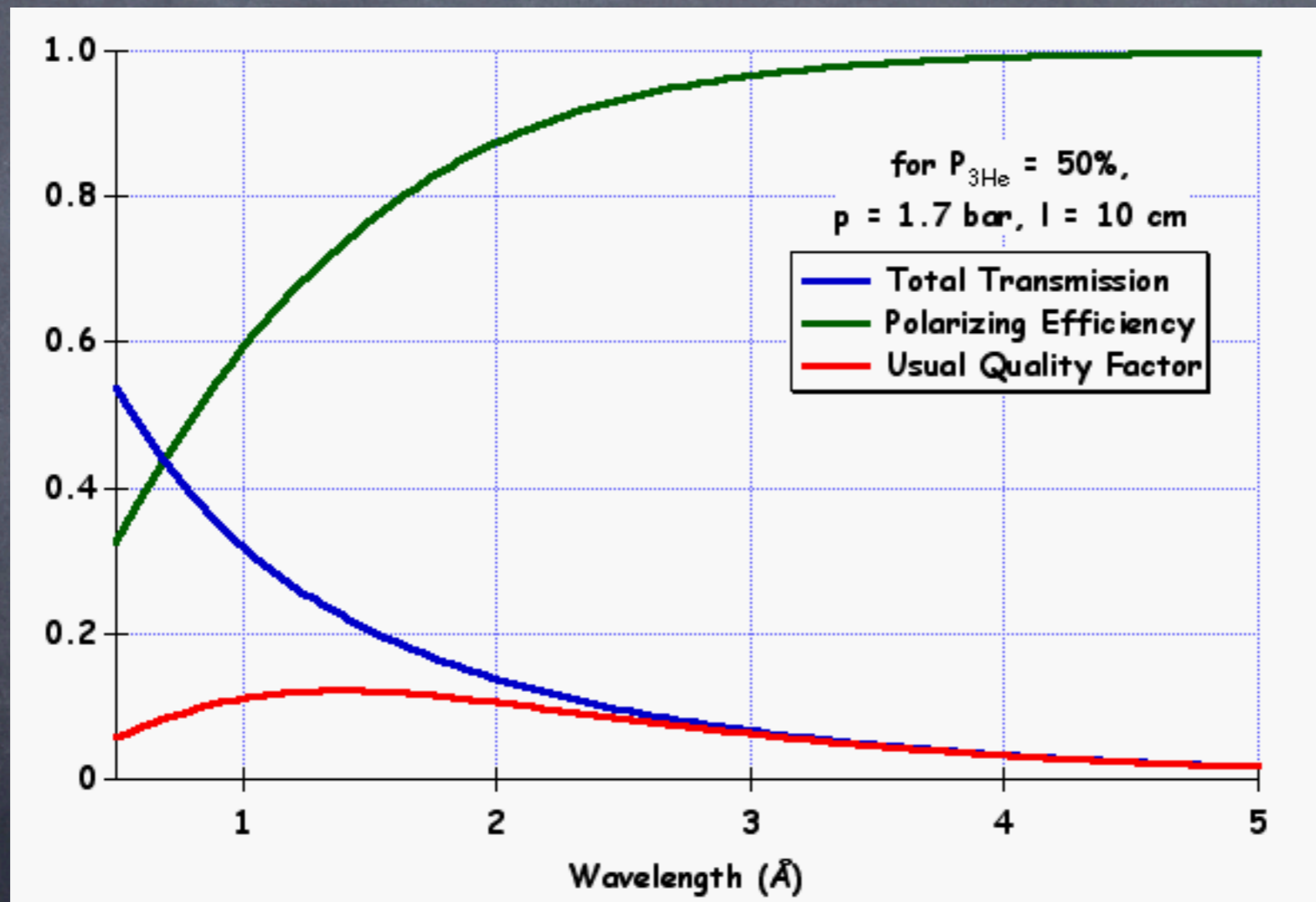


$$\frac{\Delta\lambda}{\lambda} = \frac{2d^2 (f_1 - f_2)}{\pi}$$

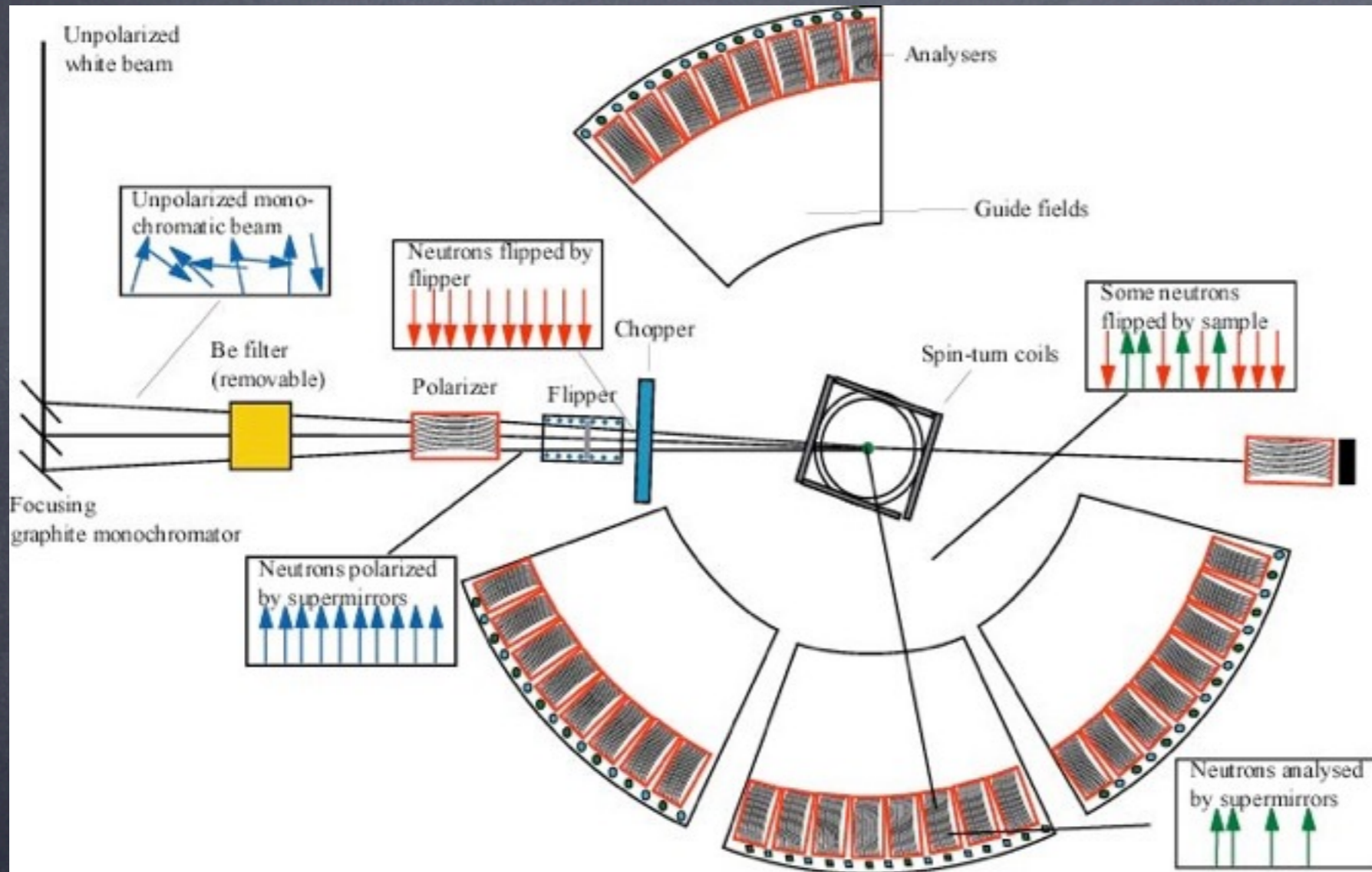
$$\theta \sim \frac{\lambda}{2d}$$



He³ absorbs only neutrons with spin antiparallel to the nuclear spin



Instruments

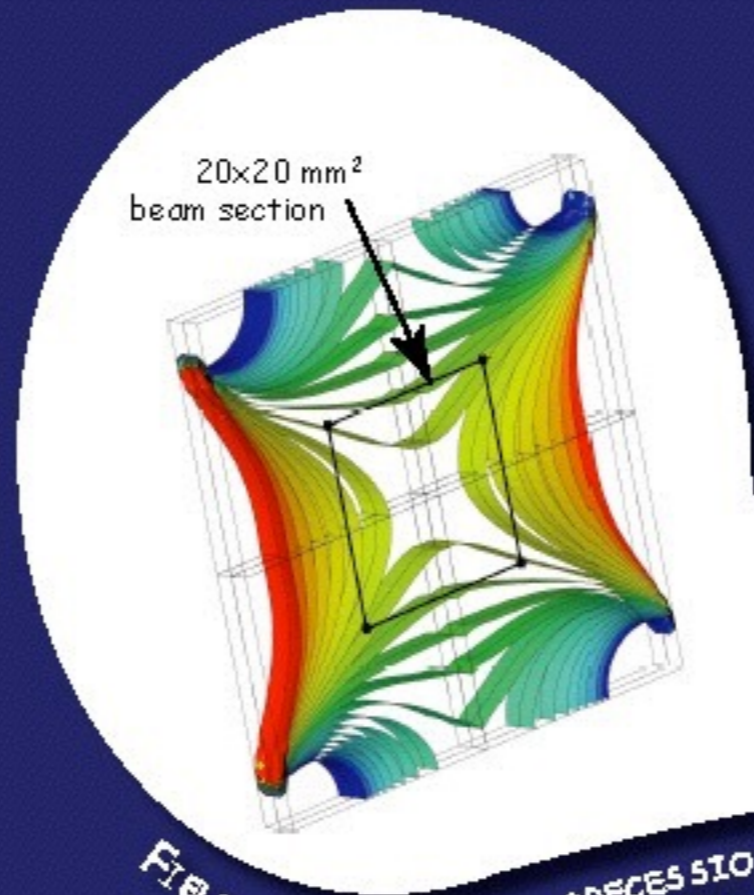


A 3rd generation Cryopad has recently been designed to carry out elastic and inelastic scattering experiments with 0.5 deg precision.

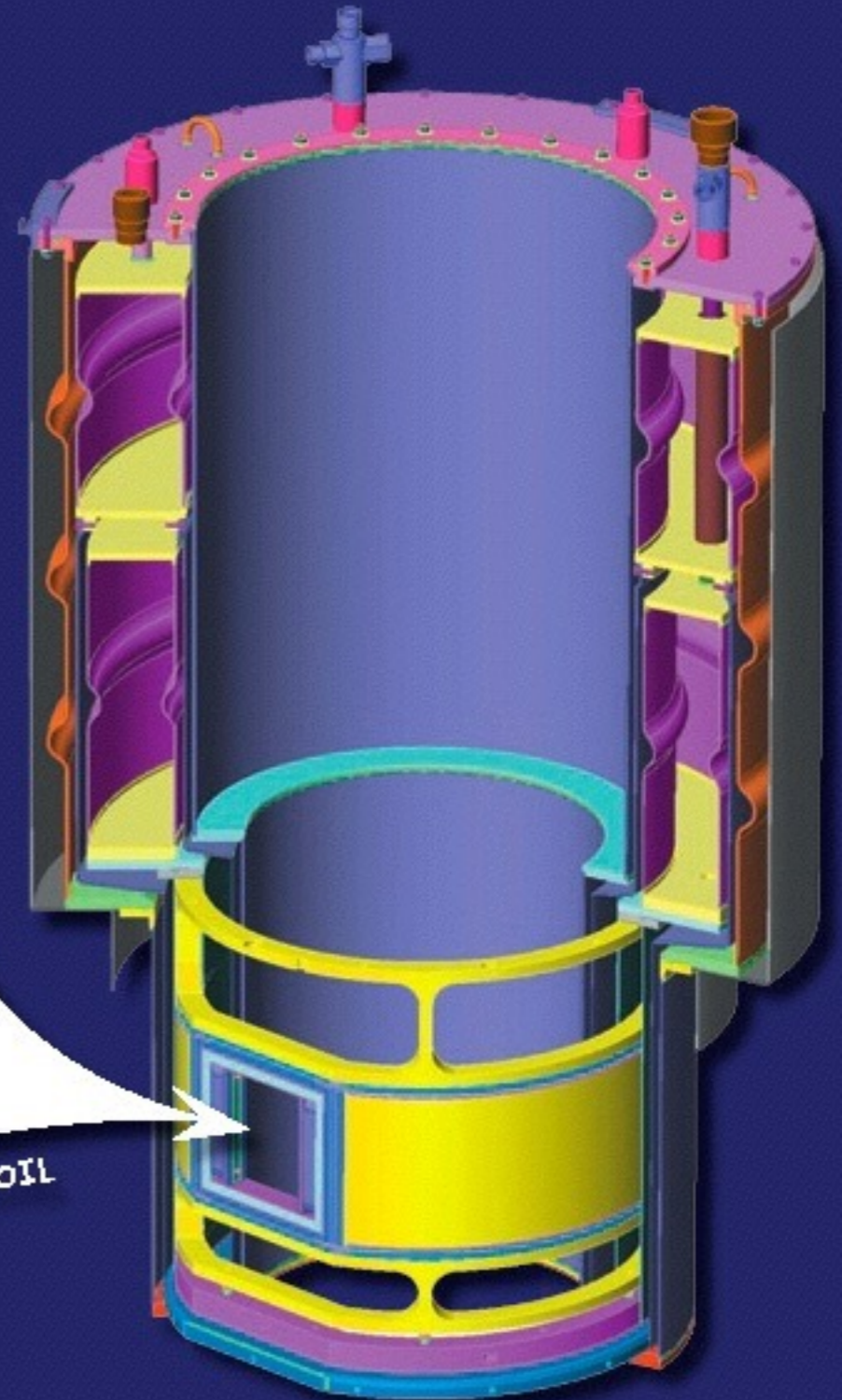
FIELD MAP ON MEISSNER SCREEN



20x20 mm²
beam section



FIELD MAP IN LARMOR PRECESSION COIL



Echo condition:

$$\int_{\pi/2}^{\pi} B_1 d\ell = \int_{\pi}^{\pi/2} B_2 d\ell$$

The measured quantity is: $S(q,t)/S(q,0)$
where

$$t \propto \lambda^3 \int B d\ell$$

For elastic scattering:

$$\varphi_{tot} = \frac{\gamma B_1 l_1}{v_1} - \frac{\gamma B_2 l_2}{v_2} = 0$$



For omega energy exchange:

$$\varphi_{tot} = \frac{\hbar \gamma B l}{m v^3} \omega + o\left(\left(\frac{\omega}{1/2 m v^2}\right)^2\right)$$



The probability of omega energy exchange:

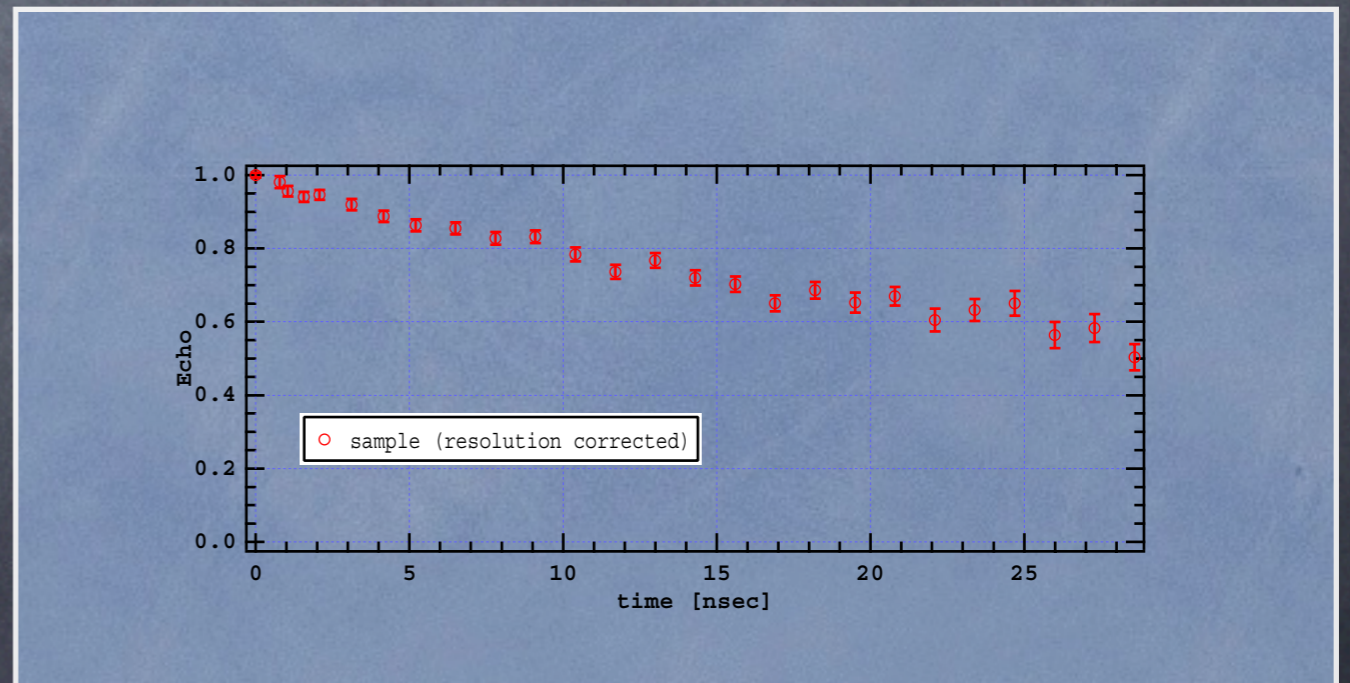
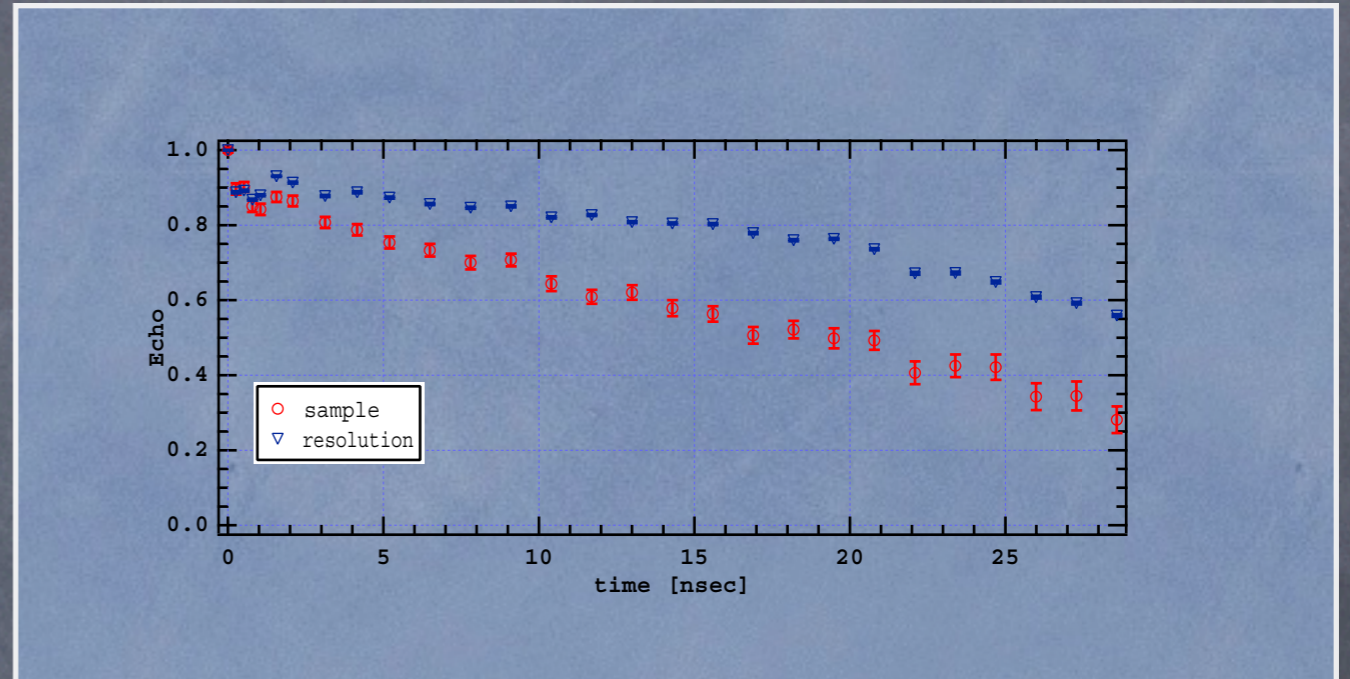
$$S(q, \omega)$$

The final polarization: $\langle \cos \varphi \rangle = \frac{\int \cos\left(\frac{\hbar \gamma B l}{m v^3} \omega\right) S(q, \omega) d\omega}{\int S(q, \omega) d\omega} = S(q, t)$

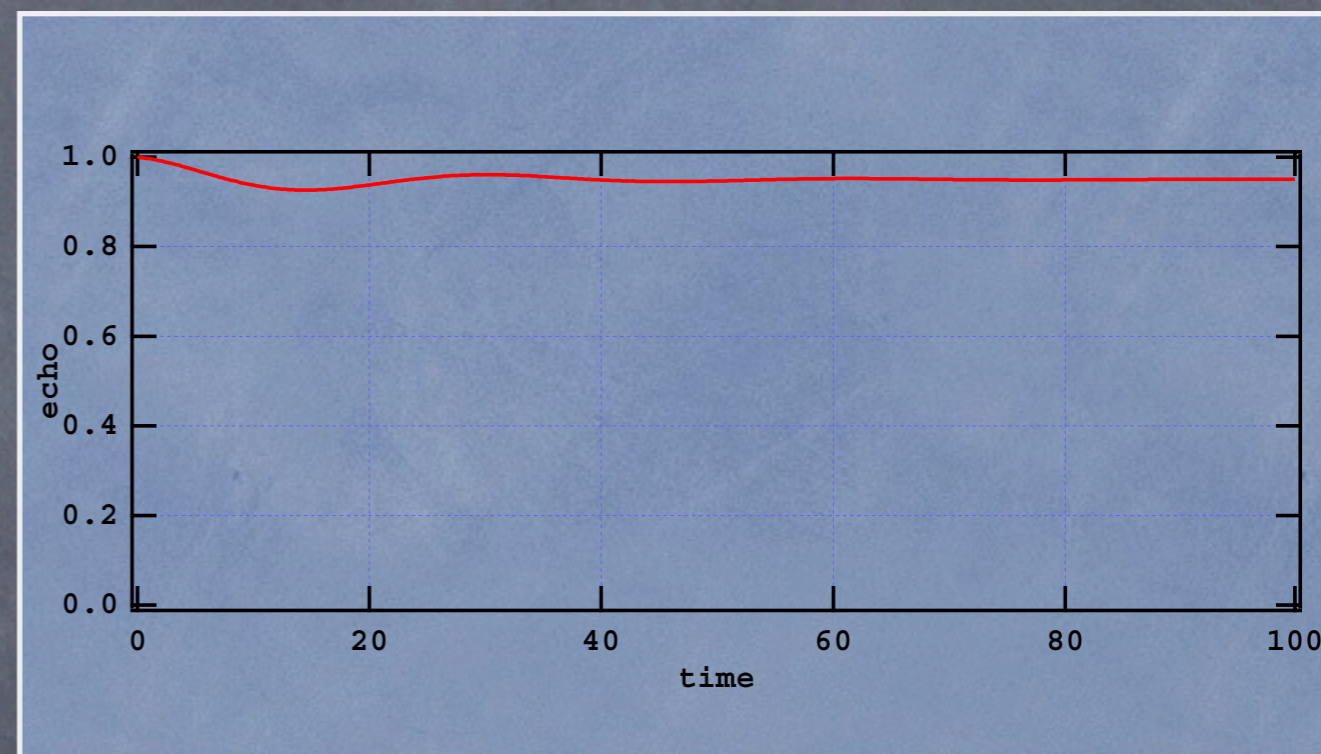
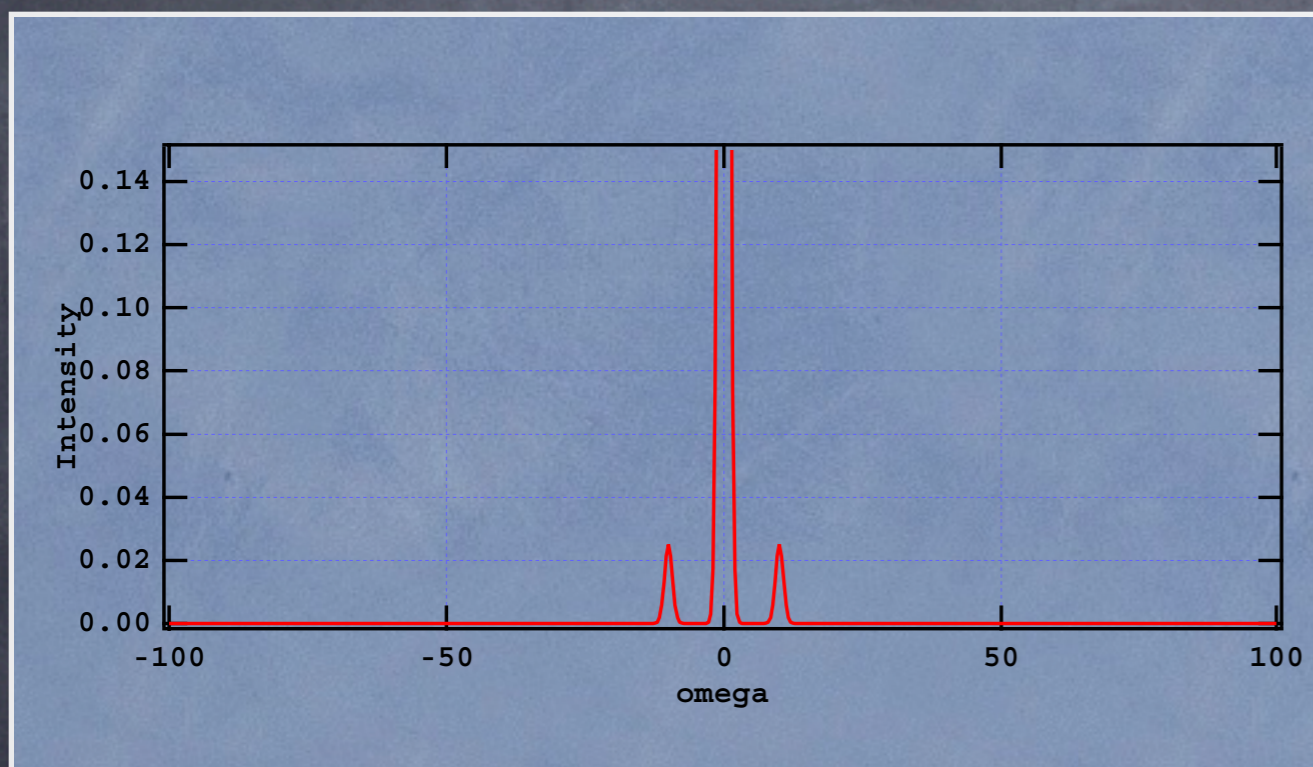
As measures the fourier
transforme

$$S(q,t)$$

the instrumental resolution is a
simple division instead of
deconvolution



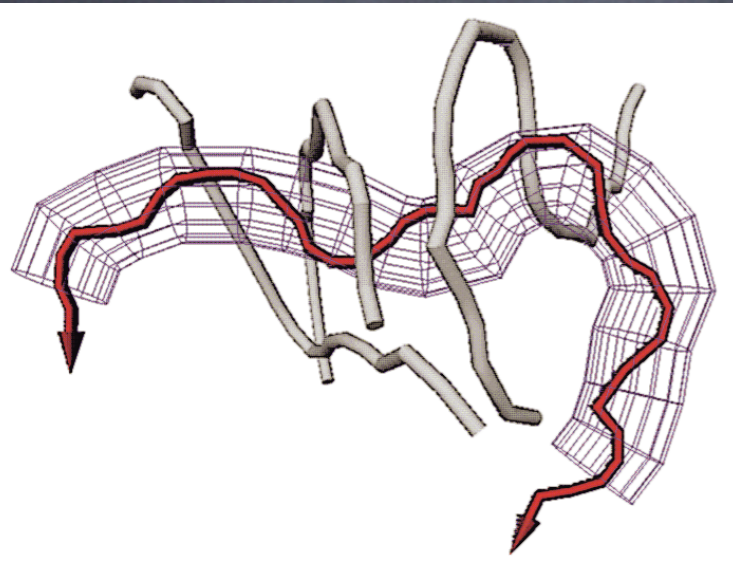
Say you have a weak but well defined excitation



$$S(q, \omega) = 0.95 \cdot \delta(\omega) + 0.025 \cdot (\delta(\omega - \omega_0) + \delta(\omega + \omega_0))$$

$$S(q, t) = 0.95 + 0.05 \cdot \cos(\omega_0 t)$$

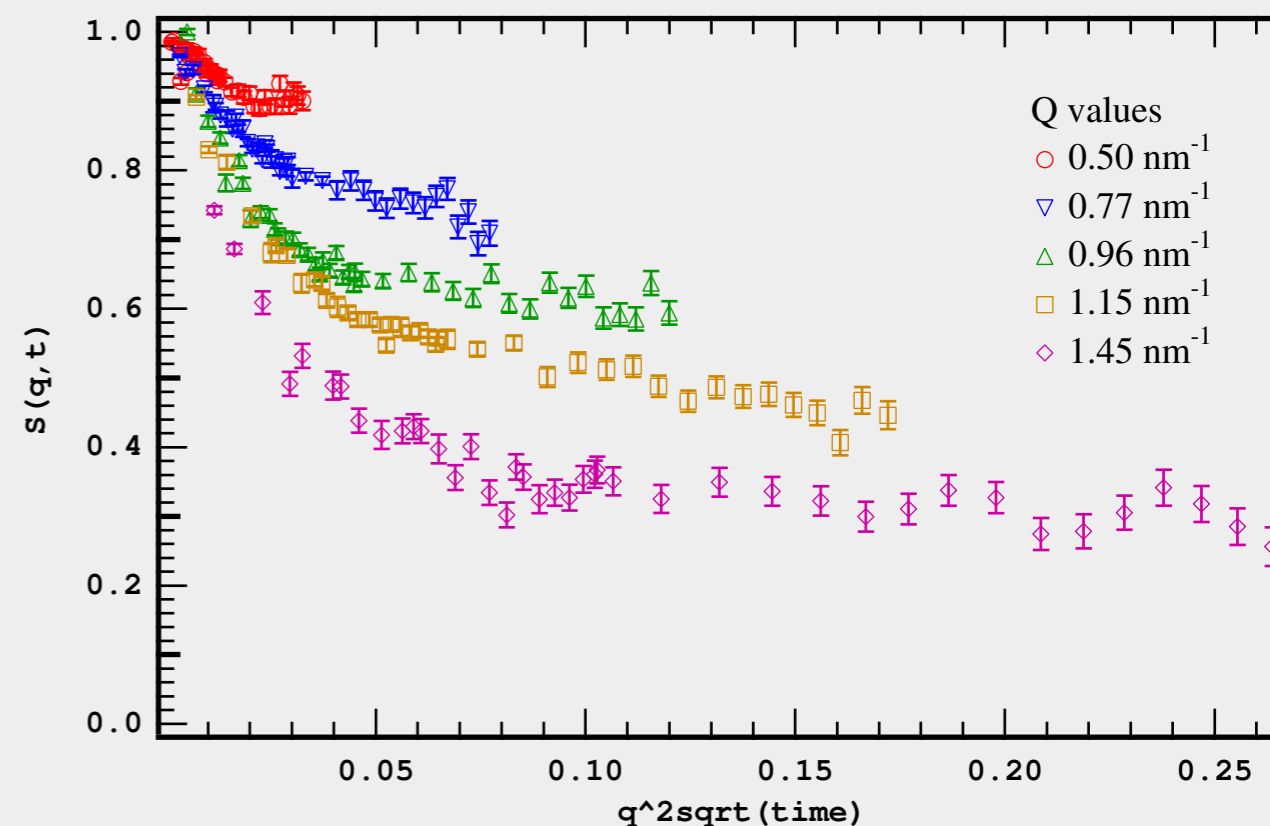
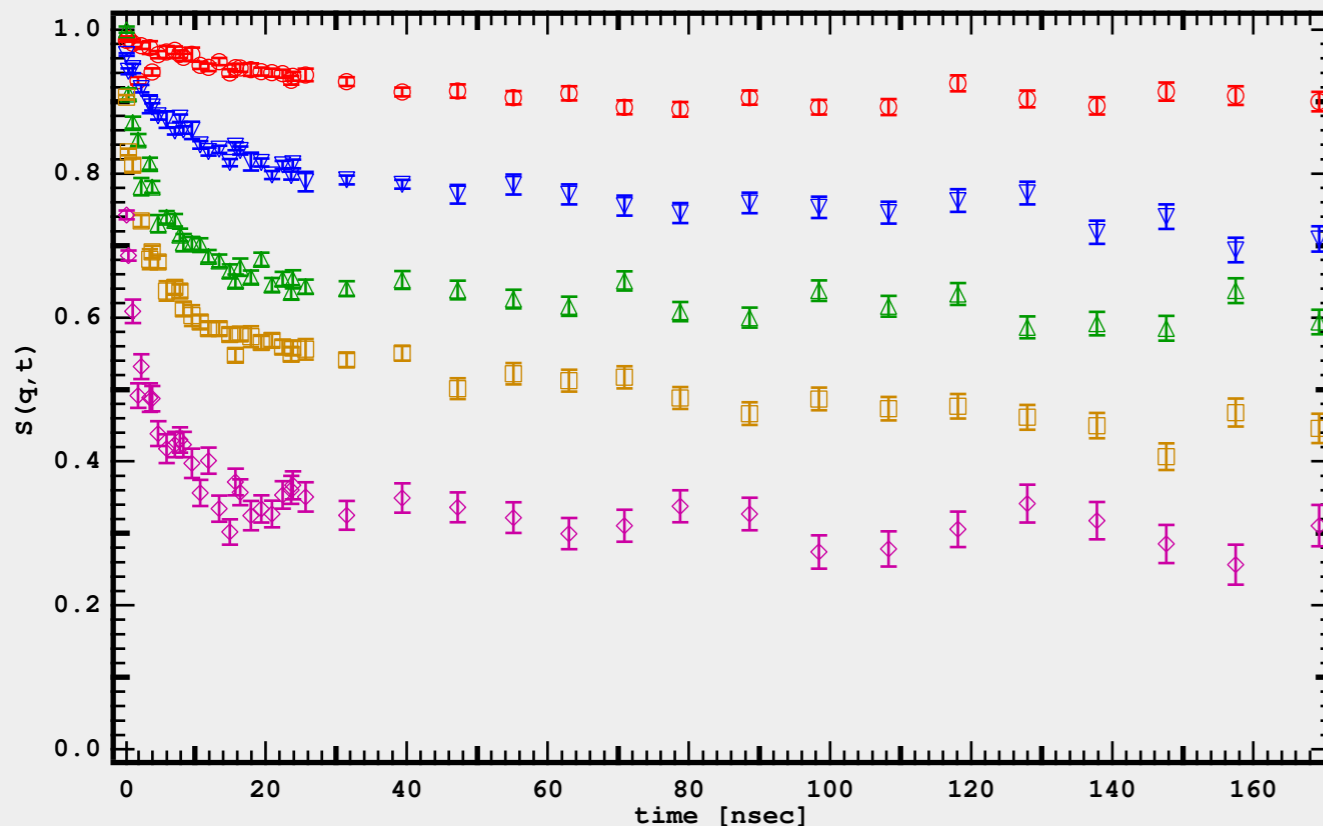
Bad signal to noise, $\Delta\lambda/\lambda \Rightarrow$ no good for Spin Echo



10% marked polymer chain(H) in deuterated matrix of the same polymer melt

at short time => Rouse dynamics $1/\tau \sim q^4$
 at longer times starts to feel the "tube" formed by the other chains (deGennes)

D. Richter, B. Ewen, B. Farago, et al., Physical Review Letters 62, 2140 (1989).

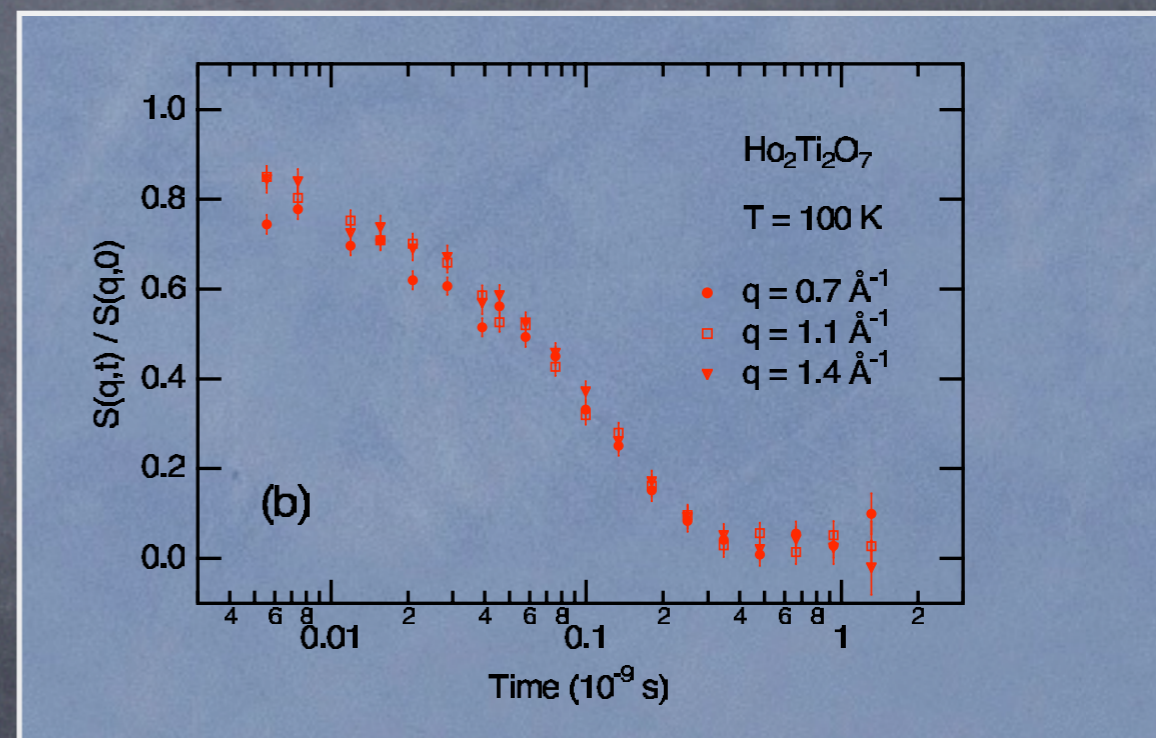
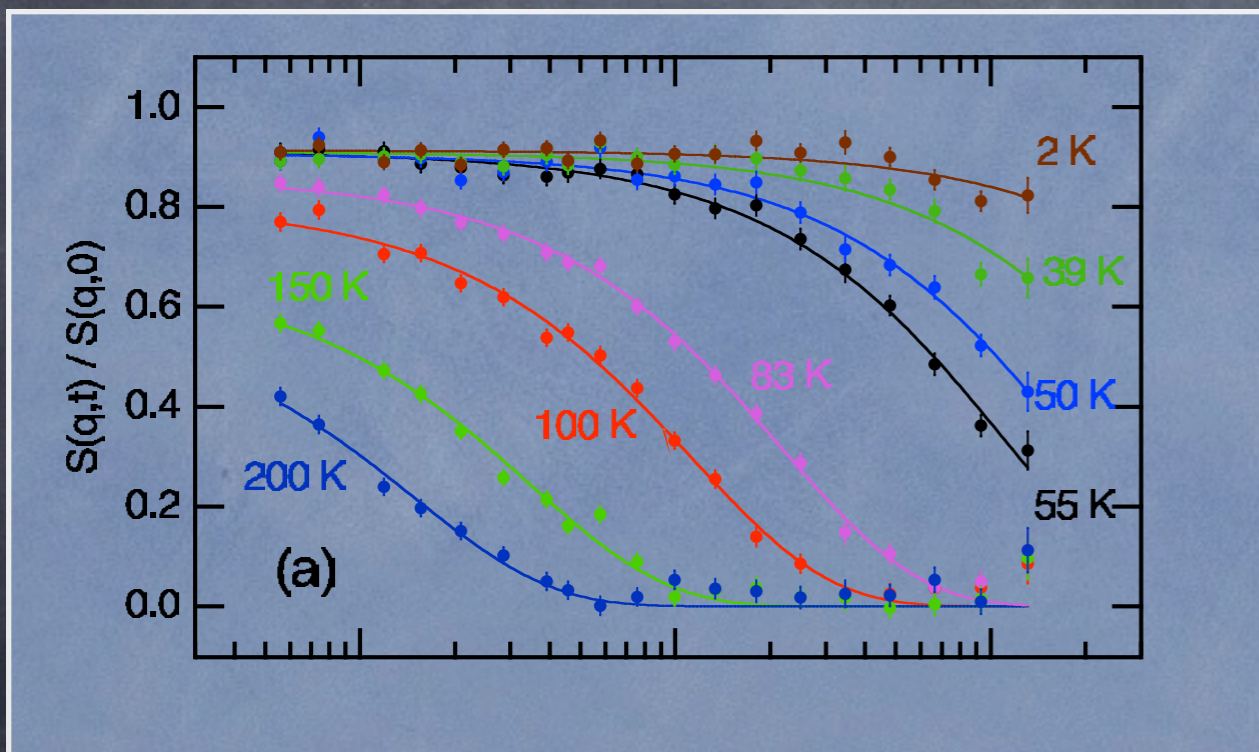


'spin ice' materials $\text{Ho}_2\text{Ti}_2\text{O}_7$, $\text{Dy}_2\text{Ti}_2\text{O}_7$ and $\text{Ho}_2\text{Sn}_2\text{O}_7$
 the spin is equivalent to the H displacement vector in water ice

Paramagnetic echo => Only magnetic scattering gives echo !

Single exponential thermally activated

Q independent relaxation



G. Ehlers, Cornelius, A L, Orendac, M, Kajnakova, M, Fennell, T, Bramwell, S T, Gardner, Journal of Physics Condensed Matter 15, L9 (2003).

It's over folks !
thanks...